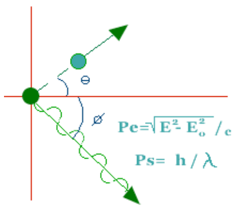
Is there a crisis in nuclear-matter theory?

FUSTIPEN workshop
Caen, 3/14/2016

Washington
University in St.Louis



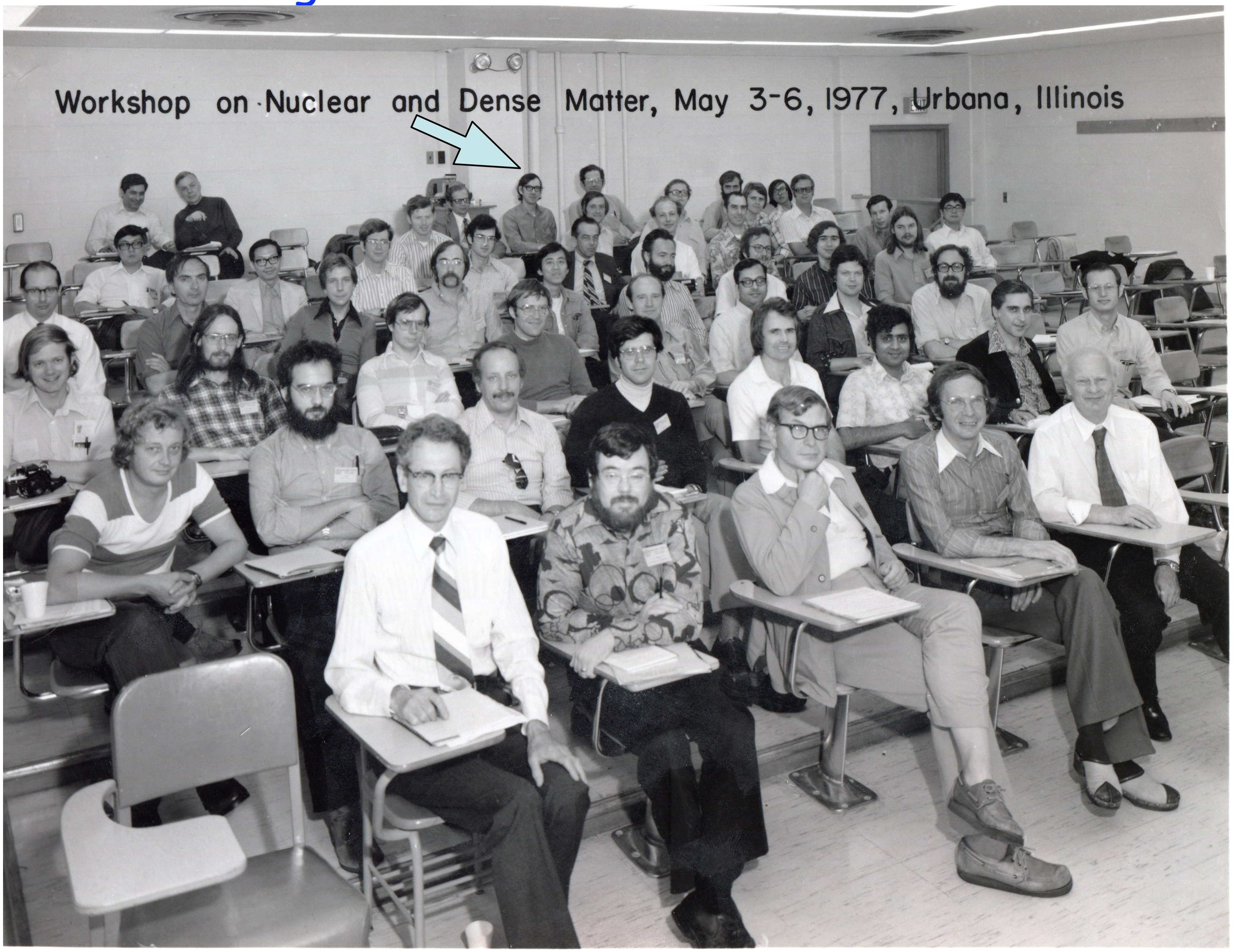
Wim Dickhoff

"DOM crowd"
Bob Charity
Helber Dussan
Hossein Mahzoon

- John Clark 80 arXiv:1512.06793
- Saturation problem of nuclear matter
- Brief history & crises
- Finite nuclei
- Dispersive optical model (DOM) results
- Saturation density and short-range correlations
- What about long-range correlations
- A different nuclear-matter problem as a way out or a real solution!
- Chiral 2N and 3N interactions
- Outlook

Those were the days...

- Precursor meeting



John Clark 80

- 1978 MBT-1 Trieste conference —> Nuclear matter "crisis"

UPDATE ON THE CRISIS IN NUCLEAR-MATTER THEORY: A SUMMARY OF THE TRIESTE CONFERENCE

Nuclear Physics **A328** (1979) 587–595

J. W. CLARK

  - LOBT/BB/BBG/2 hole-line substantially above variational result
  - "Crisis I" characterized by John
    - Resolved by Ben Day and then conclusively by the Baldo group: 3 hole-line result with gap or continuous choice for auxiliary potential resolves discrepancy
  - leads to "Crisis II"
    - Nuclear saturation properties cannot be explained in terms of non relativistic nucleons interacting only by two-body (realistic) forces
  - Crisis II "resolved" by many people in many different ways
  - Therefore it is **not** resolved as there is no universal agreement concerning the physics explanation

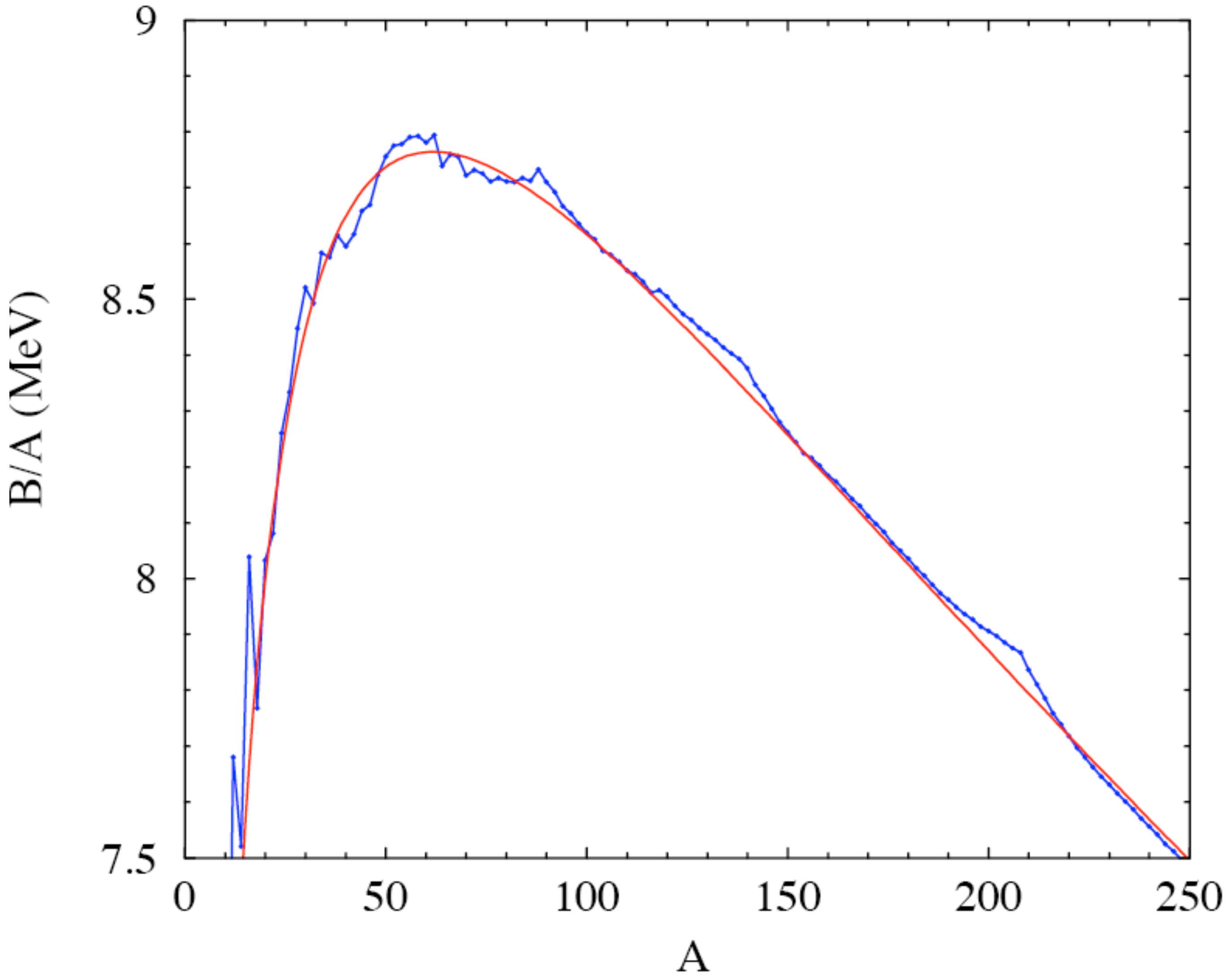
Empirical Mass Formula

Global representation of nuclear masses (Bohr & Mottelson)

$$B = b_{vol} A - b_{surf} A^{2/3} - \tfrac{1}{2} b_{sym} \frac{(N-Z)^2}{A} - \tfrac{3}{5} \frac{Z^2 e^2}{R_c}$$

- Volume term b_{vol} = 15.56 MeV
- Surface term b_{surf} = 17.23 MeV
- Symmetry energy b_{sym} = 46.57 MeV
- Coulomb energy R_c = 1.24 $A^{1/3}$ fm
- Pairing term must also be considered

Empirical Mass Formula



Plotted: most stable nucleus for a given *A*

nuclear matter

Central density of nuclei

Multiply charge density at the origin by A/Z

⇒ Empirical density = 0.16 nucleons / fm^3

⇒ Equivalent to k_F = 1.33 fm^{-1}

Nuclear Matter

N = Z

No Coulomb

$A \Rightarrow \infty$, $V \Rightarrow \infty$ but $A/V = \rho$ fixed

"Two most important numbers in nuclear physics"

b_{vol} = 15.56 MeV and k_F = 1.33 fm^{-1}

BHF (2 hole lines) + 3 hole lines

- Binding energy usually within 10 MeV from empirical volume term in the mass formula even for very strong repulsive cores
- Repulsion always completely cancelled by higher-order terms
- Minimum in density never coincides with empirical value when binding OK -> Coester band

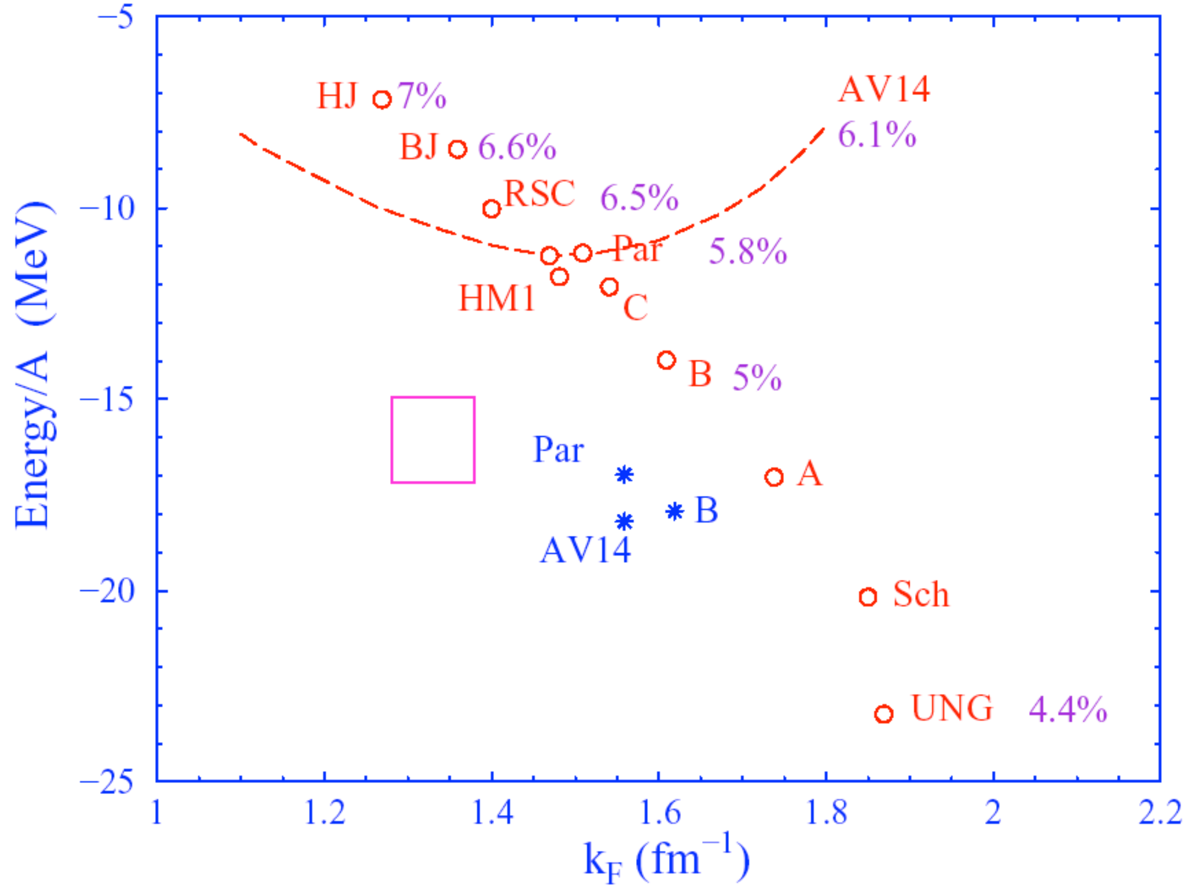
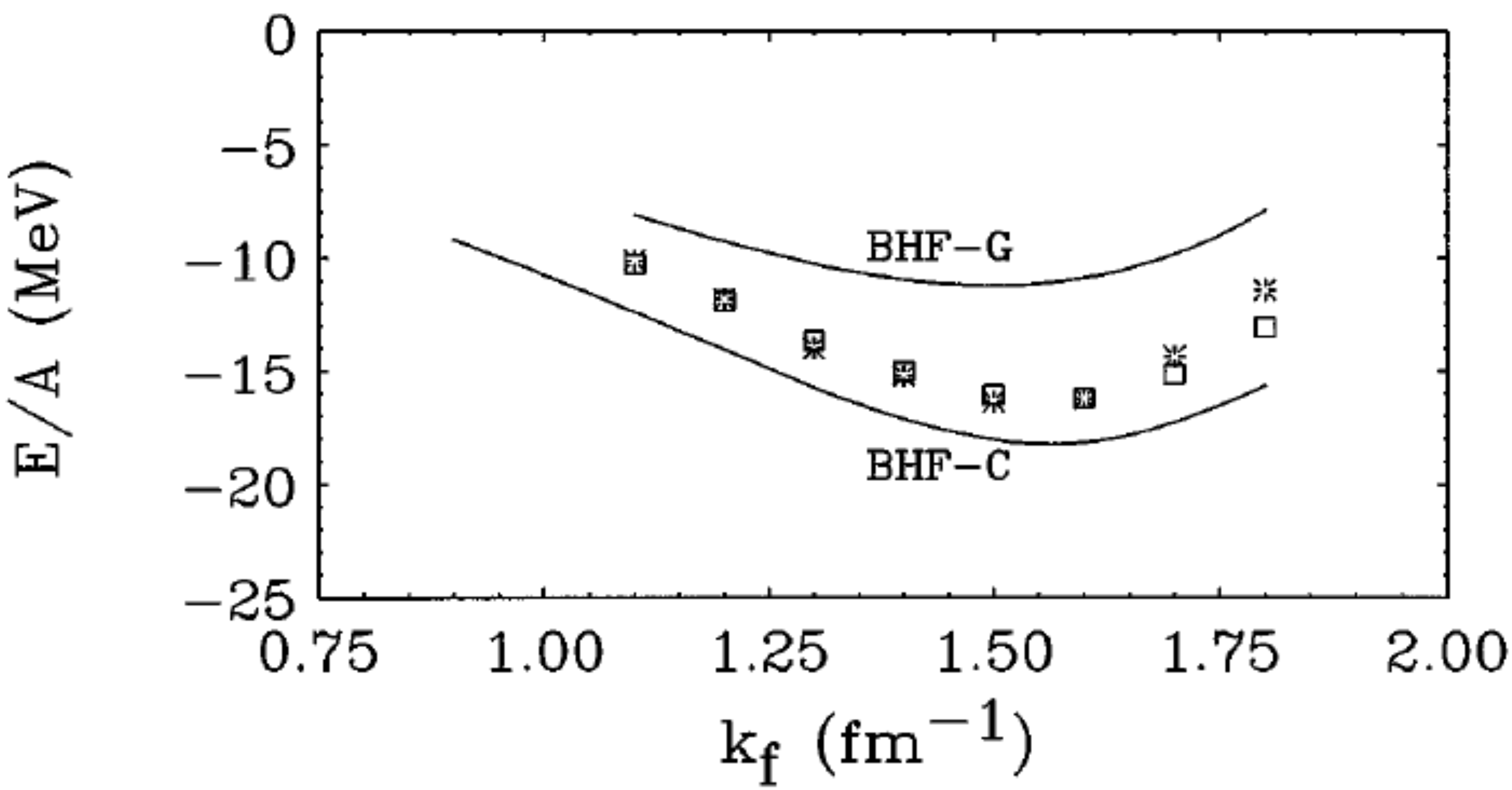


Figure adapted from Marcello Baldo (Catania)

Location of minimum determined by deuteron D-state probability

Results hole-line expansion 2+3

- Original papers B.D.Day, PRC 24, 1203 (1981) & PRL47, 226 (1981)
- Important confirmation Baldo et al. PRL81, 1584 (1998)

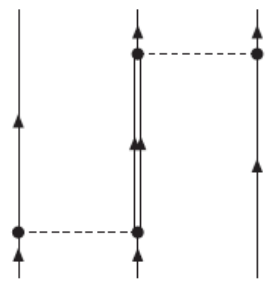


Observations (perhaps not controversial)

- Variational results and 3-hole-line results more or less in agreement
- Baldo et al. also calculated 3-hole-line terms with continuous choice for auxiliary potential and found that results do not depend on choice of auxiliary potential, furthermore 2-hole-line with continuous choice is already "almost" sufficient!
- Conclusion: convergence appears OK for a given realistic nuclear two-body interaction for the energy per particle
- Other quantities —> not always consistent (Hugenholtz-Van Hove)

- John's Crisis I resolved
- Still nuclear matter saturation problem! —> Crisis II

Possible solutions

- Include three-body interactions: inevitable on account of isobar
  - Simplest diagram:  space of nucleons -> 3-body force
  - Inclusion in nuclear matter requires phenomenology to get saturation better
  - Also needed for few-body nuclei; there is some incompatibility
  - There is no clear experimental constraint how much NN and how much NNN
- Include aspects of relativity
  - Dirac-BHF approach: ad hoc adaptation of BHF to nucleon spinors
  - Physical effect: coupling to scalar-isoscalar meson reduced with density
  - Antiparticles? Dirac sea? Three-body correlations?
  - Spin-orbit splitting in nuclei OK (also with 3N interaction)
  - Nucleons less correlated with higher density? (compare liquid ^{3}He)

Finite nuclei

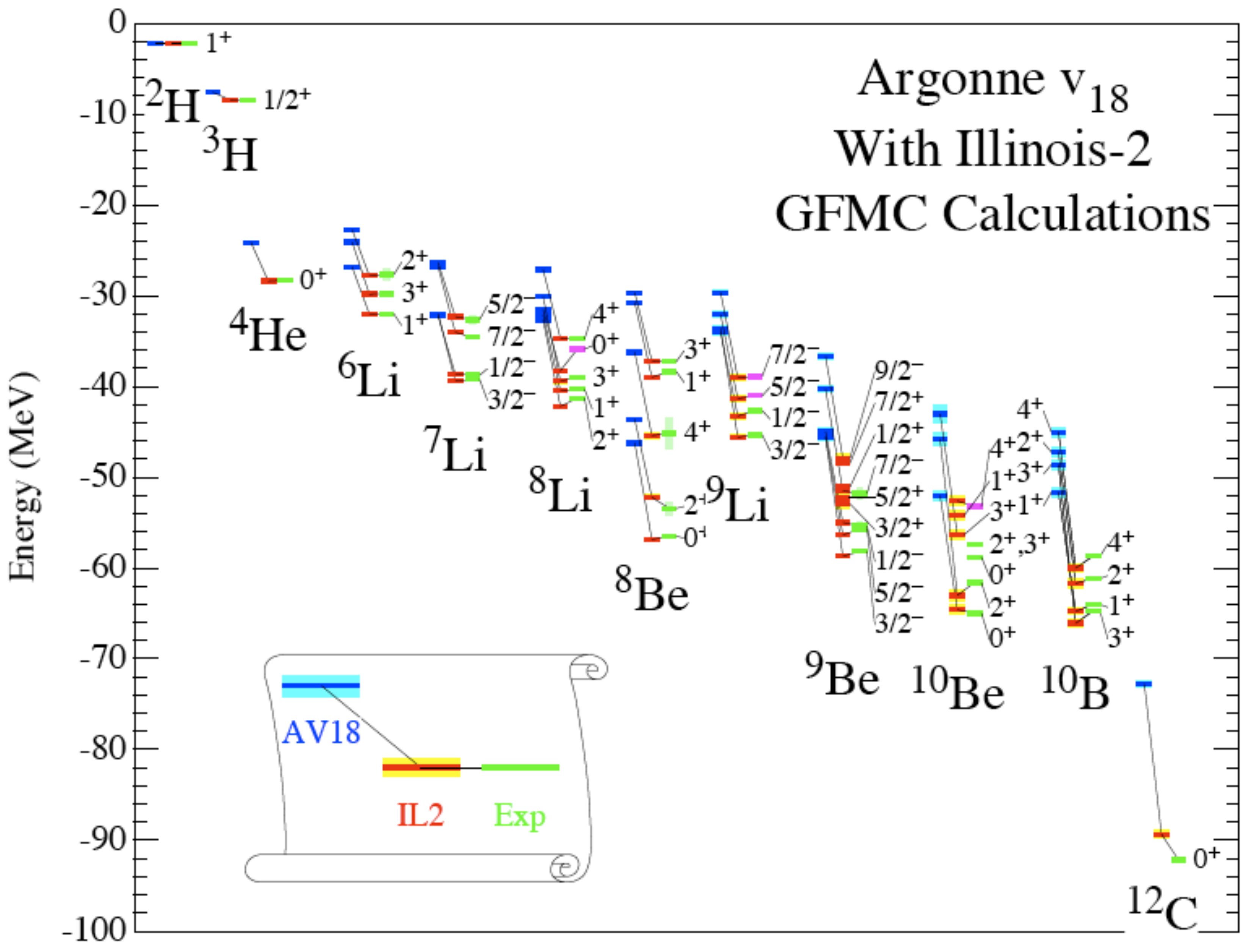
- What can we learn from finite nuclei

- Exact calculations possible for light nuclei
- Not restricted to NN interactions
- Can include NNN interactions

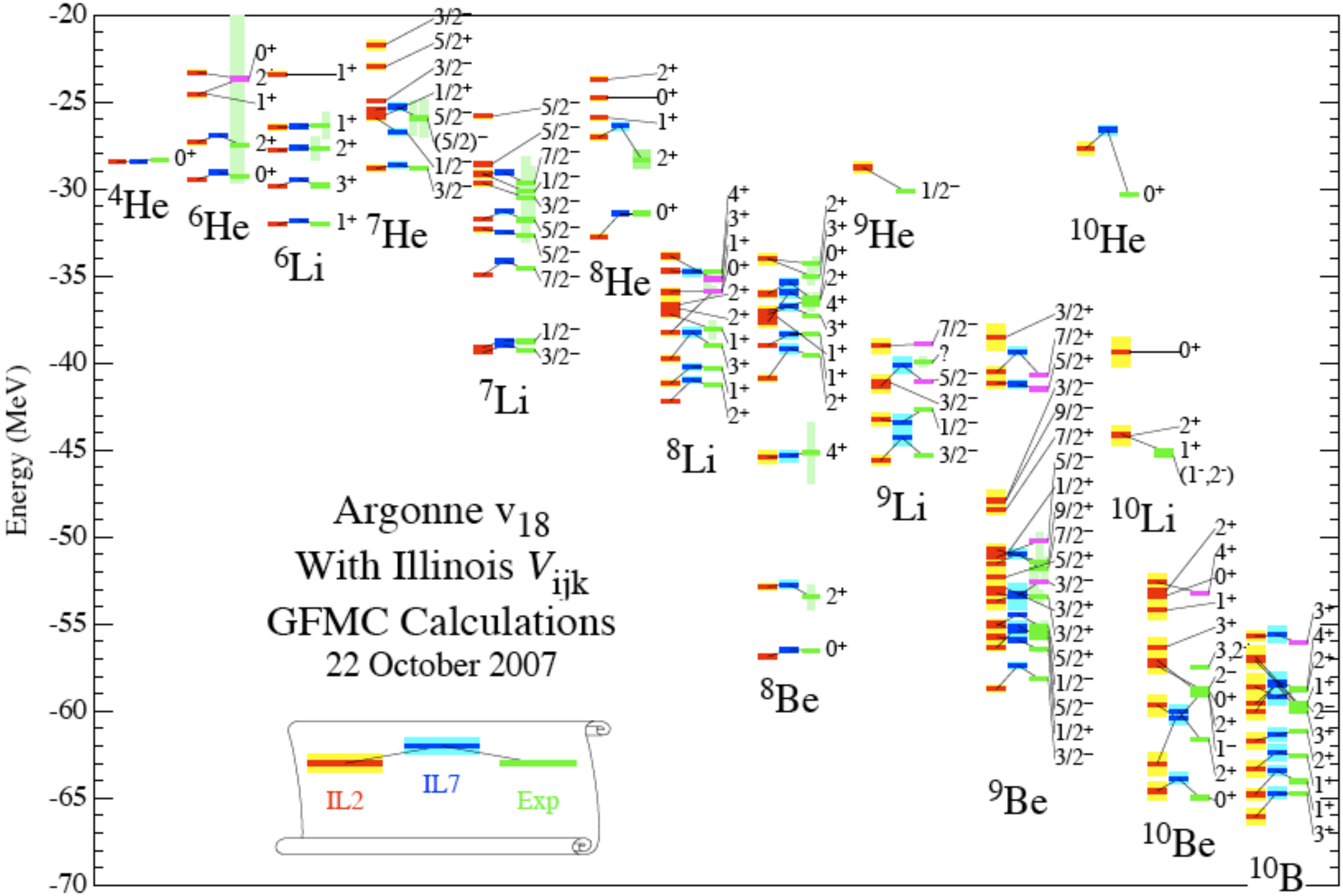
- But interactions must be local for Monte Carlo results!

- Argonne-Urbana effort

Effect of 3N attractive <—> AV18



More recent tuning 3N



Nuclear matter not so clear

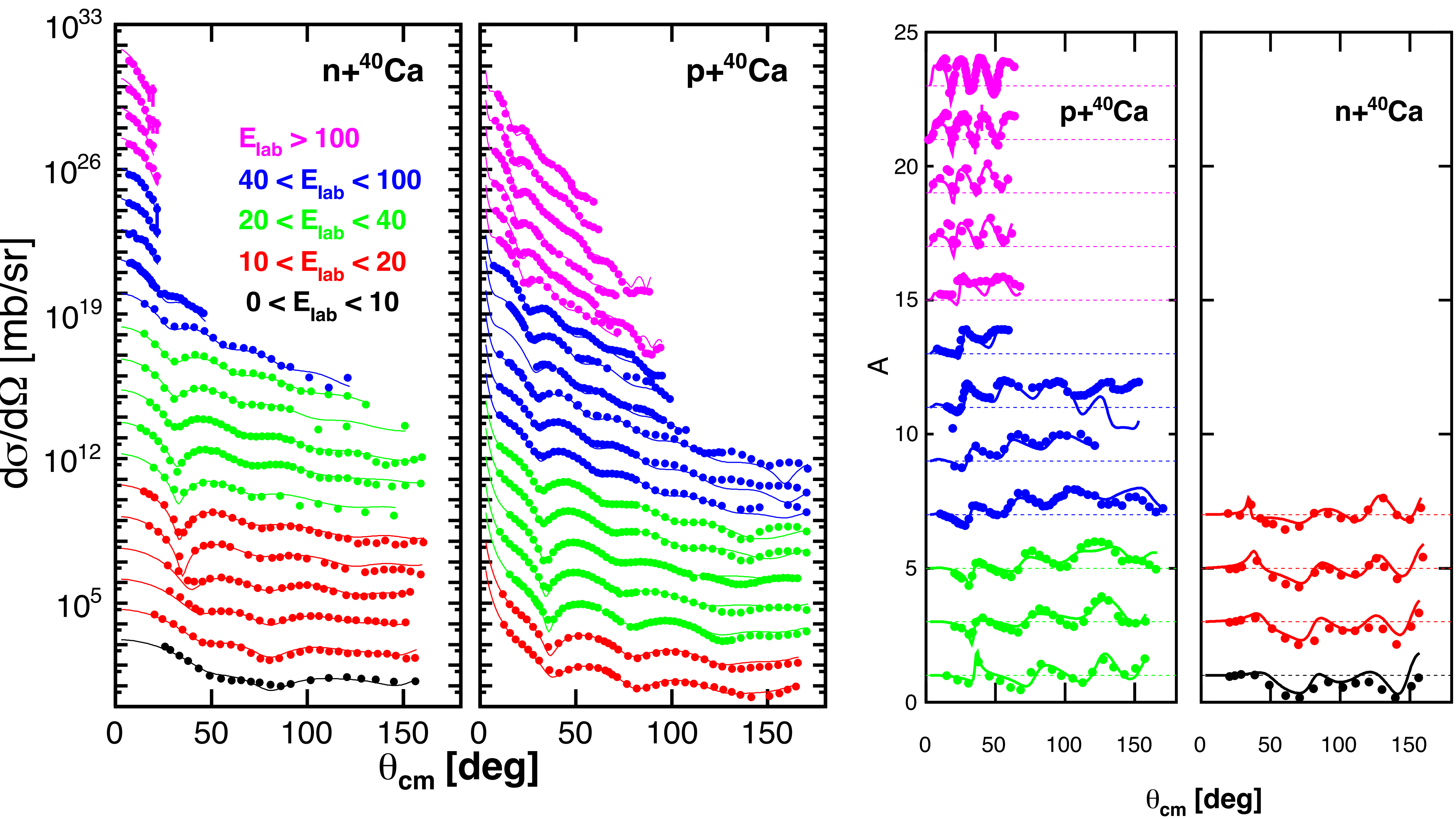
nuclear matter

Alternative way to get at NNN contribution?

- Go to experiment? But how?
- Employ an idea originally from Claude Mahaux
  - Use dispersion relation for nucleon self-energy
  - Constrain nucleon self-energy by experimental data
  - Initially elastic scattering and levels
- Recent work
  - Include charge density
  - Particle number
  - JLab (e,e'p) results for high-momentum protons
  - Nonlocal potential essential —> PRL112,162503(2014) for ^{40}Ca
- Can make a statement about NNN?!

nuclear matter

Differential cross sections and analyzing powers



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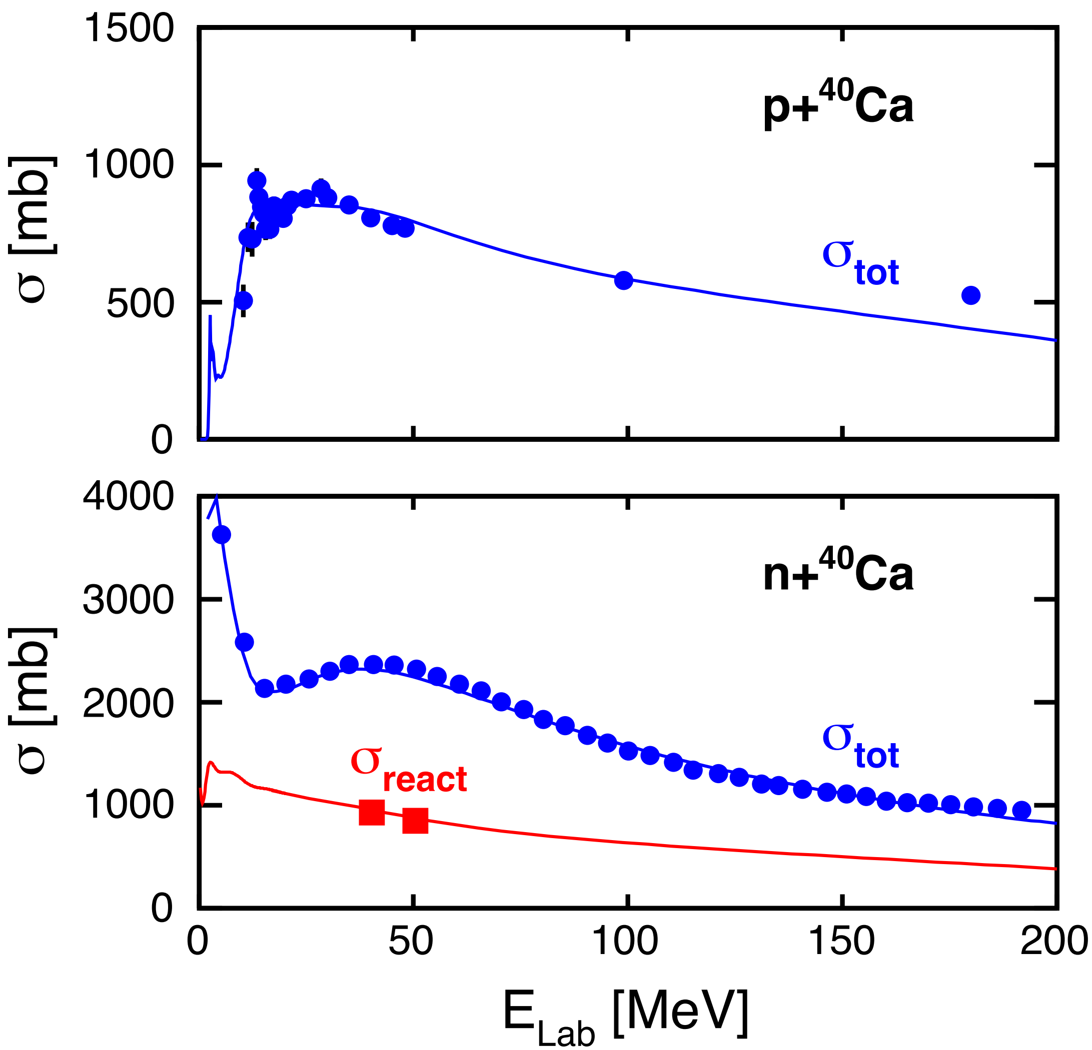
Reaction (p&n) and total (n) cross sections



nuclear matter

Critical experimental data

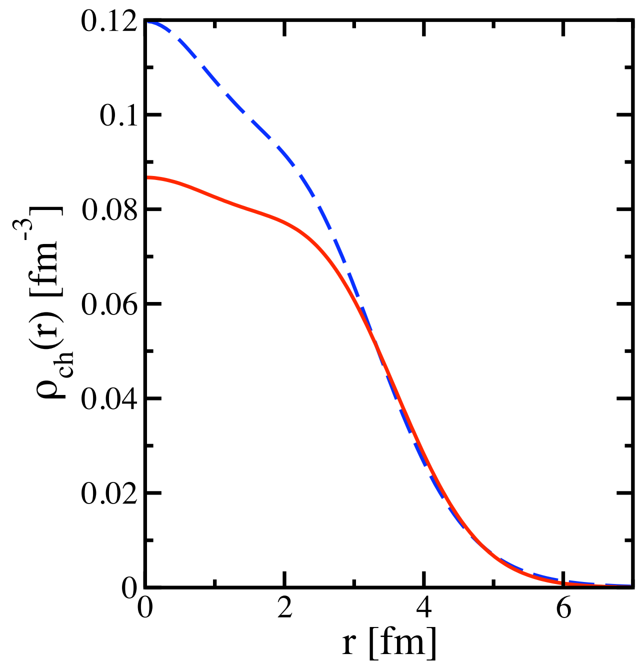
Local version

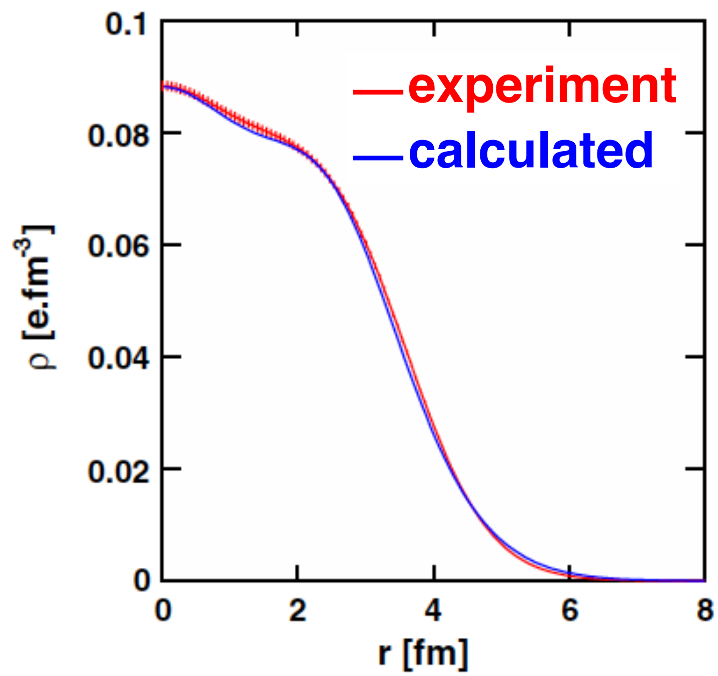
radius correct…

Charge density ^{40}Ca

Non-locality essential

PRL 112,162503(2014)



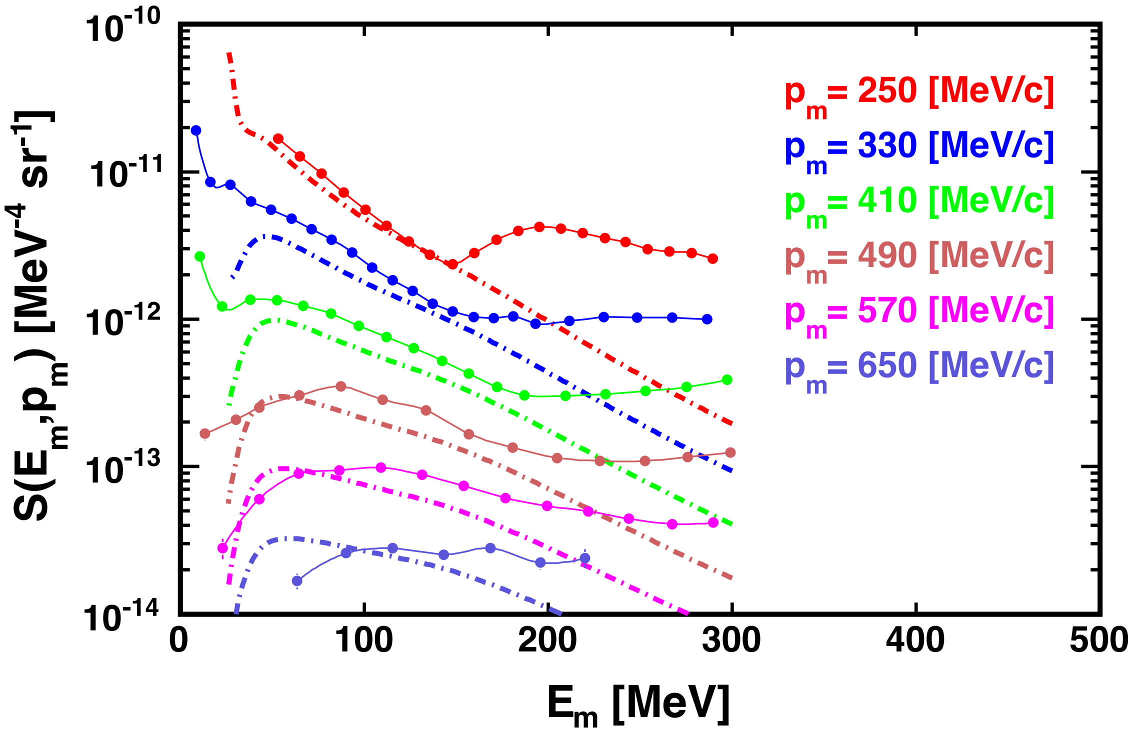


High-momentum nucleons —> JLab can also be described —> E/A

nuclear matter

Jefferson Lab data per proton

- Pion/isobar contributions cannot be described
- Rescattering contributes some cross section (Barbieri, Lapikas)
- Jlab E97-006 Phys. Rev. Lett. 93, 182501 (2004) D. Rohe et al.
- ~**10%** tail



- Or Hen et al.: High-momentum tail in heavy nuclei **20%**

nuclear matter

Energy of the ground state & NNN

- Energy sum rule (Migdal, Galitskii & Koltun)

$$E/A = \frac{1}{2A}\sum_{\ell j}(2j+1)\int_0^\infty dk k^2 \frac{k^2}{2m} n_{\ell j}(k) + \frac{1}{2A}\sum_{\ell j}(2j+1)\int_0^\infty dk k^2 \int_{-\infty}^{\varepsilon_F} dE\ E S_{\ell j}(k;E)$$

- **Not** part of fit because it can only make an exact statement for NN alone
- Result:
  - **DOM** ---> -7.91 MeV/A   T/A ---> 22.64 MeV/A
  - 10% of particles (momenta > 1.4 fm^{-1}) provide ~⅔ of the binding energy!
  - Exp.   -8.55 MeV/A
  - Three-body ---> 0.64 MeV/A "attraction" —> 1.28 MeV/A "repulsion"
  - Argonne GFMC ~ 1.5 MeV/A attraction for three-body <--> Av18

$$\begin{aligned} E_0^N &= \langle \Psi_0^N | \hat{H} | \Psi_0^N \rangle \\ &= \frac{1}{2\pi}\int_{-\infty}^{\varepsilon_F^-} dE \sum_{\alpha,\beta} \{\langle \alpha | T | \beta \rangle + E\ \delta_{\alpha,\beta}\}\ \mathrm{Im}\ G(\beta,\alpha;E) - \frac{1}{2}\langle \Psi_0^N | \hat{W} | \Psi_0^N \rangle \end{aligned}$$

with three-body interaction W

nuclear matter

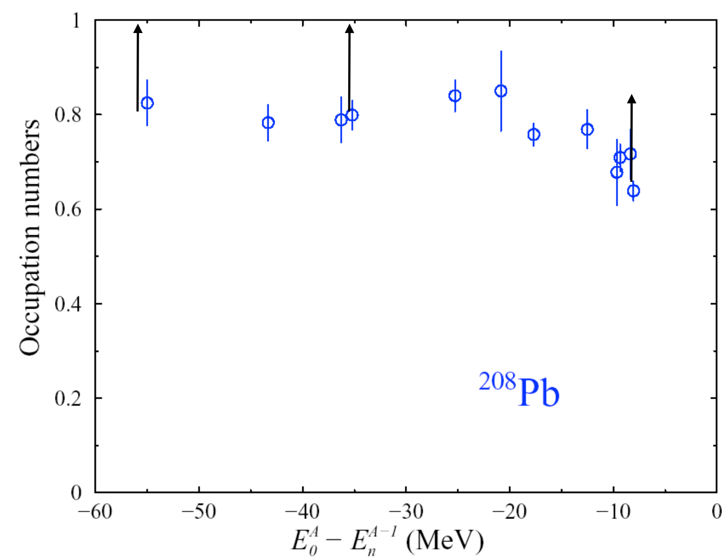
Physics of saturation

- How do we determine the saturation density
  - role of SRC
  - role of LRC
  - what are LRC in nuclei and nuclear matter
- How do we extract the binding energy at saturation
  - Can this be done with a liquid drop model?

Saturation density and SRC

- Saturation density related to nuclear charge density at the origin. Data for ^{208}Pb $\Rightarrow A/Z * \rho_{ch}(0) = 0.16$ fm^{-3}
- Charge at the origin determined by protons in s states
- Occupation of 0s and 1s totally dominated by SRC as can be concluded from an analysis of ^{208}Pb(e,e´p) data and theoretical calculations of occupation numbers in nuclei and nuclear matter (NIKHEF-Lapikas).
- Depletion of 2s proton also dominated by SRC:

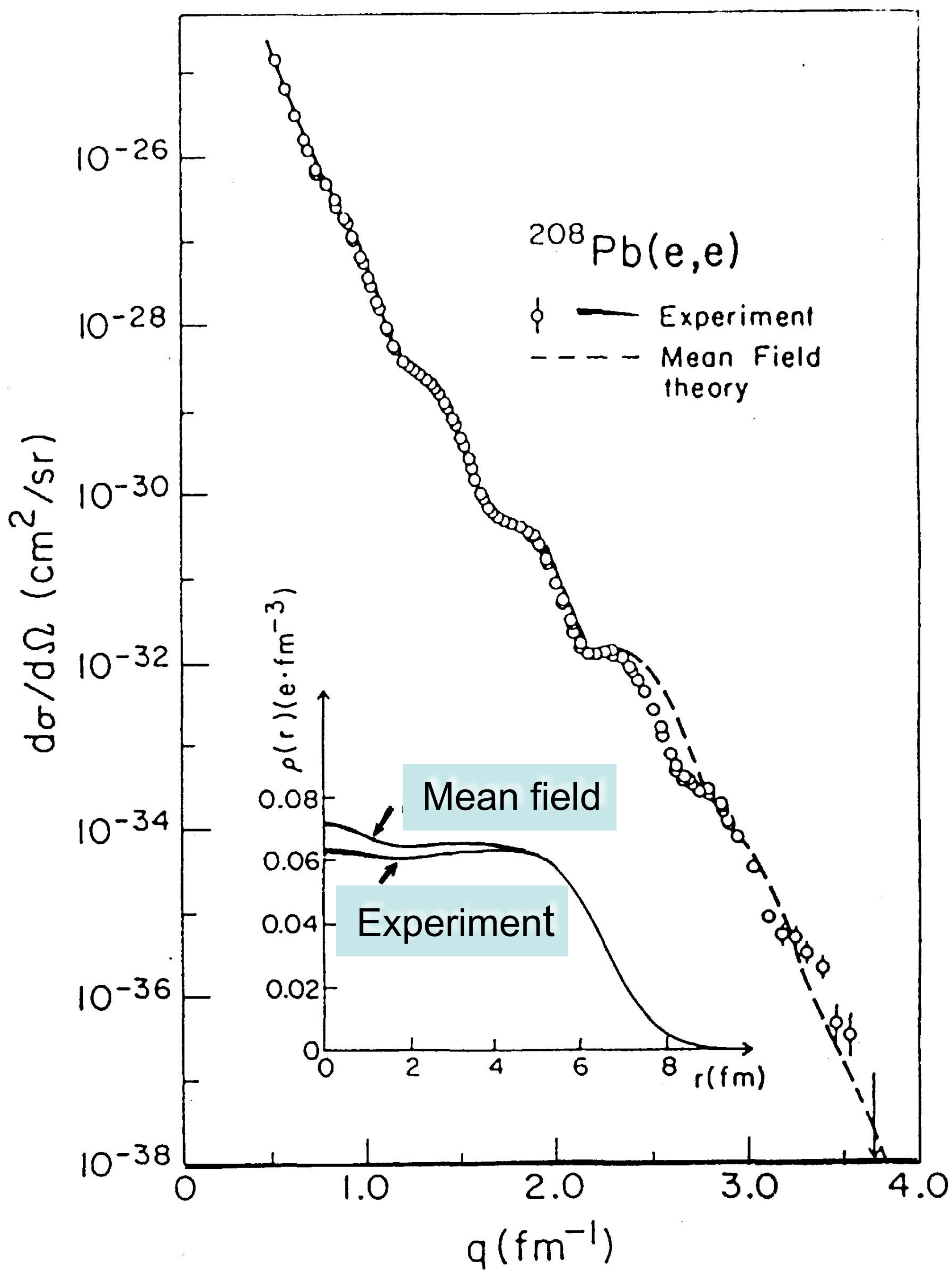
  15% of the total depletion of 25% (n_{2s} = 0.75)



- Conclusion: **Nuclear saturation dominated by SRC**

  and therefore —> presence of high-momentum components nuclear matter

Elastic electron scattering from ^{208}Pb



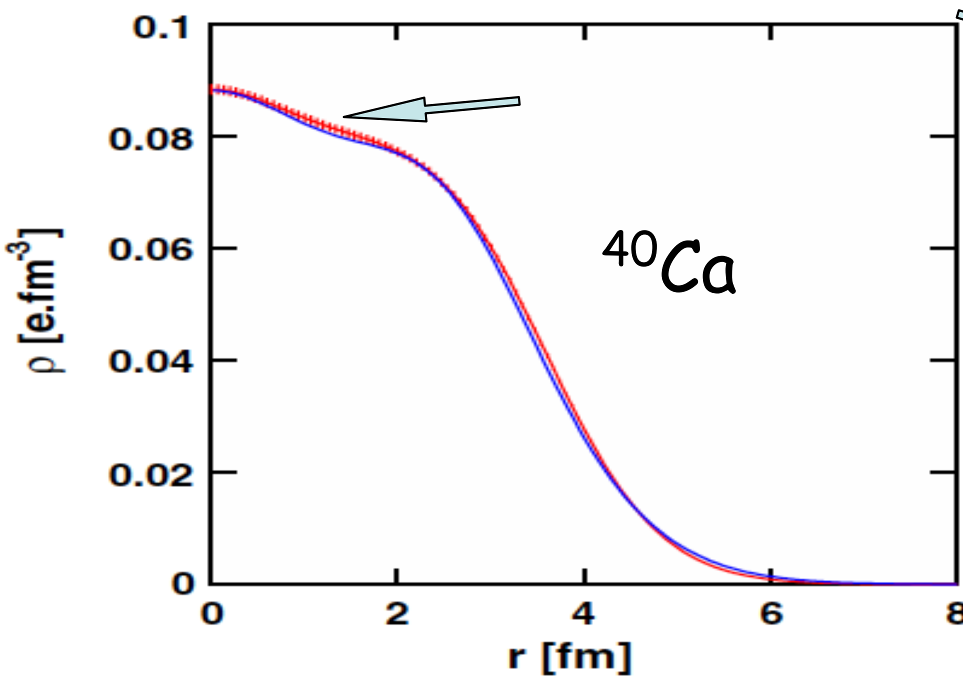
B. Frois et al.

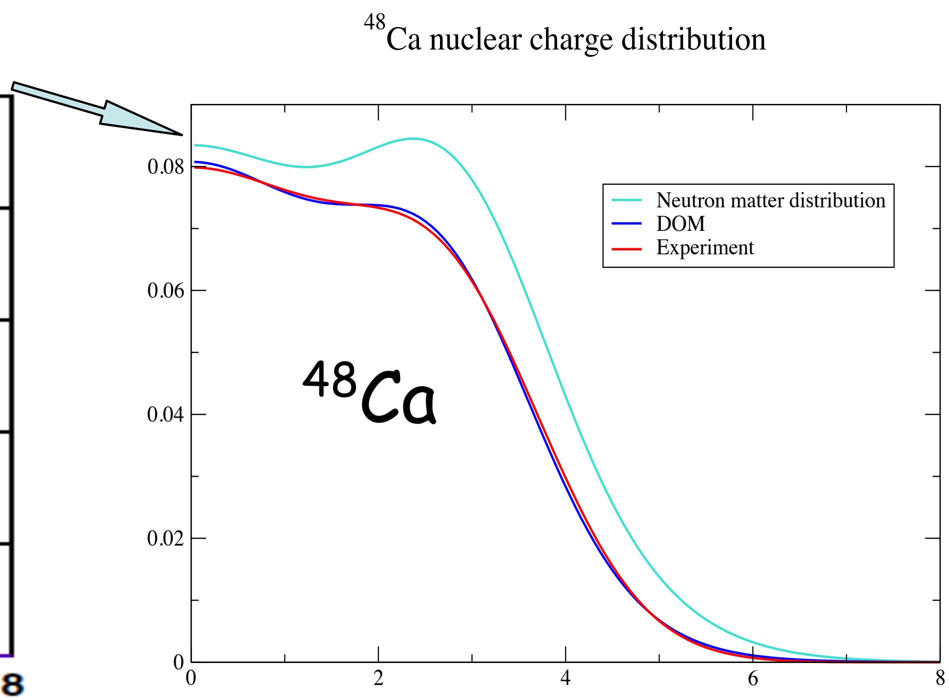
Phys. Rev. Lett. **38**, 152 (1977)

nuclear matter

Saturation density <—> Charge density

- Experimental results & empirical reproduction by DOM
- ^{48}Ca result: Hossein Mahzoon (now MSU) to be published





nuclear matter

Personal perspective 2003

Based on results from (e,e'p) reactions

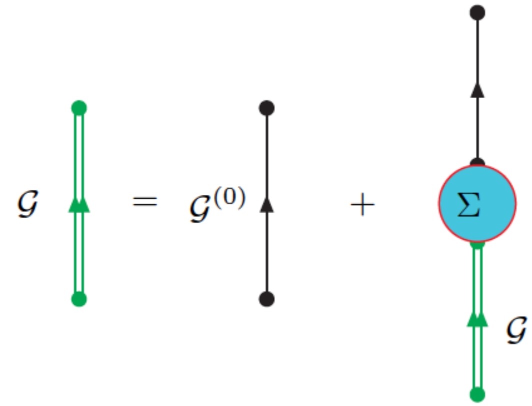
- nucleons are dressed (substantially) and this should be included in the description of nuclear matter (depletion, high-momentum components in the ground state, propagation w.r.t. correlated ground state <--> BHF?)
- SRC dominate actual value of saturation density
  - from ^{208}Pb charge density: 0.16 nucleons/fm^3
  - determined from s-shell proton occupancy at small radius
  - occupancy determined mostly by SRC
- Earlier result for SCGF of ladders do not include LRC!!

Phys. Rev. Lett. 90, 152501 (2003)

Self-consistent Green´s function and SRC (ladders) -> nuclear matter

Single-particle Green´s function $\mathcal{G}$

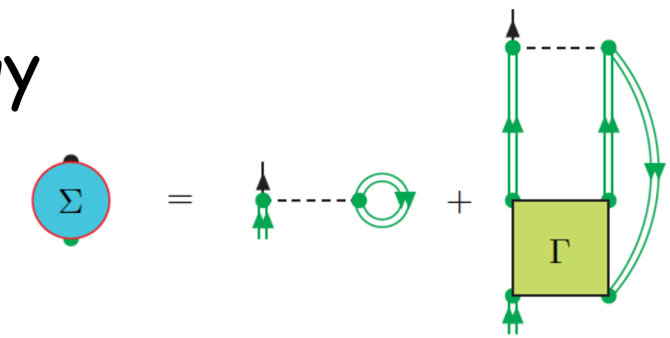
Dyson equation: $\mathcal{G} = \mathcal{G}^{(0)} + \mathcal{G}^{(0)}\Sigma\,\mathcal{G}$



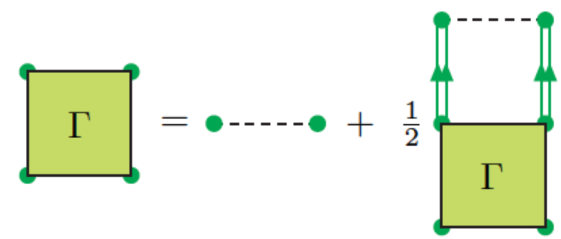
$$\mathcal{G}(k,E) = \frac{1}{E - \varepsilon_k - \Sigma(k,E)}$$

spectral function ~ $\mathrm{Im}\,\mathcal{G}(k,E)$

Self-energy



Γ-matrix



- Pairing instability possible
- Finite temperature calculation can avoid this
- T=0 extrapolation of normal self-energy OK

- Rios
- Polls
- Carbone (NNN)

Fetter & Walecka

The Bethe-Goldstone theory described above still differs in principle from the Brueckner theory because the Brueckner theory relies on a self-consistent single-particle potential. In terms of Green's functions, this result can be achieved by replacing $G^0(p)$ with a $G(p)$ that includes self-energy effects associated with Γ. Furthermore, Γ must itself be determined with G and not G^0. The equations for this self-consistent theory are shown schematically in Fig. 42.4.

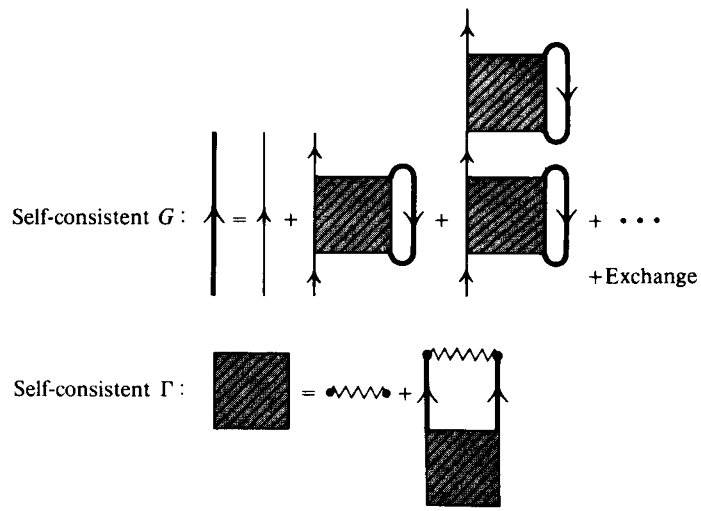


Fig. 42.4 Self-consistent theory for G and Γ.

As they stand, these equations are quite intractable because the frequency dependence of $\Sigma^\star(\mathbf{p},p_0)$ complicates the integral equation for Γ immensely. (This difficulty is sometimes known as *propagation off the energy shell.*) The simplest Brueckner-Goldstone theory can be obtained from these equations in a series of approximations. First, the self-consistency is treated only on the average, and we use a frequency-independent self-energy $\Sigma^\star_{sc}(\mathbf{p}) \equiv \Sigma^\star(\mathbf{p},\epsilon_\mathbf{p}/\hbar)$, obtained by setting $p_0 = \epsilon_\mathbf{p}/\hbar$, where $\epsilon_\mathbf{p}$ satisfies the self-consistent equation

$$\epsilon_\mathbf{p} = \epsilon^0_\mathbf{p} + \hbar\Sigma^\star(\mathbf{p},\epsilon_\mathbf{p}/\hbar) \equiv \epsilon^0_\mathbf{p} + \hbar\Sigma^\star_{sc}(\mathbf{p}) \tag{42.13}$$

In this way, the Green's function is given approximately as

$$G_{sc}(\mathbf{p},p_0) = \frac{\theta(|\mathbf{p}| - k_F)}{p_0 - \epsilon_\mathbf{p}/\hbar + i\eta} + \frac{\theta(k_F - |\mathbf{p}|)}{p_0 - \epsilon_\mathbf{p}/\hbar - i\eta} \tag{42.14}$$

Second, this Green's function is used to evaluate both the proper self-energy [Eq. (42.4)] and the scattering amplitude [Eqs. (42.5) and (42.6)]. We again obtain χ_m by omitting the hole-hole scattering, which is presumed small in the low-density limit. The only effect on the self-consistent wave function is to change the denominator in Eq. (42.6) from $mP_0/\hbar - \frac{1}{2}(\frac{1}{2}\mathbf{P} + \mathbf{q})^2 - \frac{1}{2}(\frac{1}{2}\mathbf{P} - \mathbf{q})^2 + i\eta$

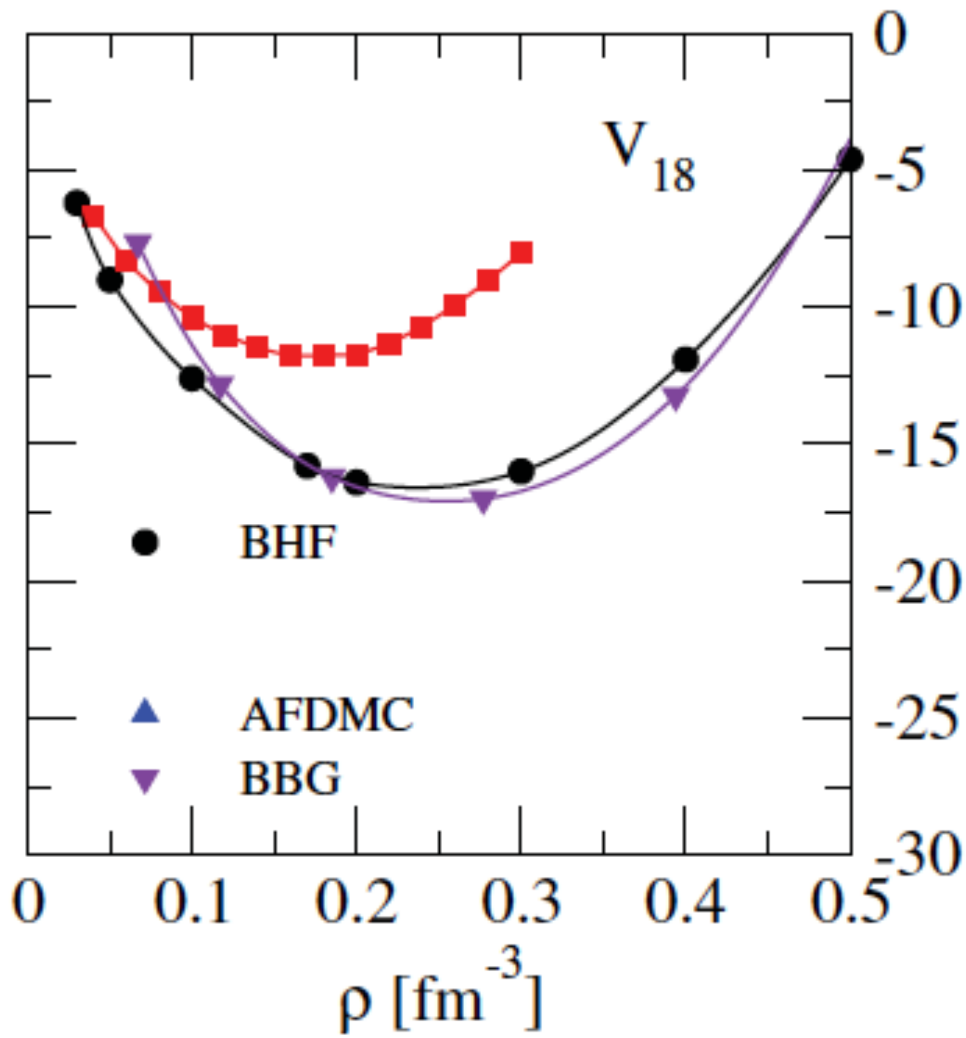
nuclear matter

Recent result SCGF & SRC compared to BHF and BBG

PHYSICAL REVIEW C 86, 064001 (2012)

Comparative study of neutron and nuclear matter with simplified Argonne nucleon-nucleon potentials

M. Baldo,[1] A. Polls,[2] A. Rios,[3] H.-J. Schulze,[1] and I. Vidaña[4]



- BBG requires a repulsive NNN at high density to improve density

nuclear matter

So why can't we get it right?

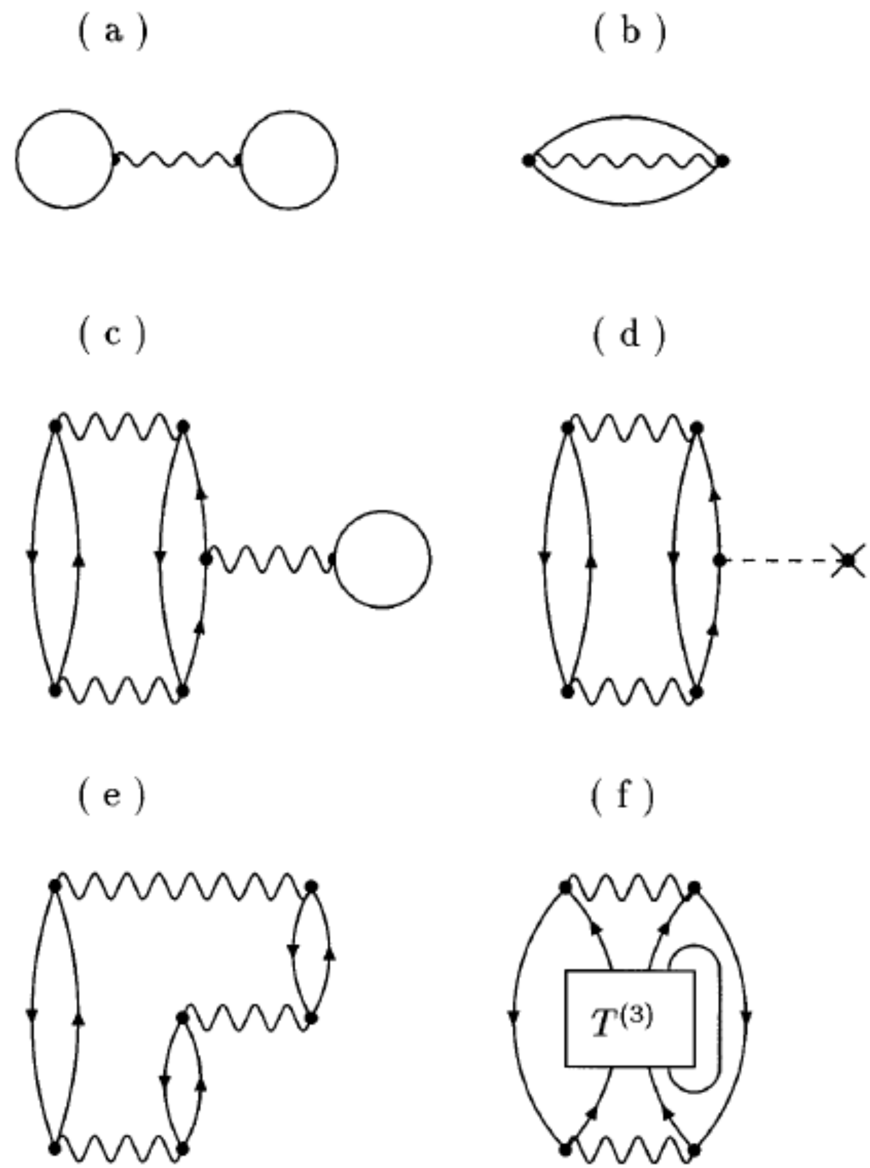
- Must be LRC?!

- Look at hole-line expansion

- Identify LRC contribution to the energy

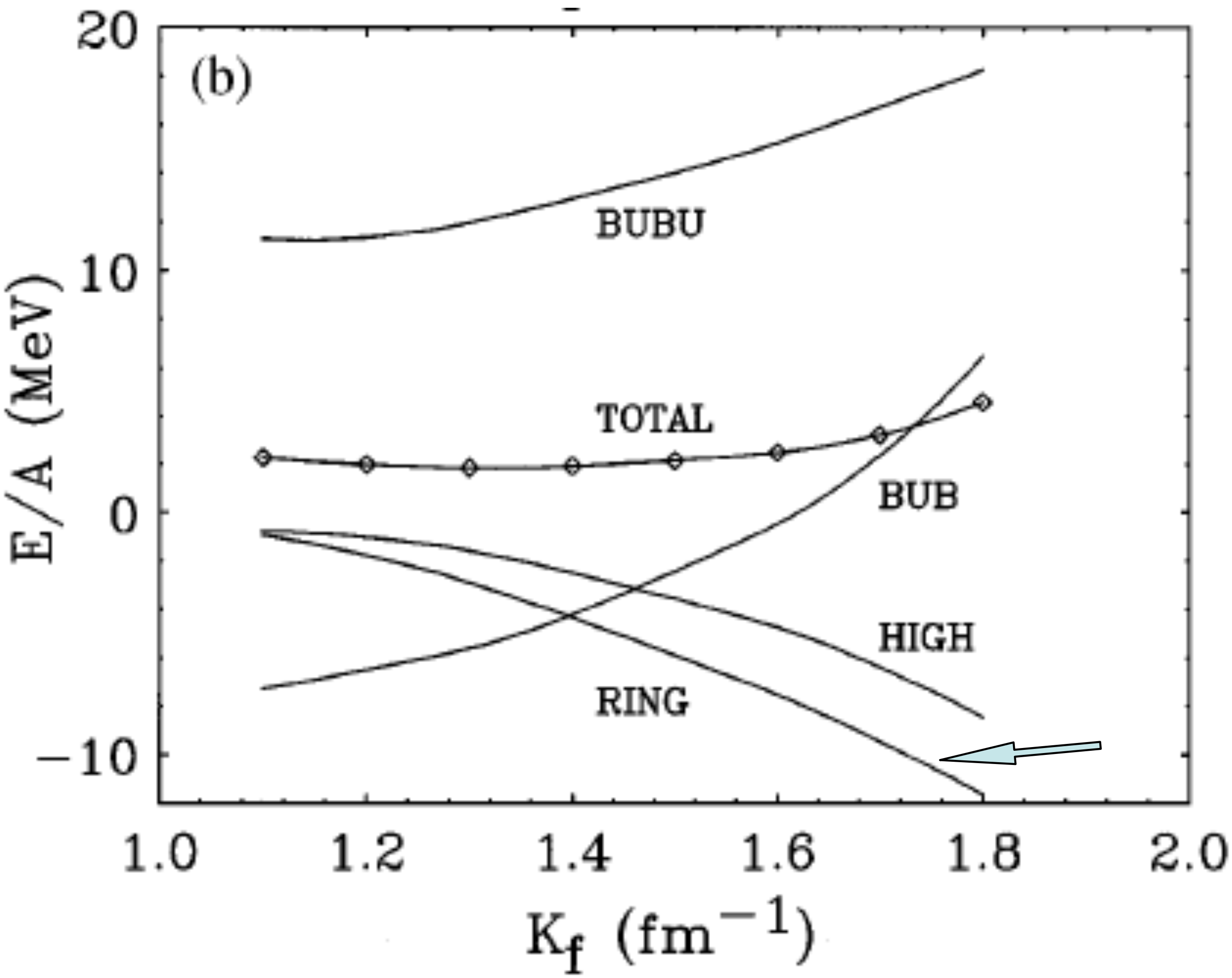
Ingredients hole-line expansion

- Wiggle: G-matrix
- a) + b) = 2 hole-line = BHF
- c) + d) +e) +f) = 3 hole-line
- c) bubble
- d) U insertion for C choice
- e) ring
- f) summed in Bethe-Faddeev



Continuous choice

- PRL 81, 1584 (1998) Baldo et al.



What about long-range correlations in nuclear matter?

- Collective excitations in finite nuclei **very different** from those in nuclear matter

- Long-range correlations **normally** associated with small q

- Contribution to the energy like $dq\, q^2 \Rightarrow$ very small (except for e-gas)

- Contributions of collective excitations to the binding energy of nuclear matter dominated by pion-exchange induced excitations and not small?!?

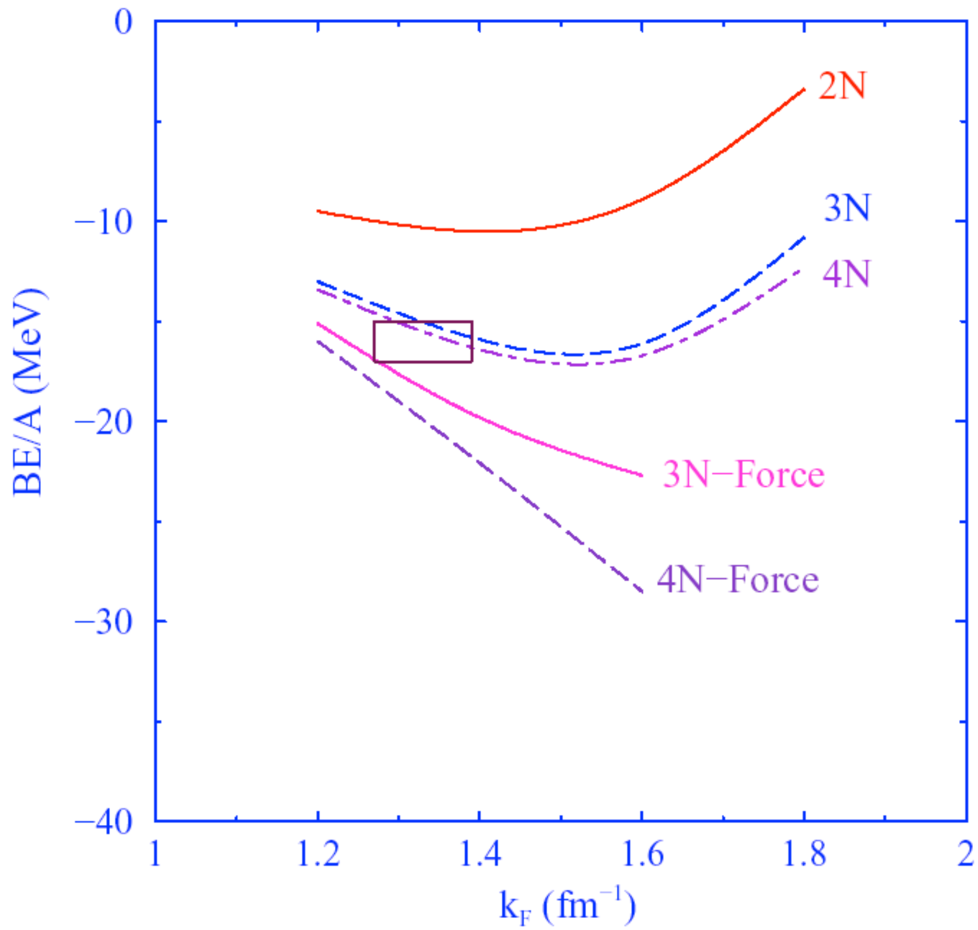
Pion-exchange channel dominates 3rd order ring

- Decomposition in spin-isospin excitations at normal density

	S	M	T	Reid
third order	0	0	0	−0.302
	1	0	0	0.149
	1	1	0	0.059
	0	0	1	0.027
	1	0	1	−3.492
	1	1	1	0.540
sum				−3.019
fourth order	0	0	0	−0.060
	1	0	0	−0.017
	1	1	0	−0.012
	0	0	1	−0.004
	1	0	1	−0.755
	1	1	1	−0.317
sum				−1.166
total				−4.185

Nucl. Phys. A389, 492 (1982)

Inclusion of Δ-isobars as 3N- and 4N-force



2N,3N, and 4N from
B.D.Day, PRC24,1203(81)

Rings with Δ-isobars :

Nucl. Phys. A389, 492 (1982)

PPNPhys 11, 529 (1983)

⇒ No sensible convergence with Δ-isobars

Must do nuclear saturation without π-collectivity

Pion collectivity: nuclei vs. nuclear matter

- Pion collectivity leads to pion condensation at higher density in nuclear matter (including Δ-isobars) => Migdal ...
- No such collectivity observed in nuclei $\Rightarrow$ LAMPF / Osaka data

- Momentum conservation in nuclear matter dramatically favors amplification of π-exchange interaction at fixed **finite** q

$$V_\pi(q) = -\frac{f_\pi^2}{m_\pi^2}\frac{q^2}{m_\pi^2 + q^2}$$

- In nuclei the same interaction is sampled over all momenta Phys. Lett. **B146**, 1(1984)

<u>Needs further study</u>

<u>$\Rightarrow$ **Exclude collective pionic contributions to nuclear matter binding energy**</u>

Two Nuclear Matter Problems

The usual one

- With π-collectivity and only nucleons
- Variational + CBF and three hole-line results OK (for E/A) but not directly relevant for comparison with nuclei!
- Add NNN —> adhoc adjustment

The relevant one?!

- Without π-collectivity
- Treat only SRC
- But at a sophisticated level by using self-consistency
- Understand lack of binding
  - LRC in finite nuclei?
- 3N-forces difficulty $\Rightarrow \pi$ ...

Even with the right NM saturation NOTHING is explained if the nuclear charge density in the interior is too large

LRC in finite nuclei

Remember:

- LRC in infinite nuclear matter —> no counterpart in finite nuclei

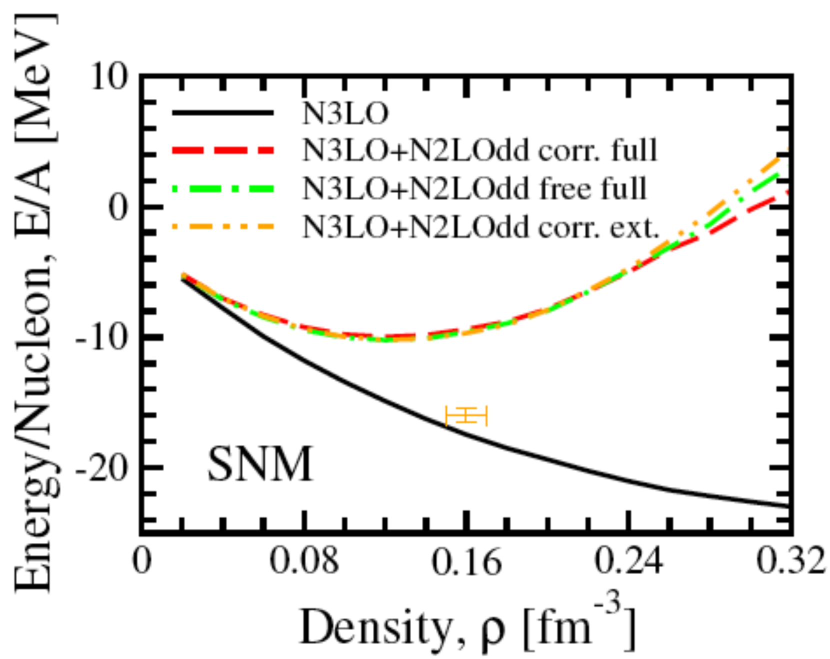
- BUT: LRC in finite nuclei —> no counterpart in nuclear matter
- They will contribute some binding!
- How much: nobody has really looked into this

Recent results for chiral interactions
Have I changed my mind?

Systematic expansion in chiral perturbation theory

- allows simultaneous construction of 2N and 3N interaction
- implemented with a very soft cut-off (500 MeV for example)
- easy to compress nuclei —> small radii & too much charge at the origin
- NNN large contribution with higher density necessary

Carbone et al
PRC90,054322(2014)



nuclear matter

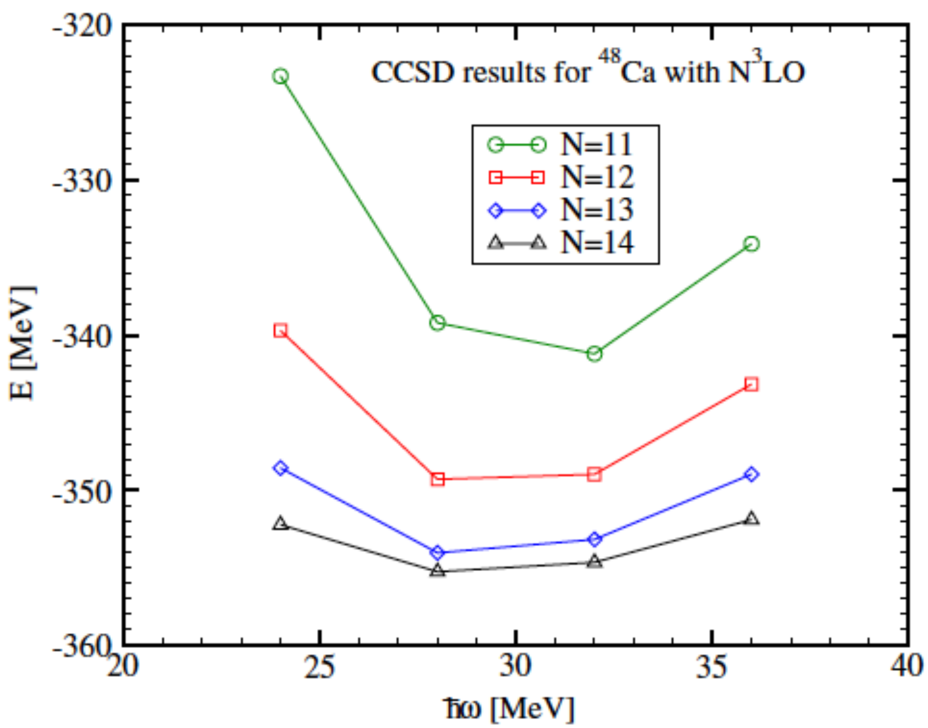
Finite nuclei and chiral interactions

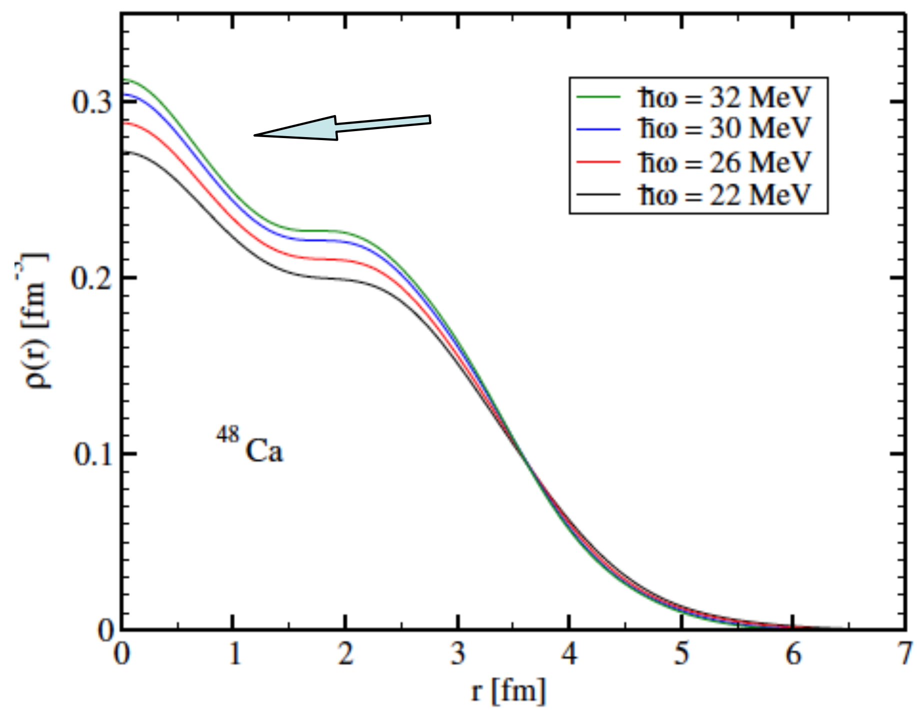
PRL **101,** 092502 (2008) PHYSICAL REVIEW LETTERS week ending 29 AUGUST 2008

Medium-Mass Nuclei from Chiral Nucleon-Nucleon Interactions

G. Hagen,[1] T. Papenbrock,[2,1] D.J. Dean,[1] and M. Hjorth-Jensen[3]

- N3LO only —> Coupled-Cluster method
- missing ~1.2 MeV binding per nucleon for ^{48}Ca

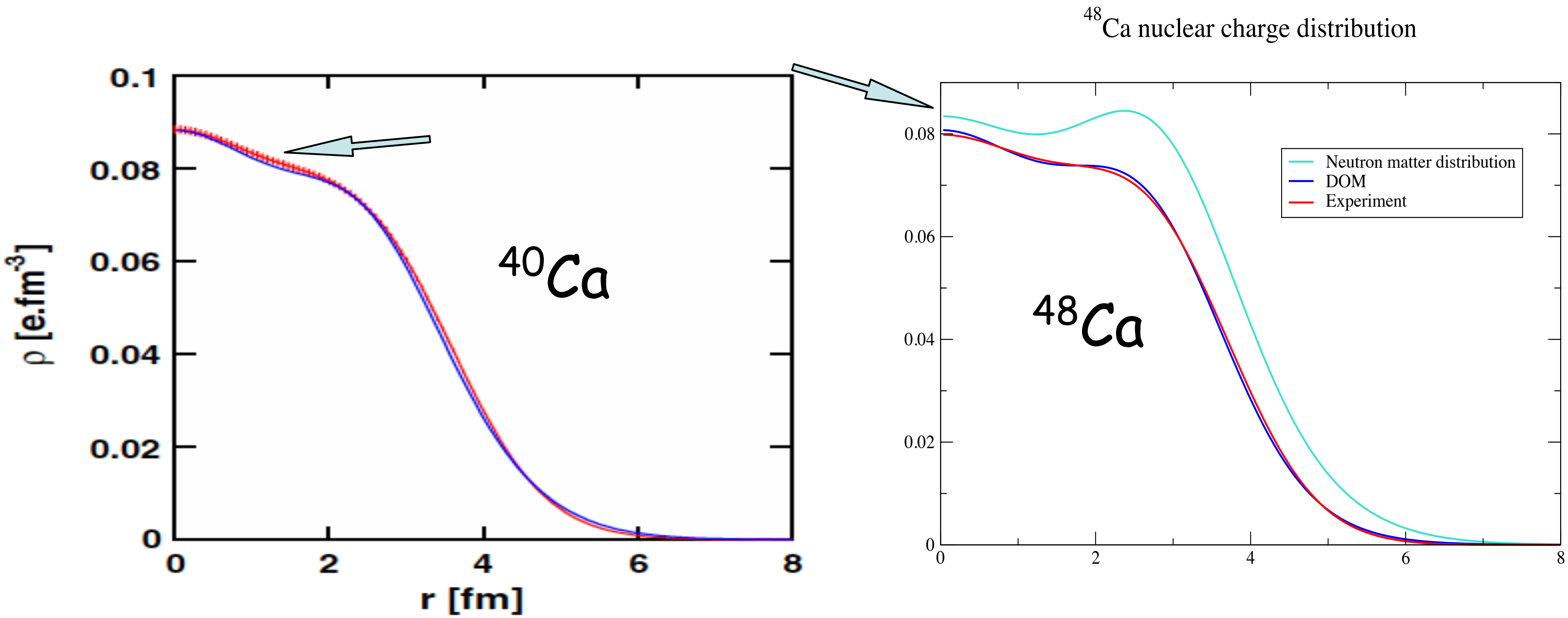




nuclear matter

Saturation density <—> Charge density

- Experimental results & empirical reproduction by DOM



nuclear matter

Finite nuclei and chiral interactions

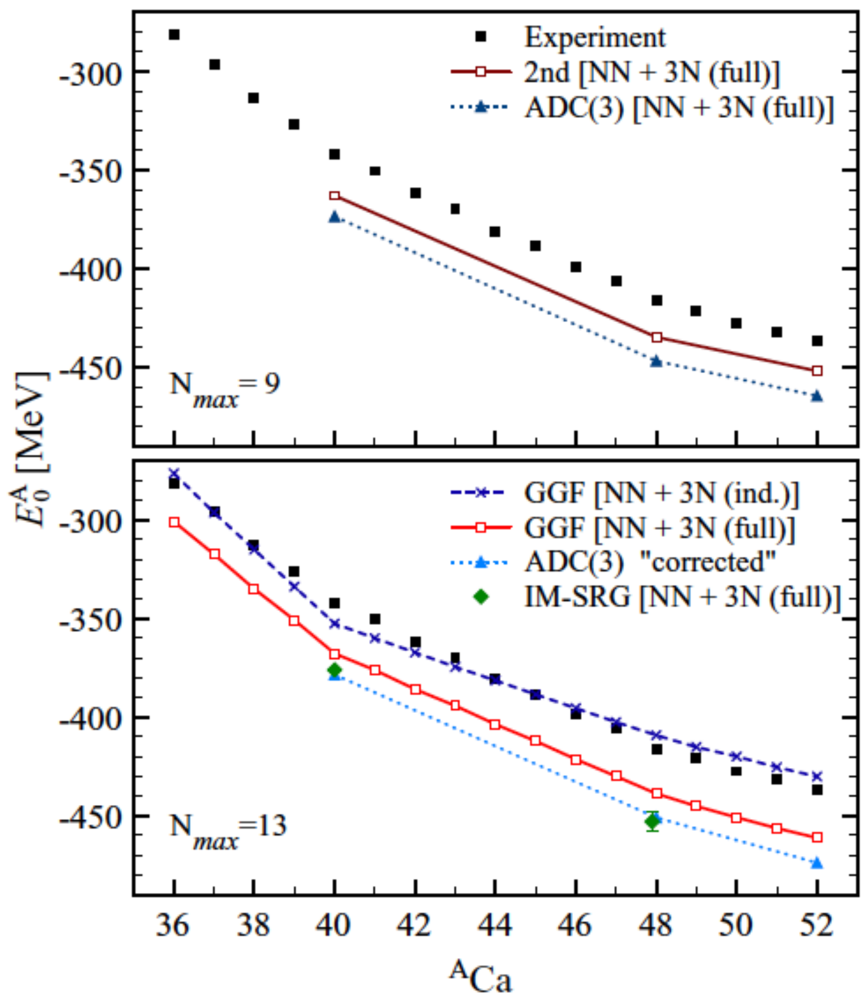
- Example

PHYSICAL REVIEW C 89, 061301(R) (2014)

Chiral two- and three-nucleon forces along medium-mass isotope chains

V. Somà,[1,2,3,*] A. Cipollone,[4] C. Barbieri,[4,†] P. Navrátil,[5] and T. Duguet[3,6,‡]

- SRG evolved N3LO
- +3N induced
- +3N chiral
- + correction ADC(3)



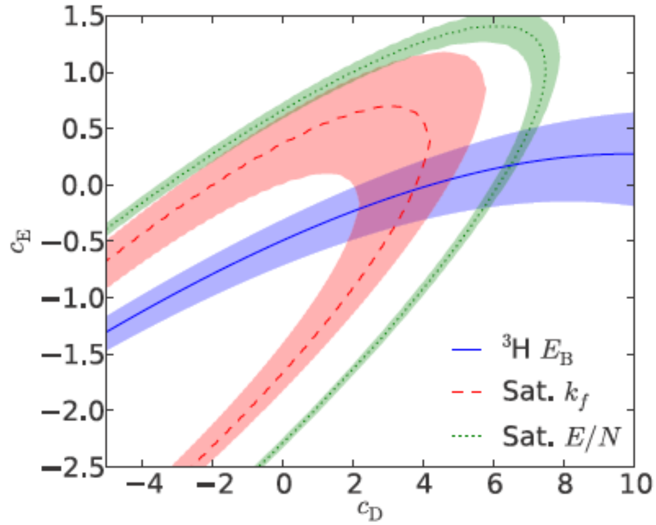
- Ca isotopes

- Overbound by ~40 MeV
- plus other problems like size

Nuclear matter saturation issues

- Old problem…
- Is it solved?
- Don't think so…

- Coupled cluster

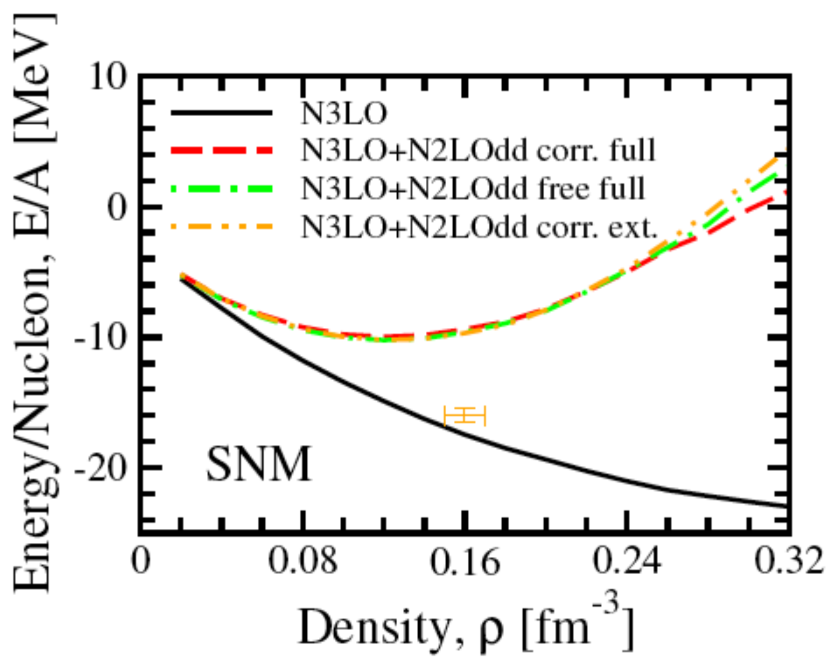


PRC **89**, 014319 (2014)
Can't do triton and saturation at the same time

- Lattice calculations

Radius of ^{16}O
$\langle r^2 \rangle^{1/2}$=2.3 fm<—> Exp 2.71 fm
PRL112, 102501 (2014)

- SCGF only "SRC" no regulators



arXiv:1408.0717 PRC90,054322(2014)
3NF —> DD2NF

Finite nuclei and chiral interactions

PHYSICAL REVIEW C **92**, 014306 (2015)

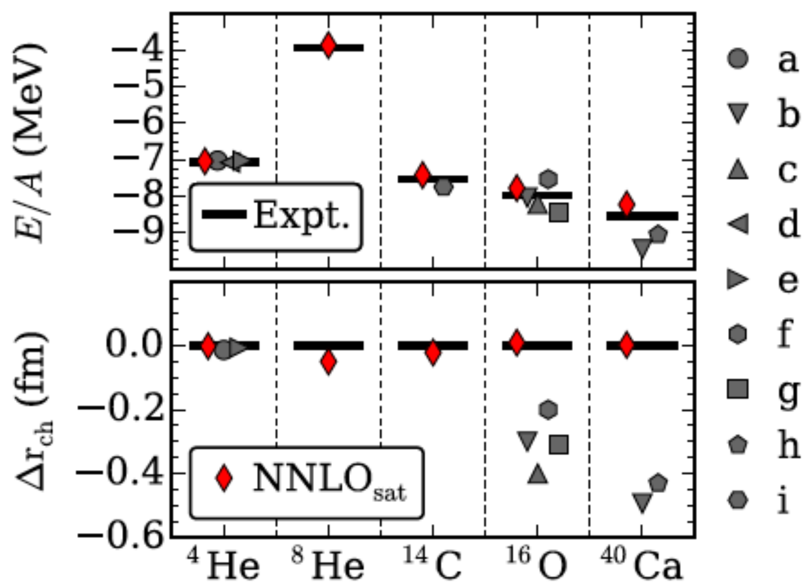
Chiral three-nucleon forces and the evolution of correlations along the oxygen isotopic chain

A. Cipollone,[1,2] C. Barbieri,[1,*] and P. Navrátil[3]

"We observe that all these deficiencies might be corrected by having extra short-range repulsion in the NN section of the Hamiltonian."

PHYSICAL REVIEW C **91**, 051301(R) (2015)

Accurate nuclear radii and binding energies from a chiral interaction



Ekström et al.

- Yet another way out? but no high-momentum nucleons…

nuclear matter

Saturation of symmetric nuclear matter: outlook

- Nuclear saturation problem
  - We know a lot …
  - We can't get it right …
  - Why not?
- Forces & methods
  - Chiral interactions + 3NF
    - Underbinds in SCGF (SRC only)
    - Coupled cluster: triton <-> nuclear matter cannot be reconciled
  - Comments
    - Not enough high-momentum content (JLab) —> chiral NN interactions too soft
    - LRC (mainly pionic) contribute to energy
    - pion physics missing (NN static **only**???)
    - interior density of heavier nuclei too high <—> saturation problem
    - empirical NNN in ^{40}Ca ~1.28 MeV/A —> PRL 112, 162503 (2014)
- What to do?
  - Make chiral interactions consistent with JLab data (a little harder) —> good for finite nuclei as well
  - Continue to develop the techniques to deal with such a harder interaction (to be done for nuclei)
  - Revisit the formulation of the nuclear matter problem
    - Why?
      - Pion-exchange in matter ≠ pion-exchange in a finite system
      - Liquid drop notion only good for very short-range physics
      - LRC normally small q —> no energy
      - Nuclear matter pions —> finite q —> increasing binding with density —> messes up saturation
      - see PRL90, 152501 (2003)
      - LRC in nuclei —> binding? how much?

nuclear matter