Cluster Production in Boltzmann Equation Model - Past and Future

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Dynamical Cluster Formation and Correlations
in Heavy-Ion Collisions
within Transport Models and in Experiments

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Boltzmann Equation Model (BEM/pBUU)

Degrees of freedom (X):
nucleons, deuterons, tritons, helions ($A \leq 3$), Δ, N^*, pions

Fundamentals:

- Relativistic Landau theory (Chin/Baym)
  Energy functional (ϵ)
- Real-time Green's function theory
  Production/absorption rates ($\mathcal{K}^<$, $\mathcal{K}^>$)

$$\frac{\partial f}{\partial t} + \frac{\partial \epsilon}{\partial \mathbf{p}} \frac{\partial f}{\partial \mathbf{r}} - \frac{\partial \epsilon}{\partial \mathbf{r}} \frac{\partial f}{\partial \mathbf{p}} = \mathcal{K}^< (1 \mp f) - \mathcal{K}^> f$$

production absorption rate

Single-Particle Energies & Functional

$$\frac{\partial f}{\partial t} + \frac{\partial \epsilon}{\partial \mathbf{p}} \frac{\partial f}{\partial \mathbf{r}} - \frac{\partial \epsilon}{\partial \mathbf{r}} \frac{\partial f}{\partial \mathbf{p}} = \mathcal{K}^{<} (1 \mp f) - \mathcal{K}^{>} f$$

The single-particle energies ϵ are given in terms of the net energy functional $E\{f\}$ by,

$$\boxed{\epsilon(\mathbf{p}) = \frac{\delta E}{\delta f(\mathbf{p})}}$$

In the local cm, the mean potential is

$$U_{opt} = \epsilon - \epsilon_{kin}$$

and $\epsilon_{kin} = \sqrt{p^2 + m^2}$

Energy Functional

The functional: $$E = E_{vol} + E_{gr} + E_{iso} + E_{Coul}$$

where $$E_{gr} = \frac{a_{gr}}{\rho_0} \int d\mathbf{r}\, (\nabla \rho)^2$$

For covariant volume term, ptcle velocities parameterized in local frame: $$v^*(p,\rho) = \frac{p}{\sqrt{p^2 + m^2 \Big/ \left(1 + c\, \frac{\rho}{\rho_0}\, \frac{1}{(1+\lambda\, p^2/m^2)^2}\right)^2}}$$

precluding a supraluminal behavior, with ρ - baryon density.

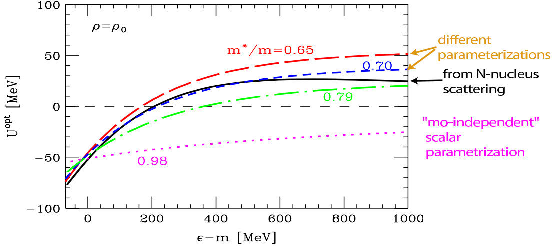
The 1-ptcle energies are then

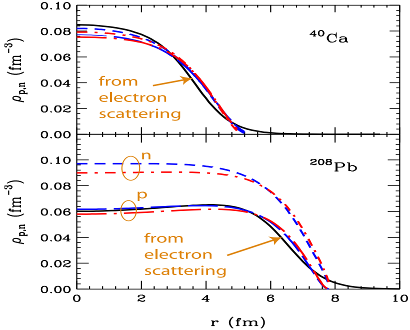
$$\epsilon(p,\rho) = m + \int_0^p dp'\, v^* + \Delta\epsilon(\rho)$$

Parameters in the velocity varied to yield different optical potentials characterized by values of effective mass, $m^* = p_F / v_F$.

Structure Interface

Potential from p-scattering (Hama *et al.* PRC41(90)2737) & parameterizations





Ground-state densities from electron scattering and from functional minimization.

From $E(f) = min$:

$$0 = \epsilon\left(p^F(\rho)\right) - 2\,a_{gr}\,\nabla^2\left(\frac{\rho}{\rho_0}\right) - \mu$$

$\Rightarrow$ Thomas-Fermi eq.

Many-Body Theory

Transport eq. for nucleons follows from the eq. of motion for the 1-ptcle Green's function (KB eq.). Transport eq. for deuterons ($A = 2$) from the eq. for 2-ptcle Green's function??

Wigner function in second quantization

$$f(\mathbf{p};\mathbf{R},T) = \int d\mathbf{r}\, \mathrm{e}^{-i\mathbf{pr}} \, \langle \hat{\psi}_H^\dagger(\mathbf{R}-\mathbf{r}/2,T)\hat{\psi}_H(\mathbf{R}+\mathbf{r}/2,T)\rangle$$

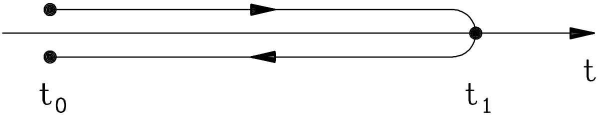
where $\langle\cdot\rangle \equiv \langle\Psi|\cdot|\Psi\rangle$ and $|\Psi\rangle$ describes the initial state.

Evolution driven by a Hamiltonian. Interaction Hamiltonian:

$$\hat{H}^1 = \frac{1}{2}\int d\mathbf{x}\, d\mathbf{y}\, \hat{\psi}^\dagger(\mathbf{x})\hat{\psi}^\dagger(\mathbf{y}) v(\mathbf{x}-\mathbf{y})\hat{\psi}(\mathbf{y})\hat{\psi}(\mathbf{x})\,,$$

Evolution Contour

$$
\begin{aligned}
\langle \hat{O}_H(t_1) \rangle &= \langle T^a \left[\exp\left(-i \int_{t_1}^{t_0} dt' \, \hat{H}_I^1(t') \right) \right] \hat{O}_I(t_1) \\
&\quad \times T^c \left[\exp\left(-i \int_{t_0}^{t_1} dt' \, \hat{H}_I^1(t') \right) \right] \rangle \\
&= \langle T \left[\exp\left(-i \, \oint_{t_0}^{t_0} dt' \, \hat{H}_I^1(t') \right) \hat{O}_I(t_1) \right] \rangle \, ,
\end{aligned}
$$



Expectation value expanded perturbatively in terms of V and noninteracting 1-ptcle Green's functions on the contour

$$
iG_0(\mathbf{x}, t, \mathbf{x}', t') = \langle T \left[\hat{\psi}_I(\mathbf{x}, t) \hat{\psi}_I^\dagger(\mathbf{x}', t') \right] \rangle
$$

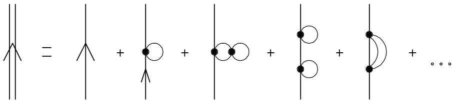
Single-Particle Evolution

Wigner function corresponds to a particular case of the Green's function on contour:

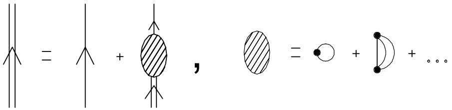
$$f(\mathbf{p};\mathbf{R},T) = \int d\mathbf{r}\, e^{-i\mathbf{p}\mathbf{r}}\,(\mp i)G^{<}(\mathbf{R}+\mathbf{r}/2,T,\mathbf{R}-\mathbf{r}/2,T)$$

If we find an equation for G, this will also be an equation for f.

Dyson eq. from perturbation expansion:



$G = G_0 + G_0\,\Sigma\,G$



Outcome of Evolution

Formal solution of the Dyson eq:

$$\mp i G^<(x,t;x',t') = \int \mathrm{d}x_1\,\mathrm{d}t_1\,\mathrm{d}x_1'\,\mathrm{d}t_1'\,G^+(x,t;x_1,t_1) \times (\mp i)\Sigma^<(x_1,t_1;x_1',t_1')\,G^-(x,t;x_1,t_1)$$

and

$$\mp i\Sigma^<(x,t;x',t') = \langle \hat{j}^\dagger(x',t')\,\hat{j}(x,t)\rangle_{irred}$$

where the source j is

$$\hat{j}(x,t) = \left[\hat{\psi}(\mathbf{x},t),\hat{H}^1\right]$$

Quasiparticle Limit

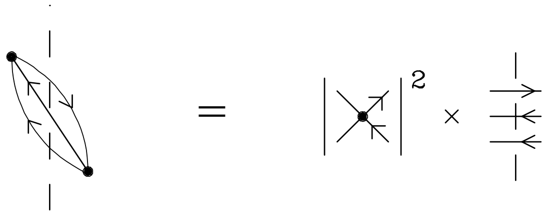
Under slow spatial and temporal changes in the system, the Green's function expressible in terms of the Wigner function f and 1-ptcle energy ϵ_p

$$\mp i G^{<}(x,t;x',t') \approx \int \mathrm{d}p\, f\Big(p; \frac{x+x'}{2}, \frac{t+t'}{2}\Big)\, \mathrm{e}^{i(p(x-x')-\epsilon_p(t-t'))}$$

Then also Boltzmann eq:

$$\frac{\partial f}{\partial t} + \frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{p}} \frac{\partial f}{\partial \mathbf{r}} - \frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{r}} \frac{\partial f}{\partial \mathbf{p}} = -i\Sigma^{<}(1-f) - i\Sigma^{>} f$$

$\mp i\Sigma^{<}$:



2-Particle Green's Function

Transport eq. for deuterons ($A = 2$) from the eq. for 2-ptcle Green's function??

$$iG_2^< \;=\; \langle \hat{\psi}^\dagger(\mathbf{x}_1'\,t')\,\hat{\psi}^\dagger(\mathbf{x}_2'\,t')\hat{\psi}(\mathbf{x}_2\,t)\,\hat{\psi}(\mathbf{x}_1\,t)\rangle$$

For the contour function:

$$G_2 = \mathcal{G} + \mathcal{G}\,v\,G_2$$

where $\mathcal{G}$ – irreducible part of G_2 (w/o two 1-ptcle lines connected by the potential v; anything else OK)

In terms of retarded Green's function $G_2^<$:

$$iG_2^< = \left(1 + v\,G_2^+\right)\, i\mathcal{G}^< \,\left(1 + v\,G_2^-\right)$$

Deuteron Quasiparticle Limit

In the limit of slow spatial and temporal changes, deuteron contribution to the 2-ptcle Green's function:

$$iG_2^< = \langle \hat{\psi}^\dagger(\mathbf{x}_1'\, t')\, \hat{\psi}^\dagger(\mathbf{x}_2'\, t')\hat{\psi}(\mathbf{x}_2\, t)\, \hat{\psi}(\mathbf{x}_1\, t)\rangle$$

$$\simeq \int d\mathbf{p}\, f_d(\mathbf{p}\,\mathbf{R}\,T)\, \phi_d^*(r')\, \phi_d(r)\, \mathrm{e}^{i\mathbf{p}\left(\frac{\mathbf{x}_1+\mathbf{x}_2}{2} - \frac{\mathbf{x}_1'+\mathbf{x}_2'}{2}\right)}\, \mathrm{e}^{-i\epsilon_d\,(t-t')}$$

$$+ \cdots ,$$

where $\mathbf{R} = \frac{1}{4}(\mathbf{x}_1 + \mathbf{x}_2 + \mathbf{x}_1' + \mathbf{x}_2')$, $\mathbf{r} = \mathbf{x}_1 - \mathbf{x}_2$

ϕ_d and f_d – internal wave function and cm Wigner function

$\cdots \equiv$ continuum

Transport eq from integral quantum eq of motion:

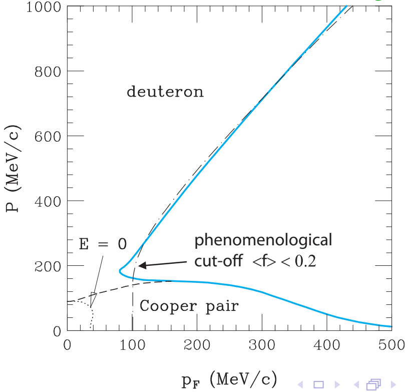
$$\frac{\partial f_d}{\partial T} + \frac{\partial \epsilon_d}{\partial \mathbf{p}} \frac{\partial f_d}{\partial \mathbf{R}} - \frac{\partial \epsilon_d}{\partial \mathbf{R}} \frac{\partial f_d}{\partial \mathbf{p}} = \mathcal{K}^< (1 + f_d) - \mathcal{K}^> f_d$$

Wave Equation

From Green's function eq, the equation for wavefunction:

$$(\epsilon_d(\mathbf{P}) - \epsilon_N(\mathbf{P}/2+\mathbf{p}) - \epsilon_N(\mathbf{P}/2-\mathbf{p}))\,\phi_d(\mathbf{p}) - (1 - f_N(\mathbf{P}/2+\mathbf{p}) - f_N(\mathbf{P}/2-\mathbf{p}))\int d\mathbf{p}'\, v(\mathbf{p}-\mathbf{p}')\,\phi_d(\mathbf{p}') = 0$$

In zero-temperature matter, discrete states lacking over a vast range of momenta



Cluster Production & Absorption

?? Production & absorption rates: $i\mathcal{K}^{\gtrless} = \phi^* \, v \, i\mathcal{G}^{\gtrless} \, v \, \phi$

Leading contribution

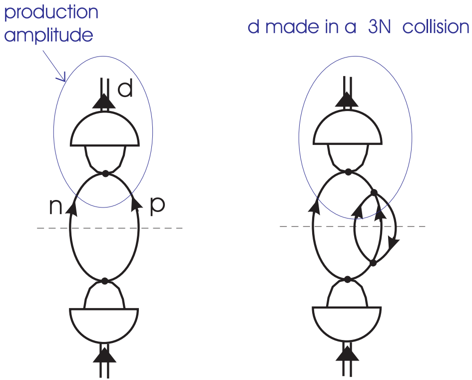
$$\mathcal{K}^< = \int d\mathbf{r} \, d\mathbf{r}' \, \phi_d^* \, v \, \langle \hat{\psi}^\dagger(\mathbf{x}_1' \, t') \, \hat{\psi}(\mathbf{x}_1 \, t) \rangle \, \langle \hat{\psi}^\dagger(\mathbf{x}_2' \, t') \, \hat{\psi}(\mathbf{x}_2 \, t) \rangle \, v \, \phi_d$$

Leading-order in the quasiparticle expansion: neutron & proton come together and make a deuteron.

If system approximately uniform and stationary, the process not allowed by energy-momentum conservation.

Process possible in a mean field varying in space, but, in nuclear case, the high-energy production rate low – tested in Glauber model.

3-Nucleon Collisions



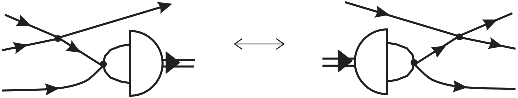
First correction to the pure 1-ptcle state, from a coupling to p-h excitations, yields a contribution to the d-production due to 3-nucleon collisions.

Still more nucleons involved in production of heavier clusters.

Deuteron Production

Detailed balance:

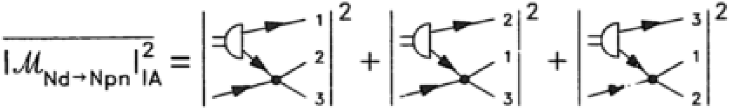
$$\overline{|\mathcal{M}^{npN \to Nd}|^2} = \overline{|\mathcal{M}^{Nd \to Nnp}|^2} \propto \mathrm{d}\sigma^{Nd \to Nnp}$$



Thus, production can be described in terms of breakup.

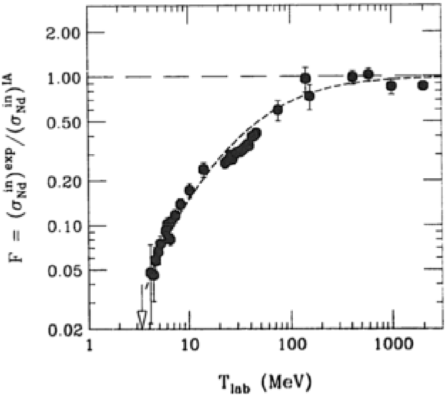
Problem: Breakup cross section only known over limited range of final states - Interpolation/extrapolation needed

Impulse approximation works at high incident energy



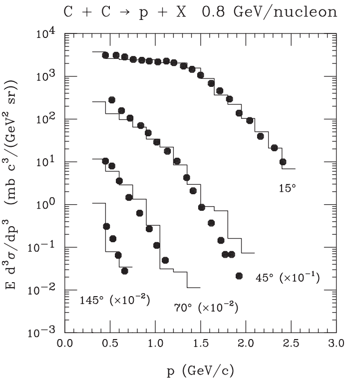
Renormalized Impulse Approximation

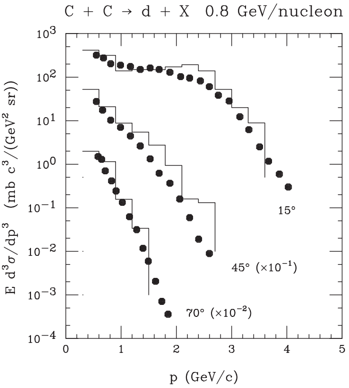
Renormalization factor for squared matrix element to get breakup cross section right as a function of energy



$$\mathrm{d}\sigma^{Nd \rightarrow Nnp} \propto F\, \sigma_{NN}\, |\phi_d(p)|^2$$

Single-Particle Spectra





proton & deuteron inclusive spectra

histograms: calculations using
$|\mathcal{M}^{npN\rightarrow Nd}|^2 = |\mathcal{M}^{Nd\rightarrow npN}|^2 \propto d\sigma_{Nd\rightarrow npN}$
and $\langle f \rangle < 0.2$ cut-off for deuterons

NSCL

$A = 3$ Particles + Tests

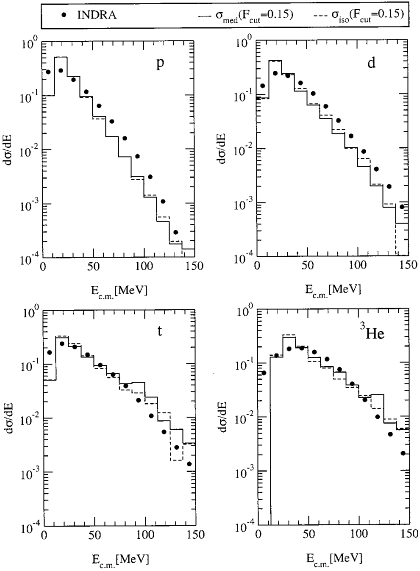
A=3-ptcles from 4N collisions

Christiane Kuhrts: solving finite-T Galitski-Feynman (GF) and modified (in-medium) Alt-Grassberger-Sandhas eqs

solid lines: finite-T GF for cross-sections and existence

dashed lines: free cross sections + $\langle f \rangle$ cut-off

symbols: INDRA data ^{129}Xe + ^{119}Sn at 50 MeV/nucleon



Cluster Yields and Entropy

Compression in central reactions accompanied by heating. Is the matter heated as much as expected for shock compression??

Experimental measure of entropy: relative cluster yields

$$E = T\,S - P\,V + \mu\,A \quad \Leftrightarrow \quad 3\,A\,T/2 \simeq T\,S - A\,T + \mu\,A$$

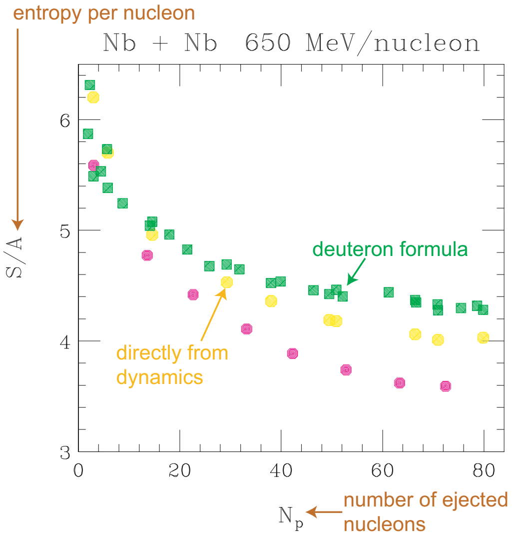
as at freeze-out ideal gas and then

$$\frac{S}{A} \simeq \frac{5}{2} - \frac{\mu}{T}$$

In equilibrium

$$\frac{N_d}{N_p} \propto \frac{\exp\left(\frac{2\mu}{T}\right)}{\exp\left(\frac{\mu}{T}\right)} \quad \Rightarrow \quad \frac{S}{A} \simeq 3.9 - \log\left(\frac{N_d}{N_p}\right)$$

Validity of Entropy Determination



Collective Expansion

Is expansion viscous or isentropic?? Is pressure carrying out work producing a collective expansion of matter?

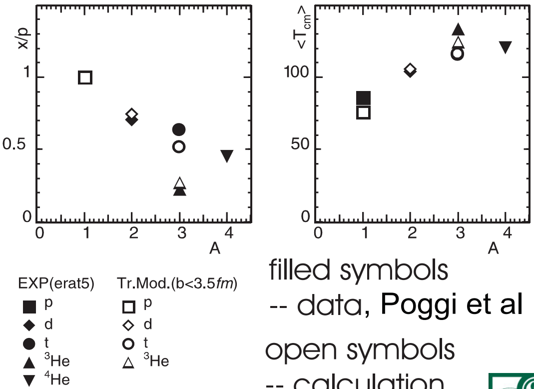
$$\langle E_x \rangle = \frac{3}{2}\, T + \frac{m_x \langle v^2 \rangle}{2} = \frac{3}{2}\, T + A_x \frac{m_N \langle v^2 \rangle}{2}$$

In isentropic expansion, average kinetic energy should increase with fragment mass.

Au+Au 250AMeV 60°<θ_{cm}<90°

yield ratio

av kin energy



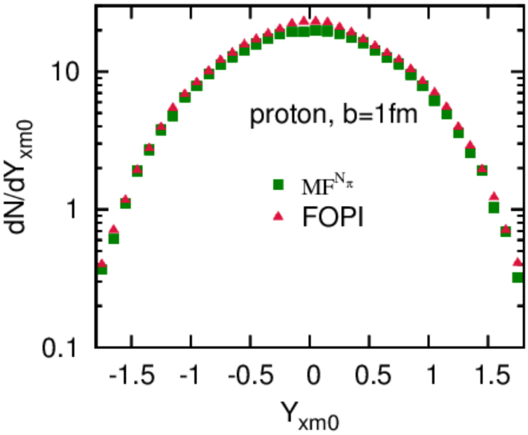
filled symbols -- data, Poggi et al

open symbols -- calculation

Energy increases linearly!

Head-On Au + Au (FOPI)

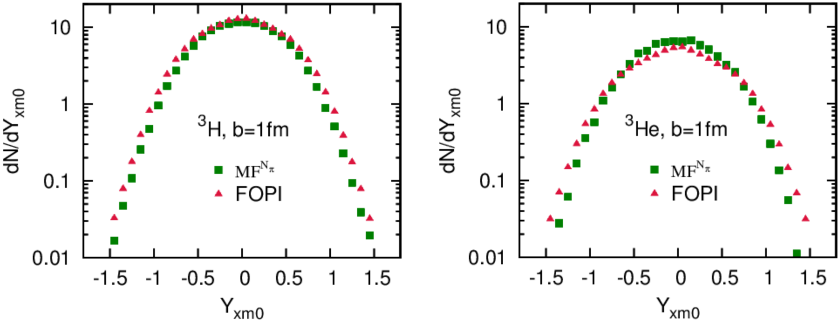
Rapidity Distribution



400 MeV/nucleon

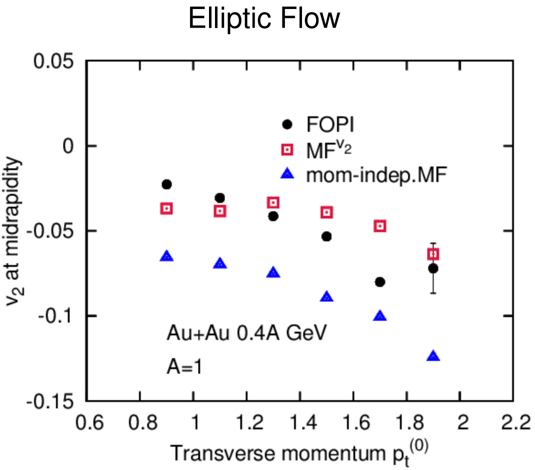
$A = 3$ in Head-On Au + Au (FOPI)

Rapidity Distributions



400 MeV/nucleon

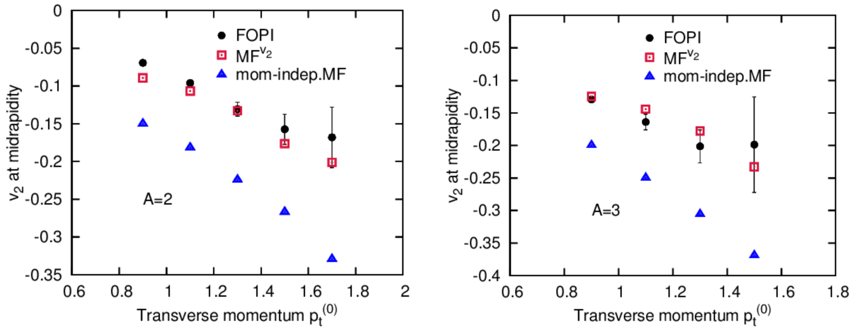
Semicentral Au + Au (FOPI)



400 MeV/nucleon

Semicentral Au + Au (FOPI)

Elliptic Flow



400 MeV/nucleon

Future of Light-Cluster Production in Transport

Production rate for cluster of mass A:

$$\mathcal{K}^{<}(\boldsymbol{p}_A) = \int \mathrm{d}\boldsymbol{p}_1' \ldots \mathrm{d}\boldsymbol{p}_{N'}' \, \mathrm{d}\boldsymbol{p}_1 \ldots \mathrm{d}\boldsymbol{p}_{N-1} \, |\mathcal{M}_{1'+\ldots+N' \to 1+\ldots+A}|^2$$
$$\times \, \delta(\boldsymbol{p}_1' + \ldots + \boldsymbol{p}_{N'}' - \boldsymbol{p}_1 - \ldots - \boldsymbol{p}_{N-1} - \boldsymbol{p}_A)$$
$$\times \, \delta(\epsilon_1' + \ldots + \epsilon_{N'}' - \epsilon_1 - \ldots - \epsilon_{N-1} - \epsilon_A)$$
$$\times \, f_{1'} \cdots f_{N'} \, (1 \pm f_1) \cdots (1 \pm f_{N-1})$$

Determination and sampling of separate $|\mathcal{M}|^2$ for every possible process... Potential nightmare! E.g.

$N + \Delta \leftrightarrow d + \pi$	AGS
$d + d + N \leftrightarrow \alpha + N$	etc.

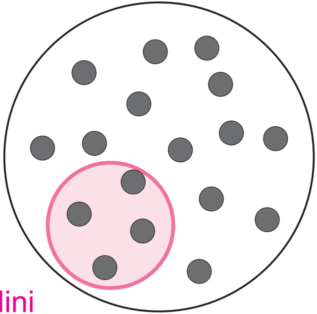
Any simplifications??

Simplified Matrix Elements

Batko, Randrup, Vetter
NPA536(92)786

$|\mathcal{M}|^2 \propto 1 \Rightarrow$ Mini Fireball

??Too much dissipation??



Mini
Compound
Nucleus

Generalized coalescence:

$$|\mathcal{M}|^2 \propto \theta(p_0 - |\frac{\boldsymbol{p}_A}{A} - \boldsymbol{p}'_1|) \cdots \theta(p_0 - |\frac{\boldsymbol{p}_A}{A} - \boldsymbol{p}'_{N'}|)$$

Branching??

Automation needed!

Conclusions

- Real-time many-body theory provides fundamentals for production of clusters in transport theory
- Few-body collisions or rapidly changing mean-field conditions are needed to spur cluster production
- Detailed balance must be obeyed for thermodynamic consistency
- Breakup data yield production rates in collisions
- Clusters emphasize collective motion and provide information on phase-space densities and entropy
- Production description needs to be simplified in extending reach of theory.

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