The nuclear contact and the photoabsorption cross section

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האוניברסיטה העברית בירושלים

The Hebrew University of Jerusalem

The Team

Ronen Weiss, Betzalel Bazak

Wine tasting, new year's eve Tzora (2014).

Outline

Short range interaction

We start with 2-body Schrodinger ...

$$\left[-\frac{\hbar^2}{m}\nabla^2 + V(\boldsymbol{r})\right]\psi = E\psi$$

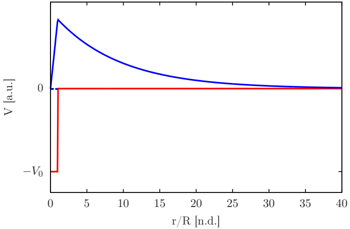
At vanishing distance, $r \longrightarrow 0$

- The energy becomes negligible $E \ll \hbar^2/mr^2$
- The w.f. ψ assumes an asymptotic **energy independent** form φ

$$\left[-\frac{\hbar^2}{m}\nabla^2 + V(r)\right]\varphi(\boldsymbol{r}) = 0 \qquad r\varphi(r) = 0|_{r=0}$$

- Valid for any A-body system.

The Limit of vanishing energy



- Short range: only the s-wave survives
- Most of the wave function is outside the range of the potential.
- The wave funcion depends on a single length scale - the **scattering length a**
- The potential can be replaced by the boundary condition `[Bethe-Peierles]`

$$\boxed{\left.\frac{d\log(r\psi)}{dr}\right|_0 = \left.\frac{u'}{u}\right|_0 = -\frac{1}{a}}$$

- Valid for **any** short range potential.

The Contact - Tan's Relations

A system of spin up - spin down fermions

Tan relations connects the contact C with:

1. **Tail of momentum distribution** $|a|^{-1} \ll k \ll r_0^{-1}$

$$n_\sigma(\boldsymbol{k}) \longrightarrow \frac{C}{k^4}$$

2. **The energy relation**

$$T + U = \sum_\sigma \int \frac{d\boldsymbol{k}}{(2\pi)^3} \frac{\hbar^2 k^2}{2m} \left(n_\sigma(\boldsymbol{k}) - \frac{C}{k^4} \right) + \frac{\hbar^2}{4\pi m a} C$$

3. **Adiabatic relation**

$$\frac{dE}{d1/a} = -\frac{\hbar^2}{4\pi m} C$$

4. ...

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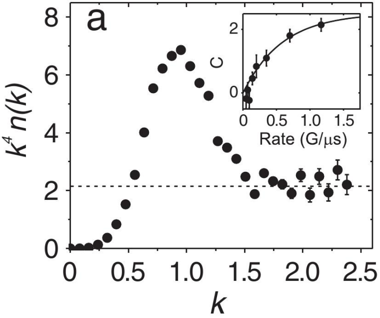
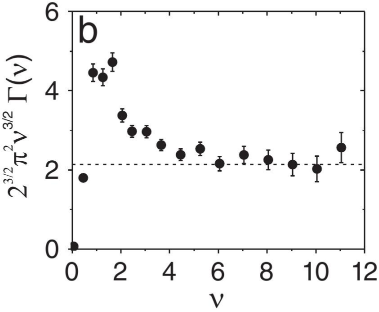
Momentum Distribution



RF line shape



Verification of Universal Relations in a Strongly Interacting Fermi Gas
J. T. Stewart, J. P. Gaebler, T. E. Drake, and D. S. Jin, Phys. Rev. Lett. 104, 235301 (2010)

The short range factorization

[Tan, Braatan & Platter, Werner & Castin,...]

- The interaction is represented through the boundary condition

$$\left[\partial \log r_{ij}\Psi/\partial r_{ij}\right]_{r_{ij}=0} = -1/a$$

- Thus, when two particles approach each other

$$\Psi \xrightarrow[r_{ij}\to 0]{} (1/r_{ij} - 1/a) A_{ij}(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j})$$

- The contact C represents the probability of finding an interacting pair within the system

$$C \equiv 16\pi^2 \sum_{ij} \langle A_{ij}|A_{ij}\rangle$$

- Where

$$\langle A_{ij}|A_{ij}\rangle = \int \prod_{k\neq i,j} d\boldsymbol{r}_k \, d\boldsymbol{R}_{ij} \, A^{\dagger}_{ij}\left(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}\right) \cdot A_{ij}\left(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}\right)$$

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The Nuclear Contact(s)

Scales

- NN interaction range $\mu_\pi^{-1} = \hbar/m_\pi c \approx 1.4$ fm
- NN scattering lengths $a_t = 5.4$ fm , $a_s \approx 20$ fm thus $\mu_\pi |a| \geq 3.8$
- The nuclear radius is $R \approx 1.2 A^{1/3}$ fm
- The interparticle distance $d \approx 2.4$ fm thus $\mu_\pi d \approx 1.7$

Conclusions

- The Tan conditions are not strictly applicable in nuclear physics.
- The interaction range is significant.
- There could be different interaction channels - not only s-wave.
- Therefore, we need replace the asymptotic form

$$\Psi \xrightarrow[r_{ij}\to 0]{} (1/r_{ij} - 1/a) A_{ij}(\mathbf{R}_{ij}, \{\mathbf{r}_k\}_{k\neq i,j})$$

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$$\Psi \xrightarrow[r_{ij} \to 0]{} \sum_\alpha \varphi_\alpha(\boldsymbol{r}_{ij}) A_{ij}^\alpha(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k \neq i,j})$$

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The Nuclear Contact(s)

- In nuclear physics we have **3** possible particle pairs

$$ij = \{pp,\ nn,\ pn\}$$

- For each pair there are different channels

$$\alpha = (s, \ell)jm$$

- For each pair we define the contact matrix

$$C_{ij}^{\alpha\beta} \equiv 16\pi^2 N_{ij} \langle A_{ij}^{\alpha} | A_{ij}^{\beta} \rangle$$

- For $\ell = 0$ we need consider only **4** contacts

$$P = \{(pp)_{S=0}, (nn)_{S=0}, (np)_{S=0}, (np)_{S=1}\}$$

- Adding isospin symmetry the number of contacts is reduced to **2**,

$$C_s \longleftrightarrow \{(pp)_{S=0}, (nn)_{S=0}, (np)_{S=0}\}$$
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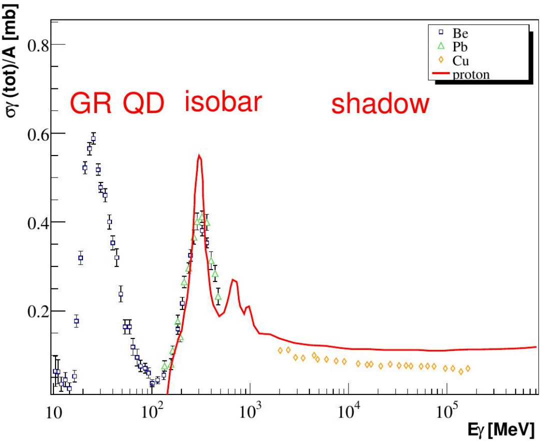
The nuclear contact relations/applications

1. **The nuclear photoabsorption cross-section - The quasi-deutron model**
   R. Weiss, B. Bazak, N. Barnea, PRL **114**, 012501 (2015)
2. **The 1-body and 2-body momentum distributions**
   R. Weiss, B. Bazak, N. Barnea, PRC **92**, 054311 (2015)
3. **Generalized treatment of the photoabsorption cross-section**
   R. Weiss, B. Bazak, N. Barnea, EPJA (2016)
4. **Electron scattering**
   O. Hen et al., PRC **92**, 045205 (2015)
5. **Symmetry energy**
   BJ Cai, BA Li, PRC **93**, 014619 (2016)
6. ...

Photoabsorption of Nuclei



R. Al Jebali, PhD Thesis, U. Glasgow (2013)

Photoabsorption of Nuclei

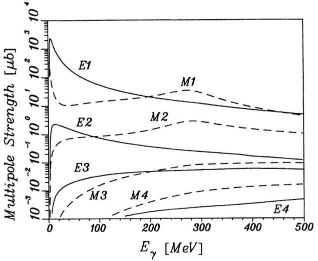
Up to $\hbar\omega \approx 200$ MeV the cross-section $\sigma_A(\omega)$ is dominated by the **dipole** operator

$$\sigma_A(\omega) = 4\pi^2 \alpha \omega R(\omega)$$

R is the response function

$$R(\omega) = \sum\!\!\!\!\!\!\int_f \left| \langle \Psi_f \left| \epsilon \cdot \hat{D} \right| \Psi_0 \rangle \right|^2 \delta(E_f - E_0 - \omega)$$

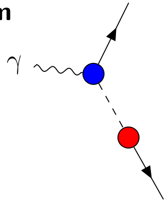
The Deuteron cross-section



H. Arenhovel, and M. Sanzone, Few-Body Syst. (1991).

The Quasi-Deuteron picture

J. S. Levinger
"The high energy nuclear photoeffect", Phys. Rev. **84**, 43 (1951).



- The photon carries **energy** but (almost) **no momentum**
- It is captured by a single **proton.**
- The proton is ejected without any FSI.
- Momentum conservation $\Rightarrow$ a nucleon with opposite momentum must be ejected $\boldsymbol{k} \approx -\boldsymbol{k}_p$.
- Dipole dominance $\Rightarrow$ this partner must be a **neutron**.
- $\hbar\omega \longrightarrow \infty \Rightarrow \sigma(\omega)$ depends on a **universal** short range pn wave-function.
- The resulting cross-section is given by

$$\sigma_A(\omega) = L\frac{NZ}{A}\sigma_d(\omega)$$

- L is known as the Levinger Constant

The Quasi-Deuteron in the zero range model

The initial and final wave functions Ψ_0, Ψ_f

- When a **pn** pair are close together Ψ_0 is factorized into

$$\Psi_0(\mathbf{r}_1, ..., \mathbf{r}_A) = \sum_\alpha \varphi_\alpha(\boldsymbol{r}_{pn}) A^\alpha_{pn}\left(\boldsymbol{R}_{pn}, \{\boldsymbol{r}_j\}_{j\neq p,n}\right) + \ldots$$

- A pn pair is emitted with

$$\mathbf{k}_n \approx -\mathbf{k}_p \equiv \mathbf{k}$$

- They can form either an $S = 0$ or an $S = 1$ spin state
- The $A - 2$ spectators are frozen
- For s-wave dominance, α is either singlet or triplet.

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The Quasi-Deuteron

The transition matrix element

- Considering now the transition matrix element
- Comparing to the deutron matrix element
- Assuming s-wave dominance, and $\varphi_\alpha \approx \varphi_d$,

$$\langle \Psi_f^\alpha | \boldsymbol{\epsilon} \cdot \hat{\boldsymbol{D}} | \Psi_0 \rangle \approx \sqrt{\frac{C_\alpha a_t}{8\pi}} \langle \psi_{d,f} | \boldsymbol{\epsilon} \cdot \hat{\boldsymbol{D}} | \psi_{d,0} \rangle$$

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$$\sigma_A(\omega) = \frac{a_t}{4\pi}\frac{(C_s + C_t)}{2}\sigma_d(\omega) = \frac{a_t}{4\pi}\bar{C}_{pn}\sigma_d(\omega)$$

The Cross-Section

Back to Levinger

- The cross-section of **any** nucleus is proportional to the dueteron cross-section $\sigma_d(\omega)$

$$\sigma_A(\omega) = \frac{a_t}{4\pi}\bar{C}_{pn}\sigma_d(\omega)$$

- Comparing this to Levinger formula

$$\sigma_A(\omega) = L\frac{NZ}{A}\sigma_d(\omega)$$

- We see that the Levinger constant L is a close relative of the nuclear contacts,

$$\boxed{L = \frac{a_t}{4\pi}\frac{A}{NZ}\bar{C}_{pn}}$$

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The Levinger Constant and the Nuclear Contact

- In his original paper Levinger has estimated $L = 6.4$
- In view of the available data we can conclude

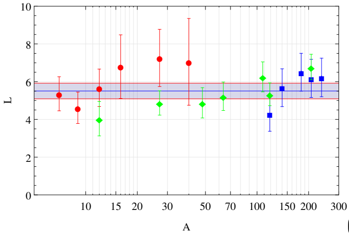
$$L = 5.50 \pm 0.21$$

- $N = Z = A/2$
- Normalize by the Fermi monemtum

$$\frac{\bar{C}_{pn}}{k_F A} = \frac{\pi}{k_F a_t} (5.50 \pm 0.21)$$

- $1/k_F a_t \approx 0.15$

$$\bar{C}_{pn}/k_F A \approx 2.55 \pm 0.10$$



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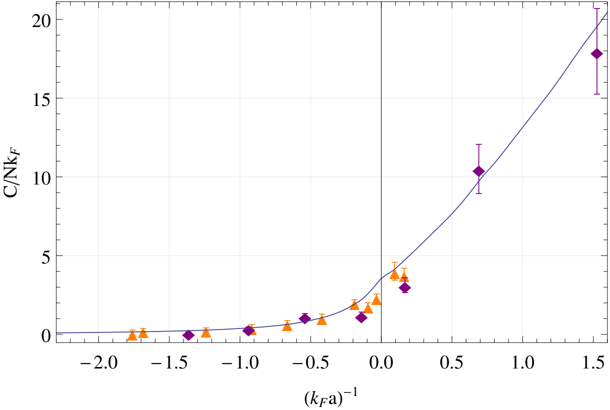
A. P. Tavares and M. L. Terranova, J. Phys. G **18**, 521 (1992).

Comparison to Atomic Physics



Atomic data - J. T. Stewart, et al., PRL 104, 235301 (2010)
G.B. Partridge, et al., PRL 95, 020404 (2005)

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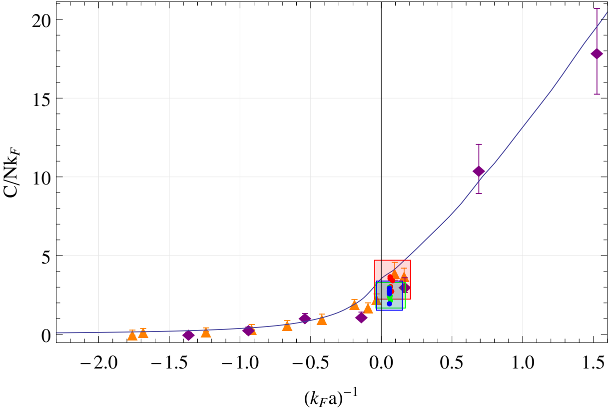

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Momentum distributions

1-body neutron and proton momentum distributions

$$n_n(\boldsymbol{k}), \; n_p(\boldsymbol{k})$$

2-body nn, np, pp momentum distributions

$$F_{nn}(\boldsymbol{k}), \; F_{pn}(\boldsymbol{k}), \; F_{pp}(\boldsymbol{k})$$

Momentum distributions

The proton momentum distribution

$$n_p^{JM}(\boldsymbol{k}) = Z \int \prod_{l \neq p} \frac{d^3 k_l}{(2\pi)^3} \left| \tilde{\Psi}(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_p = \boldsymbol{k}, \ldots, \boldsymbol{k}_A) \right|^2$$

Using the asymptotic wave-function

$$\Psi \xrightarrow[r_{ij} \to 0]{} \sum_\alpha \varphi_\alpha(\boldsymbol{r}_{ij}) A_{ij}^\alpha(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k \neq i,j})$$

we get

$$\begin{aligned} n_p(\boldsymbol{k}) = & \frac{1}{2J+1} \sum_{\alpha,\beta} \tilde{\varphi}_{pp}^{\alpha\dagger}(\boldsymbol{k}) \tilde{\varphi}_{pp}^{\beta}(\boldsymbol{k}) Z(Z-1) \langle A_{pp}^\alpha | A_{pp}^\beta \rangle \\ & + \frac{1}{2J+1} \sum_{\alpha,\beta} \tilde{\varphi}_{pn}^{\alpha\dagger}(\boldsymbol{k}) \tilde{\varphi}_{pn}^{\beta}(\boldsymbol{k}) NZ \langle A_{pn}^\alpha | A_{pn}^\beta \rangle \end{aligned}$$

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$$n_p(\boldsymbol{k}) = \sum_{\alpha,\beta} \underbrace{\tilde{\varphi}_{pp}^{\alpha\dagger}(\boldsymbol{k}) \tilde{\varphi}_{pp}^{\beta}(\boldsymbol{k})}_{\text{universal 2b}} \frac{2C_{pp}^{\alpha\beta}}{16\pi^2} + \sum_{\alpha,\beta} \underbrace{\tilde{\varphi}_{pn}^{\alpha\dagger}(\boldsymbol{k}) \tilde{\varphi}_{pn}^{\beta}(\boldsymbol{k})}_{\text{universal 2b}} \frac{C_{pn}^{\alpha\beta}}{16\pi^2}$$

Momentum distributions

Similarly

$$F_{ij}(\boldsymbol{k}) = \sum_{\alpha,\beta} \tilde{\varphi}_{ij}^{\alpha\dagger}(\boldsymbol{k})\tilde{\varphi}_{ij}^{\beta}(\boldsymbol{k})\frac{C_{ij}^{\alpha\beta}}{16\pi^2}$$

comparing with

$$n_p(\boldsymbol{k}) = \sum_{\alpha,\beta} \tilde{\varphi}_{pp}^{\alpha\dagger}(\boldsymbol{k})\tilde{\varphi}_{pp}^{\beta}(\boldsymbol{k})\frac{2C_{pp}^{\alpha\beta}}{16\pi^2} + \sum_{\alpha,\beta} \tilde{\varphi}_{pn}^{\alpha\dagger}(\boldsymbol{k})\tilde{\varphi}_{pn}^{\beta}(\boldsymbol{k})\frac{C_{pn}^{\alpha\beta}}{16\pi^2}$$

the **asymptotic** relations between the 1-body and 2-body momentum distributions **follows**

$$n_p(\boldsymbol{k}) \xrightarrow[k\to\infty]{} 2F_{pp}(\boldsymbol{k}) + F_{pn}(\boldsymbol{k})$$

$$n_n(\boldsymbol{k}) \xrightarrow[k\to\infty]{} 2F_{nn}(\boldsymbol{k}) + F_{pn}(\boldsymbol{k})$$

These relations hold regardless of the specific form of φ_α and without any assumptions on $\{\alpha\}$

Momentum distributions

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$$F_{ij}(\boldsymbol{k}) = \sum_{\alpha,\beta} \tilde{\varphi}_{ij}^{\alpha\dagger}(\boldsymbol{k})\tilde{\varphi}_{ij}^{\beta}(\boldsymbol{k})\frac{C_{ij}^{\alpha\beta}}{16\pi^2}$$

comparing with

$$n_p(\boldsymbol{k}) = \sum_{\alpha,\beta} \tilde{\varphi}_{pp}^{\alpha\dagger}(\boldsymbol{k})\tilde{\varphi}_{pp}^{\beta}(\boldsymbol{k})\frac{2C_{pp}^{\alpha\beta}}{16\pi^2} + \sum_{\alpha,\beta} \tilde{\varphi}_{pn}^{\alpha\dagger}(\boldsymbol{k})\tilde{\varphi}_{pn}^{\beta}(\boldsymbol{k})\frac{C_{pn}^{\alpha\beta}}{16\pi^2}$$

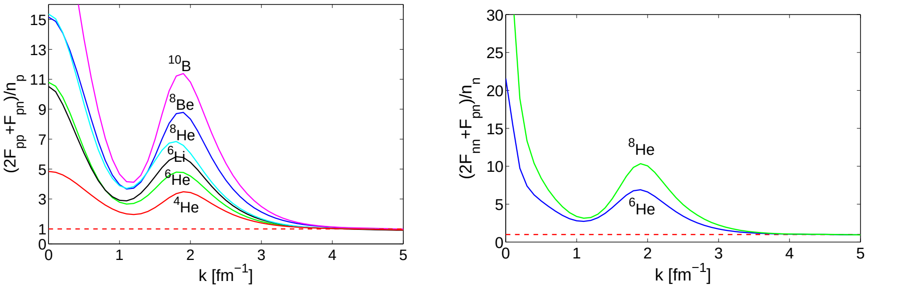
the **asymptotic** relations between the 1-body and 2-body momentum distributions **follows**

$$n_p(\boldsymbol{k}) \xrightarrow[k\to\infty]{} 2F_{pp}(\boldsymbol{k}) + F_{pn}(\boldsymbol{k})$$

$$n_n(\boldsymbol{k}) \xrightarrow[k\to\infty]{} 2F_{nn}(\boldsymbol{k}) + F_{pn}(\boldsymbol{k})$$

These relations hold regardless of the specific form of φ_α and without any assumptions on $\{\alpha\}$

Numerical verification of the momentum relations



VMC calculations of light nuclei

- Wiringa et. al. published a series of 1-body, 2-body momentum distributions
  R. B. Wiringa, *et al.*, PRC **89**, 024305 (2014)
- The data is available for nuclei in the range $2 \leq A \leq 10$.
- The calculations were done with the VMC method
- For symmetric nuclei $n_p = n_n$

The momentum relations holds for $4\ \mathrm{fm}^{-1} \leq k \leq 5\ \mathrm{fm}^{-1}$

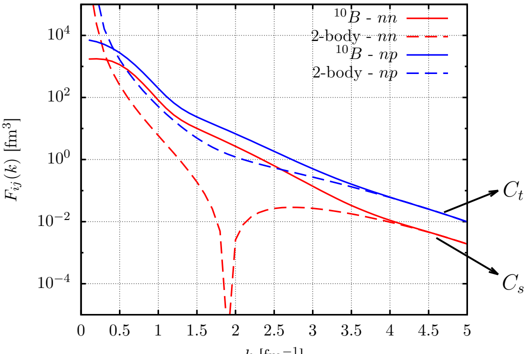
Extracting the contacts

We can extract the leading contacts using the asymptotic 2-body momentum distributions

$$F_{ij}(\boldsymbol{k}) = \sum_{\alpha} \underbrace{|\tilde{\varphi}_{ij}^{\alpha}(\boldsymbol{k})|^2}_{2-\text{body}} \frac{C_{ij}^{\alpha\alpha}}{16\pi^2}$$

For non-deuteron channels the 2-body functions are $E = 0$ scattering w.f.

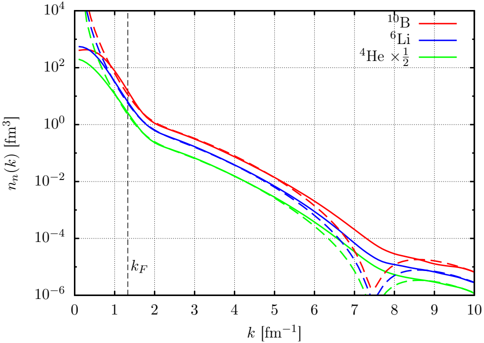
Example - VMC calculations of ^{10}B



The resulting 1-body momentum distribution is given by

$$n_n^\infty(\boldsymbol{k}) = \sum_\alpha |\tilde{\varphi}_{ij}^\alpha(\boldsymbol{k})|^2 \frac{C_{np}^{\alpha\alpha}}{16\pi^2} + 2\sum_\alpha |\tilde{\varphi}_{ij}^\alpha(\boldsymbol{k})|^2 \frac{C_{nn}^{\alpha\alpha}}{16\pi^2}$$

Comparing with the VMC data



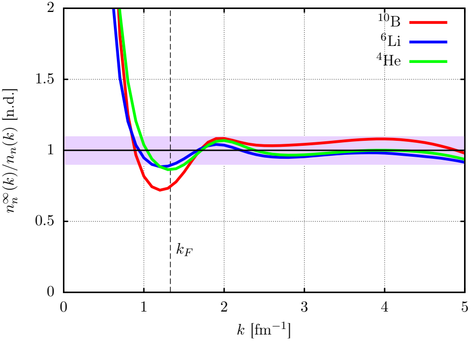
Surprisingly, the agreement holds for $k_F \le k \le 6\ \mathrm{fm}^{-1}$

Further numerical verifications

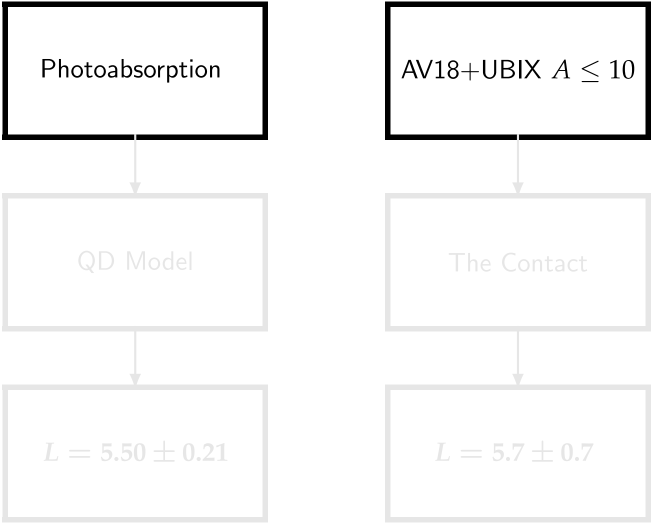
The resulting 1-body momentum distribution is given by

$$n_n^\infty(\boldsymbol{k}) = \sum_\alpha |\tilde{\varphi}_{ij}^\alpha(\boldsymbol{k})|^2 \frac{C_{np}^{\alpha\alpha}}{16\pi^2} + 2\sum_\alpha |\tilde{\varphi}_{ij}^\alpha(\boldsymbol{k})|^2 \frac{C_{nn}^{\alpha\alpha}}{16\pi^2}$$

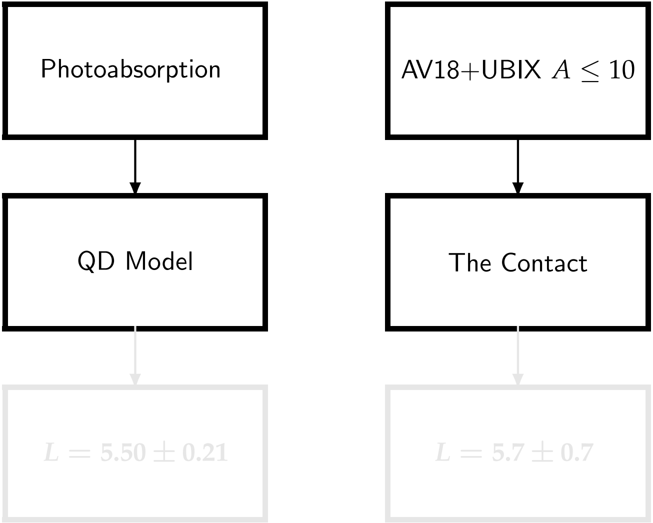
Comparing with the VMC data



Surprisingly, the agreement holds for $k_F \le k \le 6 \text{ fm}^{-1}$



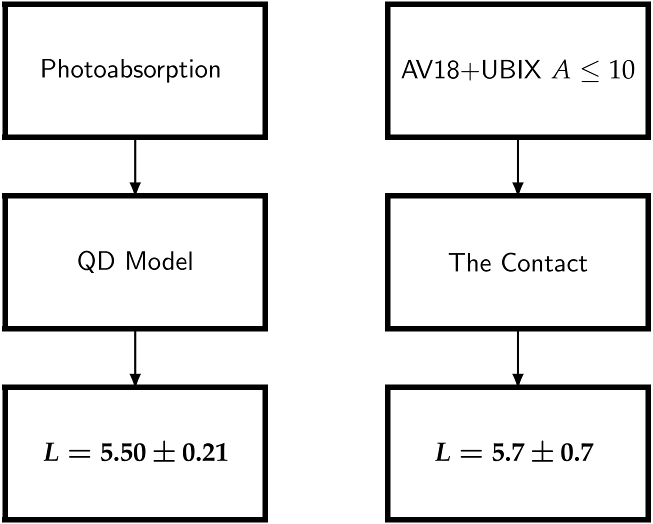
Photoabsorption
AV18+UBIX $A \leq 10$
QD Model
The Contact
$L = 5.50 \pm 0.21$
$L = 5.7 \pm 0.7$



Photoabsorption
AV18+UBIX $A \leq 10$
QD Model
The Contact
$L = 5.50 \pm 0.21$
$L = 5.7 \pm 0.7$

The Levinger Constant again

Theory and Experiment



Summary and Conclusions

Generalizing Tan's contact to nuclear physics

- Rederived the Quasi-Deuteron model utilizing the zero-range model.
- The Levinger constant and the nuclear contacts are close relatives.
- $\bar{C}_{pn}$ was deduced using previous evaluations of Levinger's constant.
- $\bar{C}_{pn}/A$ seems to be constant throughout the nuclear chart.
- Derived momentum relations for nuclear physics.
- The 1-body momentum distribution seems to be dominated (upto 10%) by 2-body correlations, from k_F up.

Summary and Conclusions (II)

Outlook

- Electron scattering.
- Neutrino scattering.
- ...
- ...
- ...

`We have only started to explore the usefulness of the contact formalism in nuclear physics !`