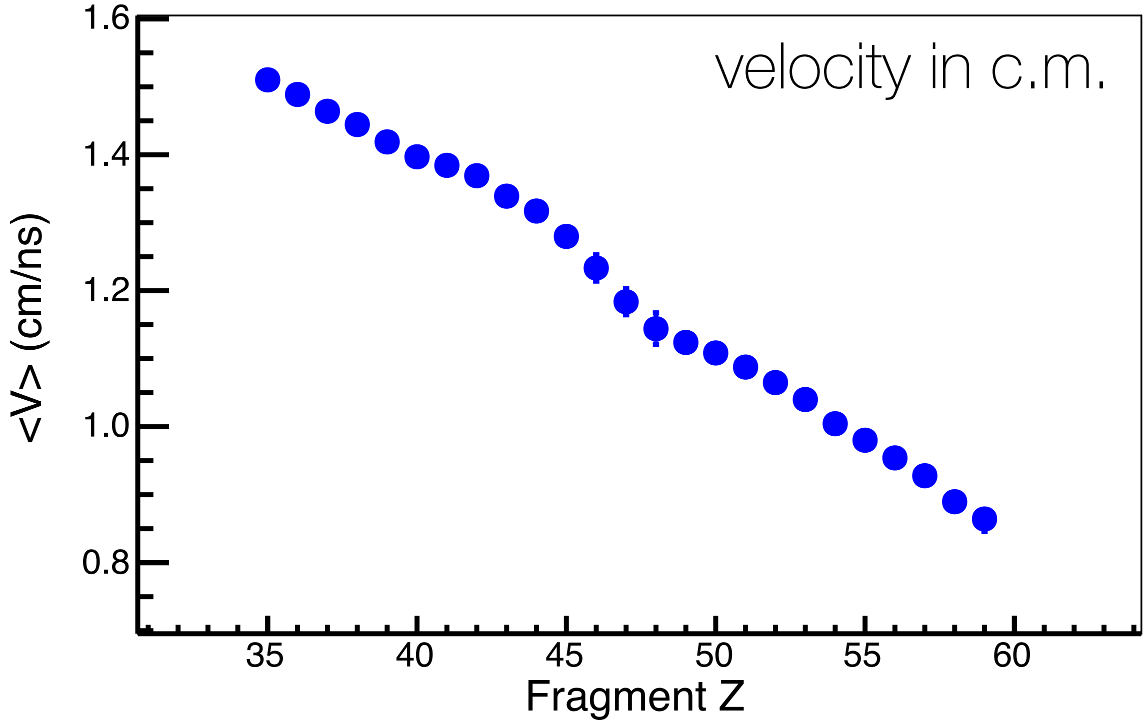
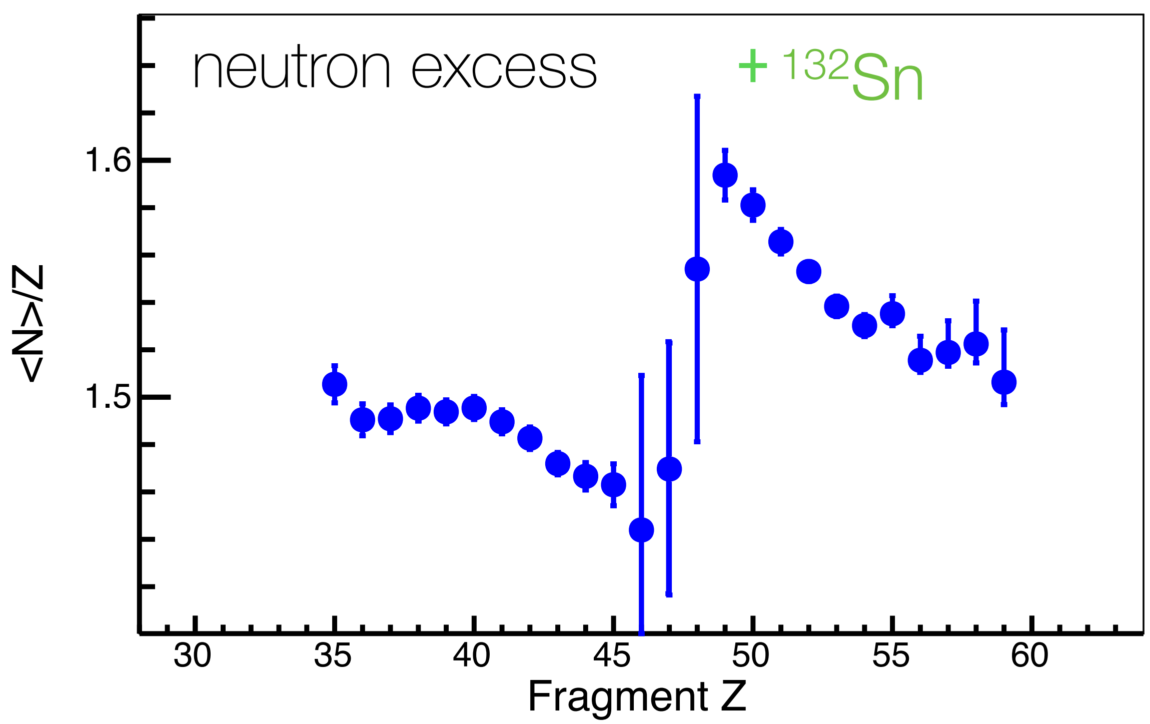
Energetics and deformation at scission

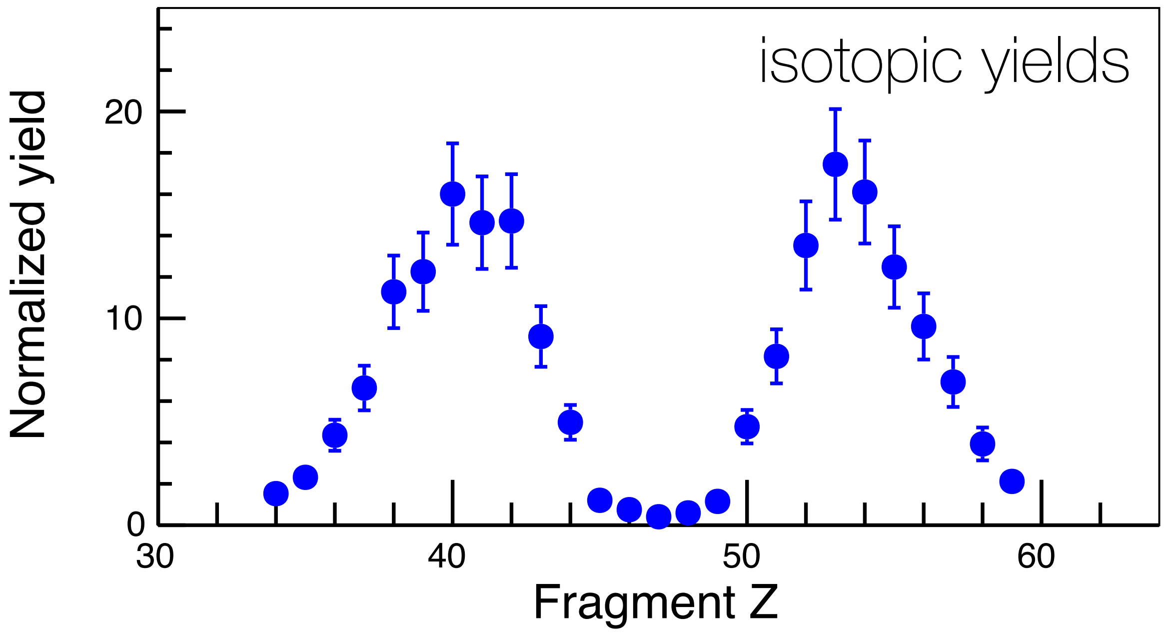
M. Caamaño
(U. Santiago de Compostela, Spain)

FUSTIPEN 2016

Experimental observables





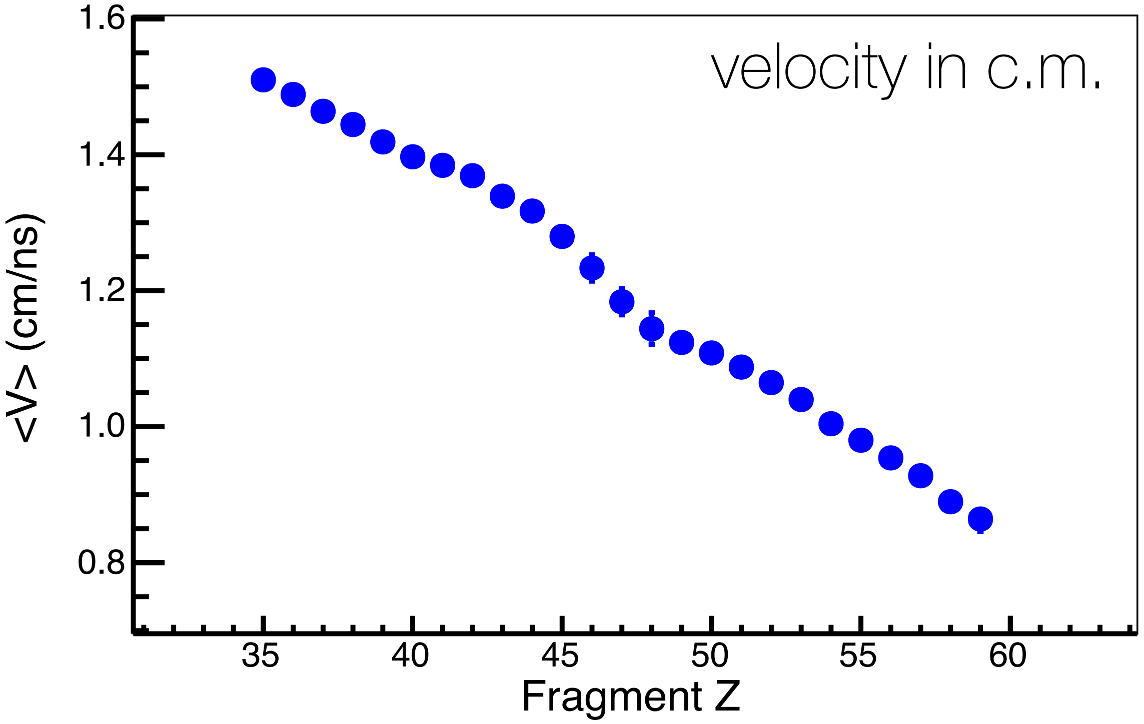


M. Caamaño, F. Farget et al., PRC 88, 024605 (2013)

Can we go further
with simple assumptions?

We focus on ^{240}**Pu** (<E*>= 9 MeV)

Experimental observables. Back to scission



$$\boxed{\frac{\langle A^{*}\rangle_{Z1}}{\langle A^{*}\rangle_{Z2}} \approx \frac{\langle \beta\gamma\rangle_{Z2}}{\langle \beta\gamma\rangle_{Z1}}}$$

C. Britt *et al.* NIM 24 (1963) 13

masses

$$\langle A^{*}\rangle_{Z1}\left(1+\frac{\langle \beta\gamma\rangle_{Z1}}{\langle \beta\gamma\rangle_{Z2}}\right) = A_{\mathrm{FS}} - \langle \nu^{\mathrm{pre}}\rangle$$

GEF code:
K.-H. Schmidt et al.
NDS 131,107 (2016)

total kinetic energy

$$TKE^{*} = u\langle A_1^{*}\rangle\left(\langle \gamma_1^{*}\rangle - 1\right) + u\langle A_2^{*}\rangle\left(\langle \gamma_2^{*}\rangle - 1\right)$$

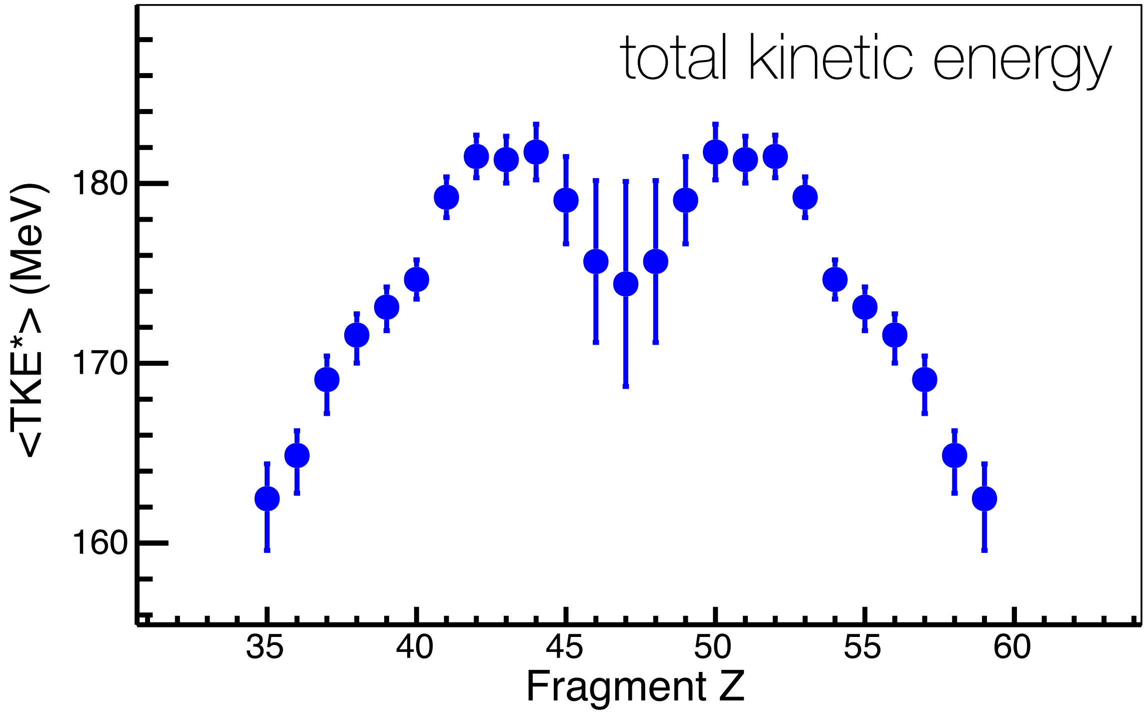
neutron multiplicity

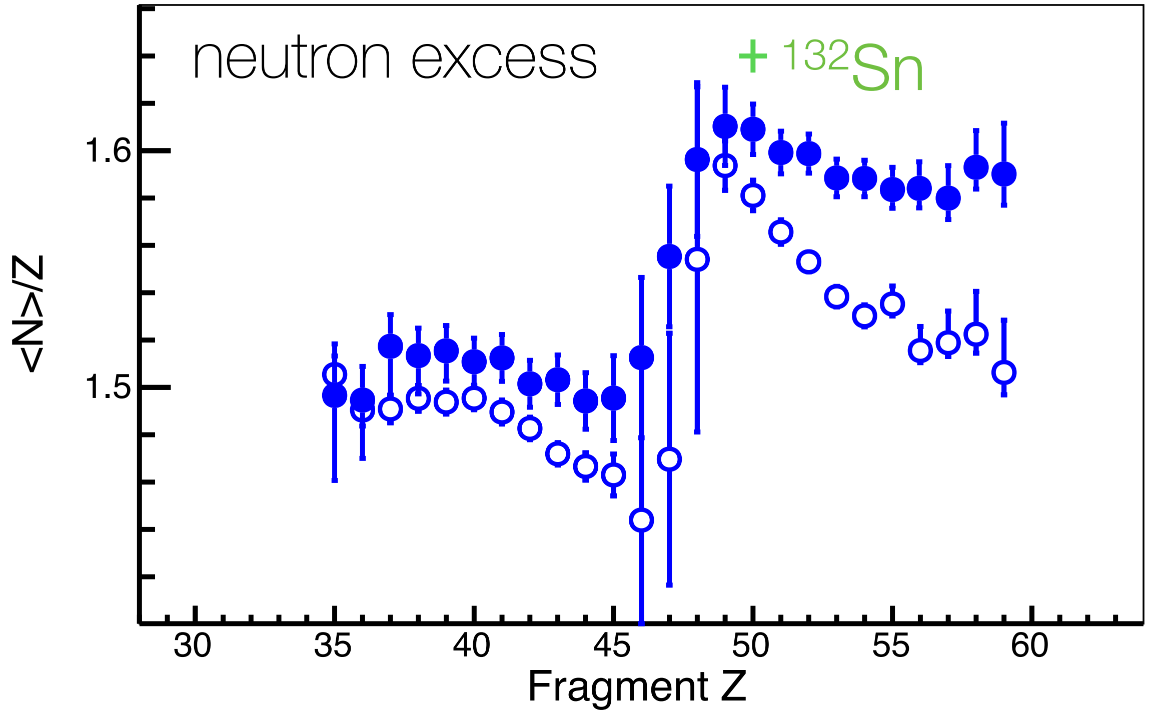
$$\langle \nu\rangle_Z = \langle A^{*}\rangle_Z - \langle A^{measured}\rangle_Z$$

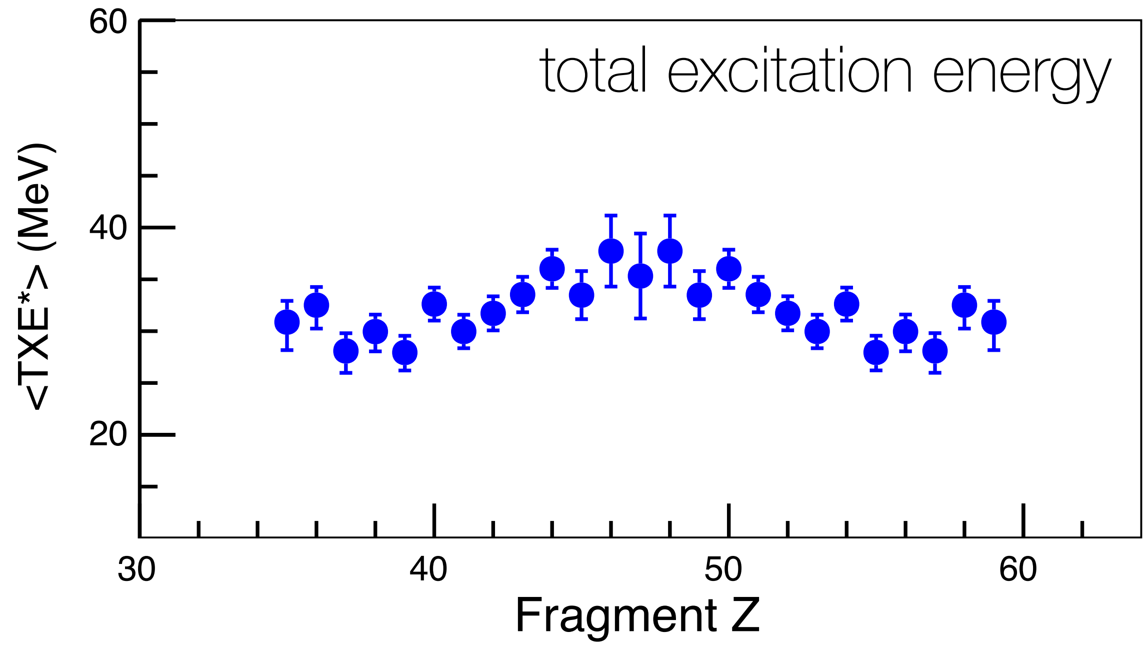
total excitation energy

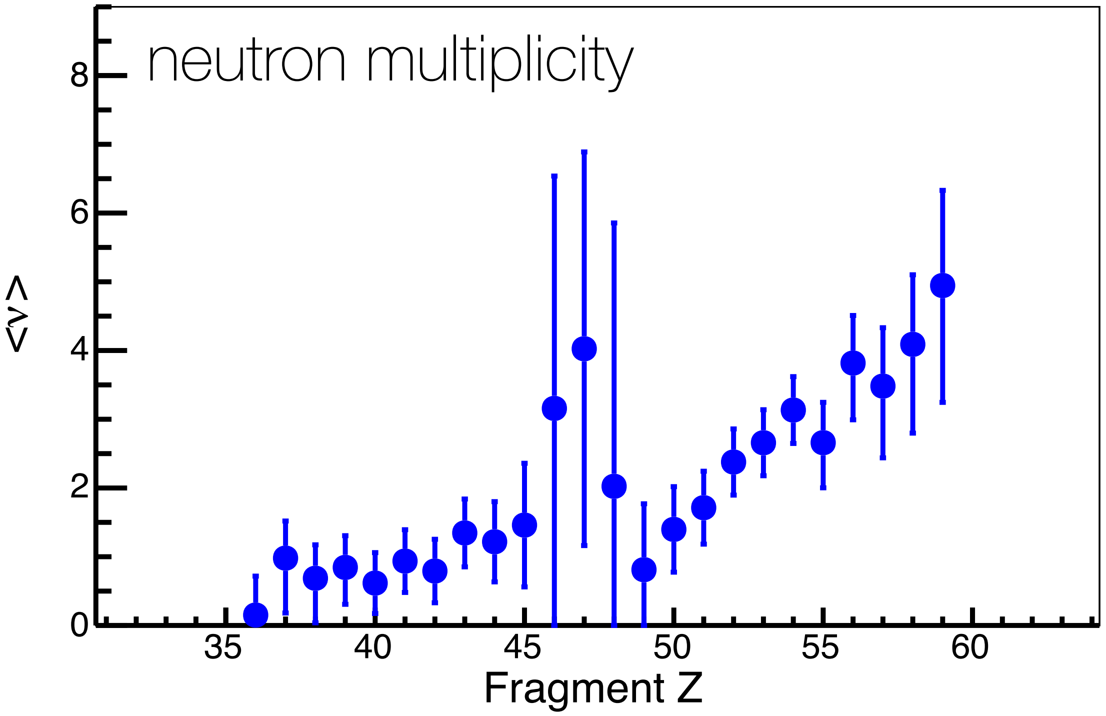
$$\langle TXE^{*}\rangle = [M_{\mathrm{FS}} + E^{*}_{\mathrm{FS}}] - [\langle M_1^{*}\rangle + \langle M_2^{*}\rangle + \langle TKE^{*}\rangle]$$

Experimental observables at scission



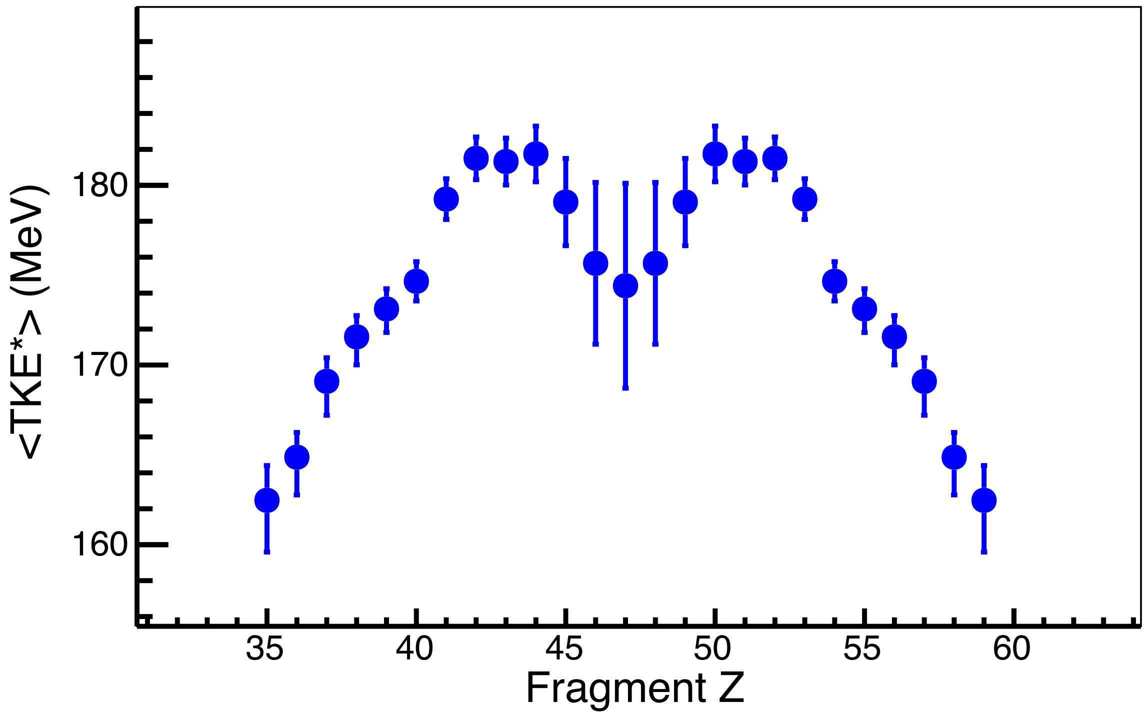


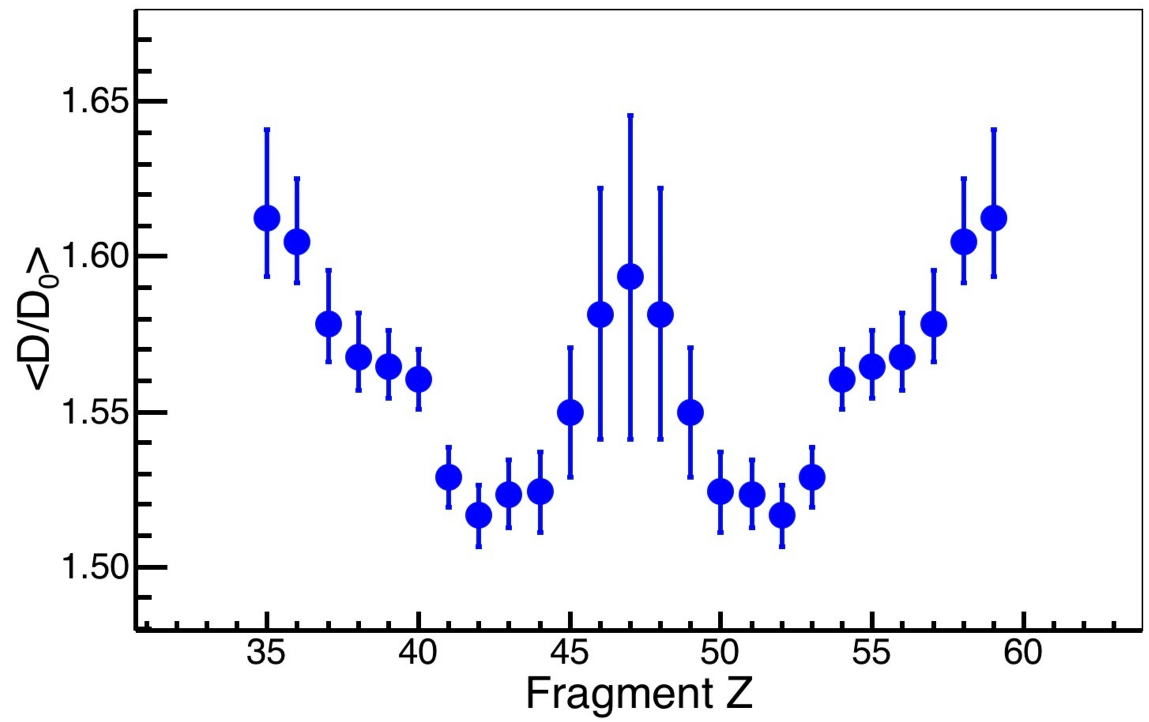


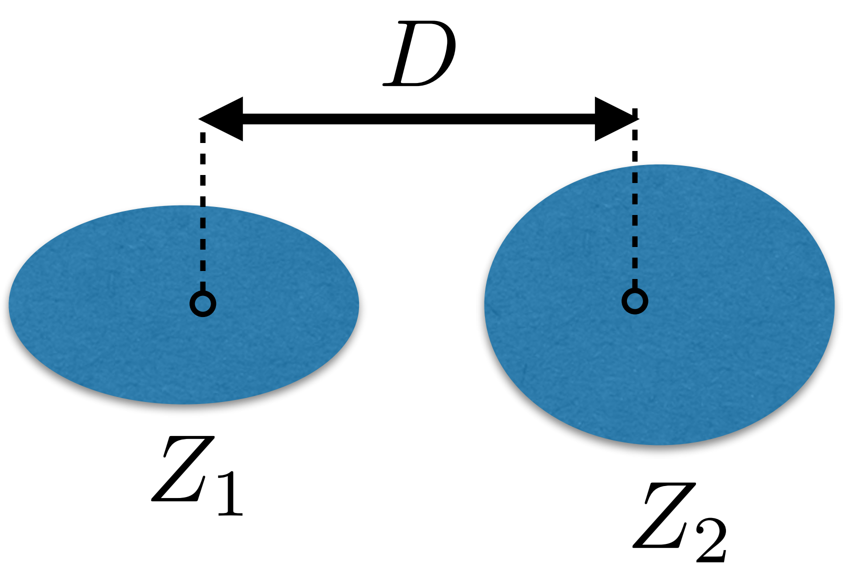


M. Caamaño, F. Farget et al., PRC 92, 034606 (2015)

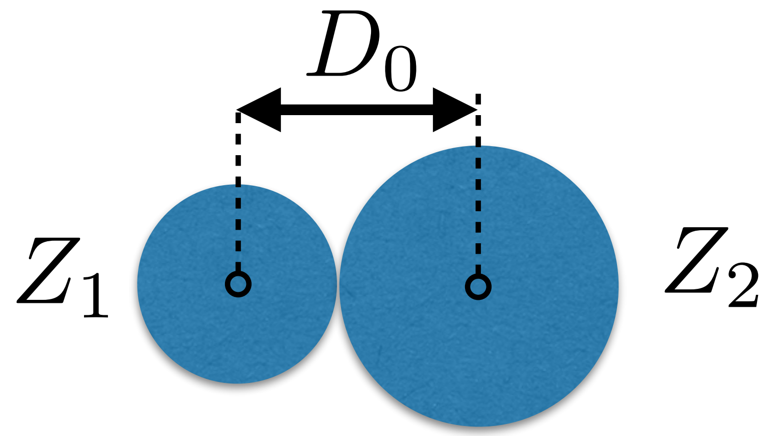
Elongation at scission







$$TKE = k\frac{Z_1 Z_2}{D}$$



$$\frac{D}{D_0} = k\frac{Z_1 Z_2}{TKE}\frac{1}{r_0 A_1^{1/3} + r_0 A_2^{1/3}}$$

M. Caamaño, F. Farget et al., PRC 92, 034606 (2015)

Energy balance at scission

Can we do more?

$$M_{\mathrm{FS}} + E^{*}_{\mathrm{FS}} = M_1 + M_2 + TKE + TXE$$

$$TKE = E^{\mathrm{k,C}}(Z_1, Z_2, \beta_1, \beta_2, d) + E^{\mathrm{k,pre}}$$

Coulomb repulsion

fragment deformation & distance

pre-scission energy

$$TXE = E^{*,\mathrm{Bf}} + E^{*,\mathrm{dis}} + \sum_{i=1}^{2} E_i^{*,\mathrm{def}}(\beta_i)$$

energy above Bf

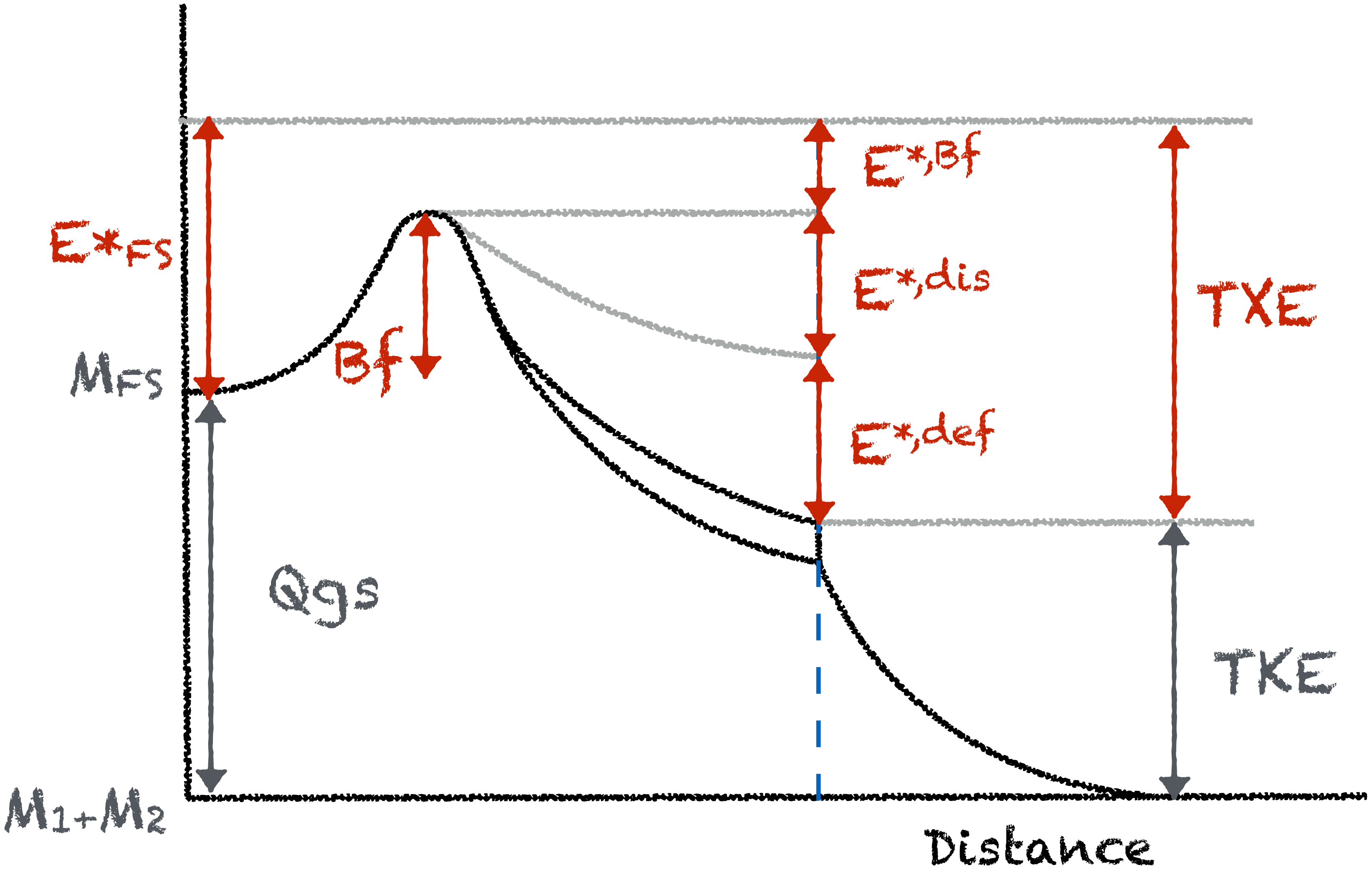
energy dissipated

deformation energy

Energy balance at scission. Excitation energy

$$TXE = E^{*,\mathrm{Bf}} + E^{*,\mathrm{dis}} + \sum_{i=1}^{2} E_i^{*,\mathrm{def}}(\beta_i)$$

energy above Bf — energy dissipated — deformation energy



Energy balance at scission. Excitation energy

$$TXE = E^{*,\mathrm{Bf}} + E^{*,\mathrm{dis}} + \sum_{i=1}^{2} E_i^{*,\mathrm{def}}(\beta_i)$$

energy above Bf

energy dissipated

deformation energy

The measurements of the E^*_{FS} of the fissioning system and its barrier are performed with the same setup

$$E^{*,\mathrm{Bf}} = E^*_{\mathrm{FS}} - \mathrm{Bf} \approx 3\ \mathrm{MeV}$$

C. Rodríguez Tajes et al., PRC 89, 025614 (2014)

The proton even-odd effect (δ_Z) is related with the amount of intrinsic energy

$$E^{*,\mathrm{Bf}} + E^{*,\mathrm{dis}} \approx -4\ln(\delta_z)$$

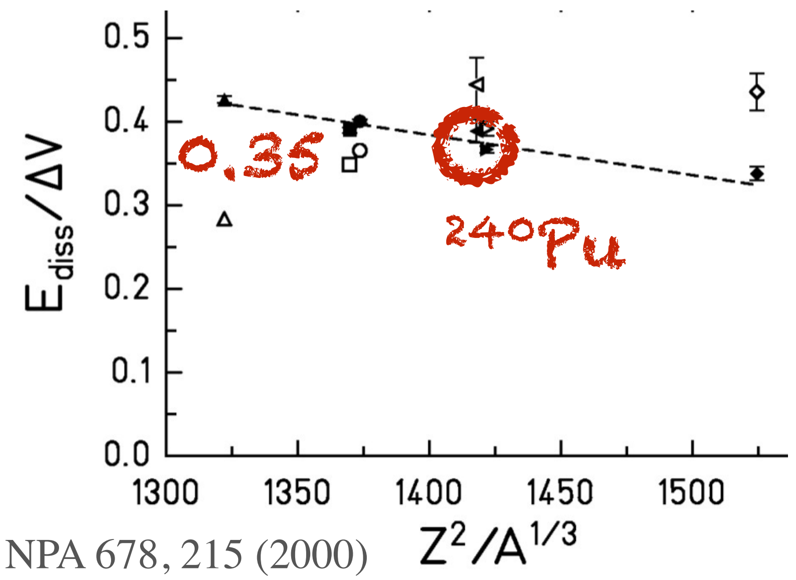
F. Gönnenwein, "The Nuclear Fission Process" (1991)

The dissipated energy can be also related with the available TXE:

$$E^{*,\mathrm{dis}} = F^{\mathrm{dis}}(TXE - E^{*,\mathrm{Bf}})$$

$$F^{\mathrm{dis}} \approx 0.35$$

GEF code: NDS 131,107 (2016)



F. Rejmund et al., NPA 678, 215 (2000)

Energy balance at scission. Excitation energy

$$\sum_{i=1}^{2} E_i^{*,\mathrm{def}}(\beta_i) = (1-F^{\mathrm{dis}})(TXE-3)$$

We need to split it between the fragments

This energy is released in post-scission evaporation:

$$\overset{\text{measured}}{TXE} = \sum_{i=1}^{2} \underset{\substack{\text{neutrons}\\ \text{binding E}}}{\overset{\text{measured}}{Q_i^\nu}} + \underset{\substack{\text{neutrons}\\ \text{kinetic E}}}{\nu_i\langle E^\nu\rangle} + \underset{\substack{\text{gamma}\\ \text{emission}}}{E_i^\gamma}$$

$$Q_i^\nu = M_i - \nu_i m_{\mathrm{n}} - M_i^{\mathrm{post}}$$

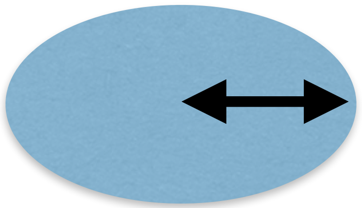
We assume that the sharing of the energy released is very similar to that of neutron binding

$$\boxed{\frac{Q_1^\nu}{Q_2^\nu} \approx \frac{Q_1^\nu + \nu_1\langle E_1^\nu\rangle + E_1^\gamma}{Q_2^\nu + \nu_2\langle E_2^\nu\rangle + E_2^\gamma}}$$

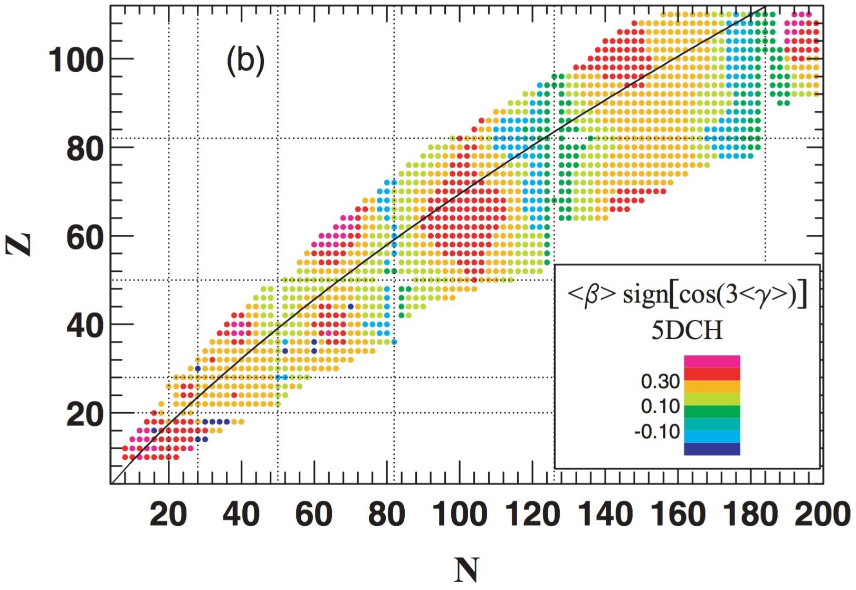
Energy balance at scission. Excitation energy

$$\boxed{E_i^{*,\mathrm{def}}(\beta_i) \approx (1-F^{\mathrm{dis}})(TXE-3)\left(\frac{Q_i^{\nu}}{Q_1^{\nu}+Q_2^{\nu}}\right)}$$

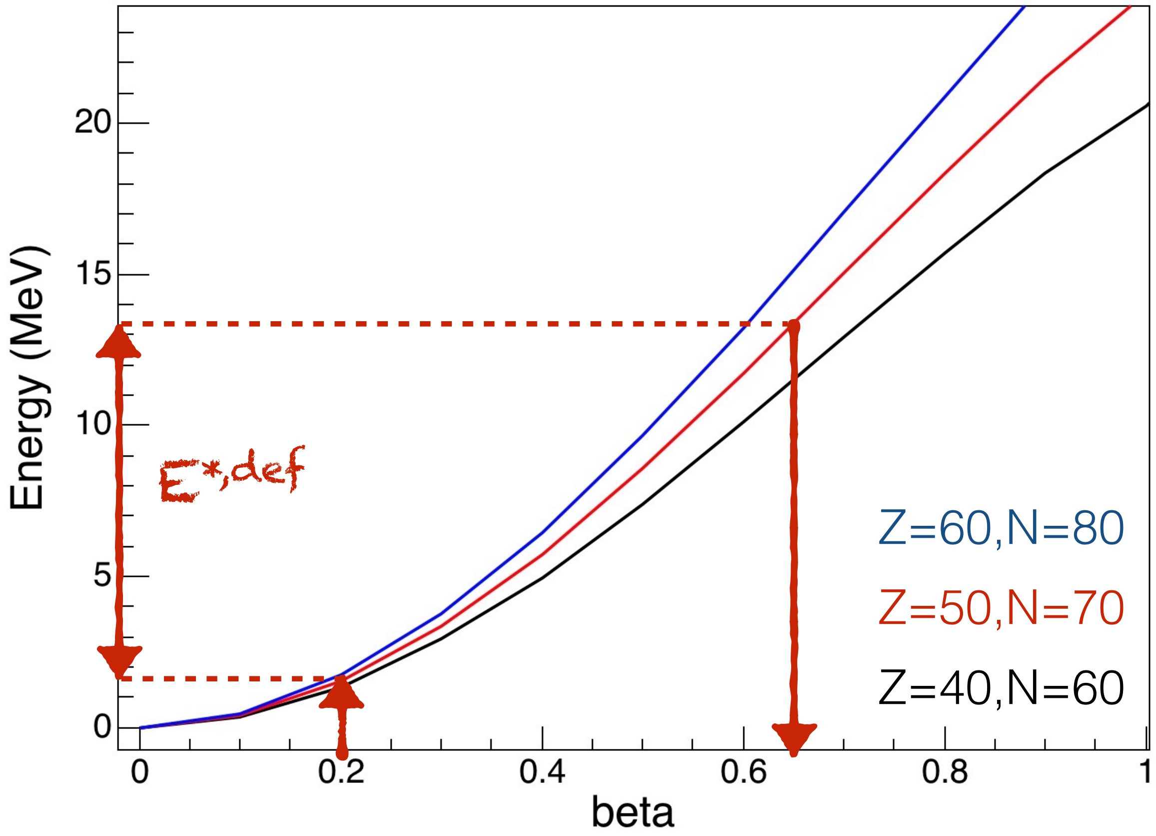
We transform the $\mathbf{E_i^{*,def}}$ into deformation with a simple factorisation around **ß** of the mass formula, taking into account the deformation at the g.s.



$$r_{\mathrm{major}} = r_0 A^{\frac{1}{3}}\left(1+\frac{2}{3}\beta\right)$$



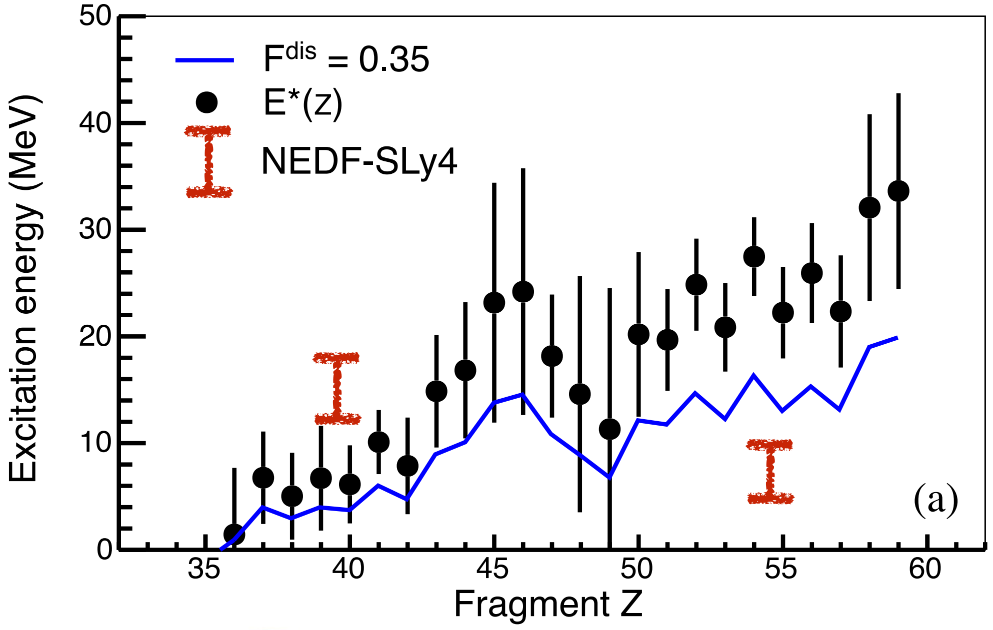
J.-P. Delaroche et al., PRC 81, 014303 (2010)

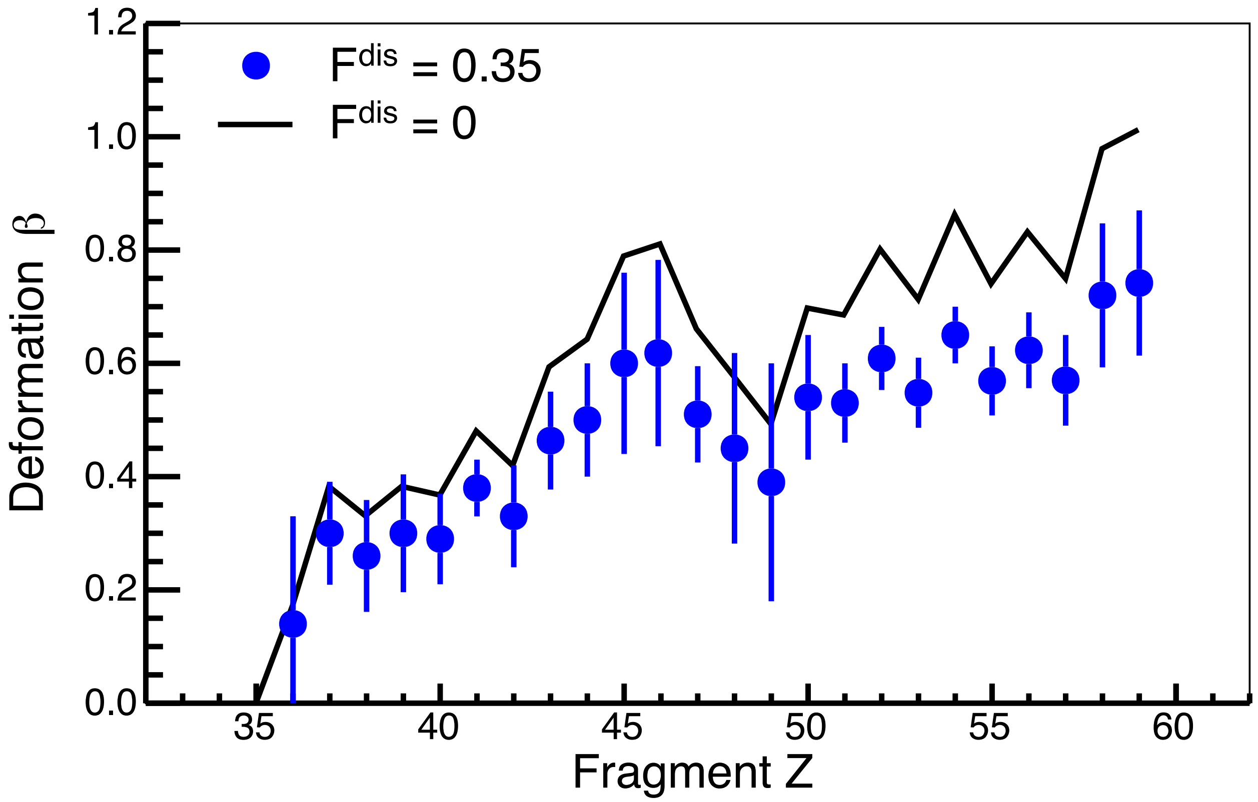


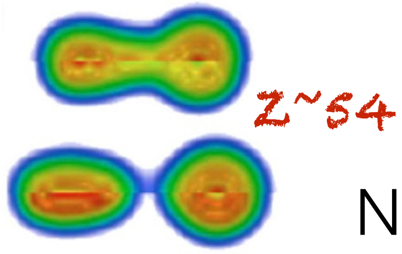
Energy balance at scission. Deformation

$$E_i^{*,\mathrm{def}}\boxed{(\beta_i)} \approx (1-F^{\mathrm{dis}})(TXE-3)\left(\frac{Q_i^{\nu}}{Q_1^{\nu}+Q_2^{\nu}}\right)$$

The value of F^{dis} is a weak point in our calculations, however, with $F^{\mathrm{dis}} = 0$ we have an upper limit for the fragment deformation.



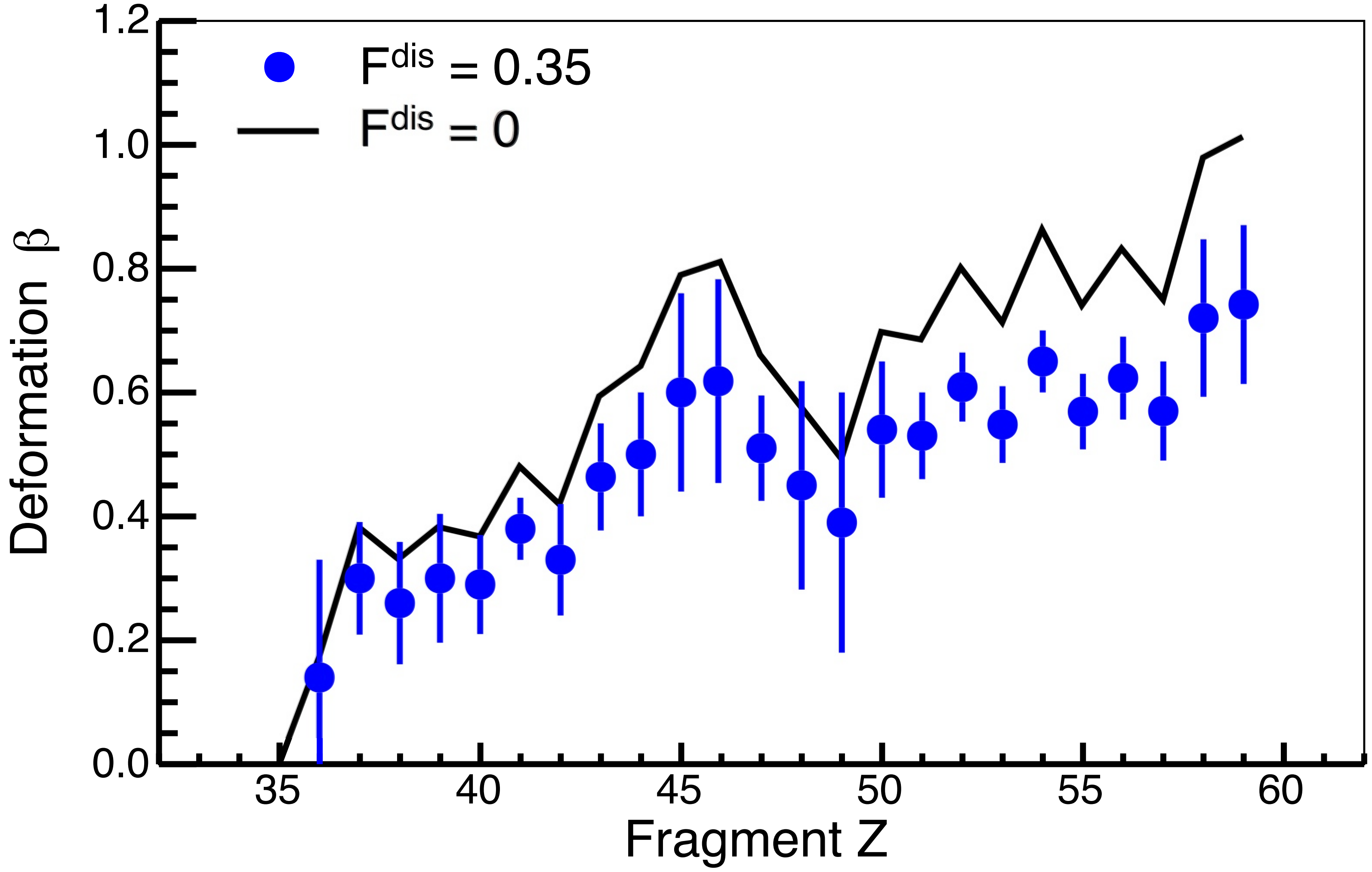




NEDF calculations

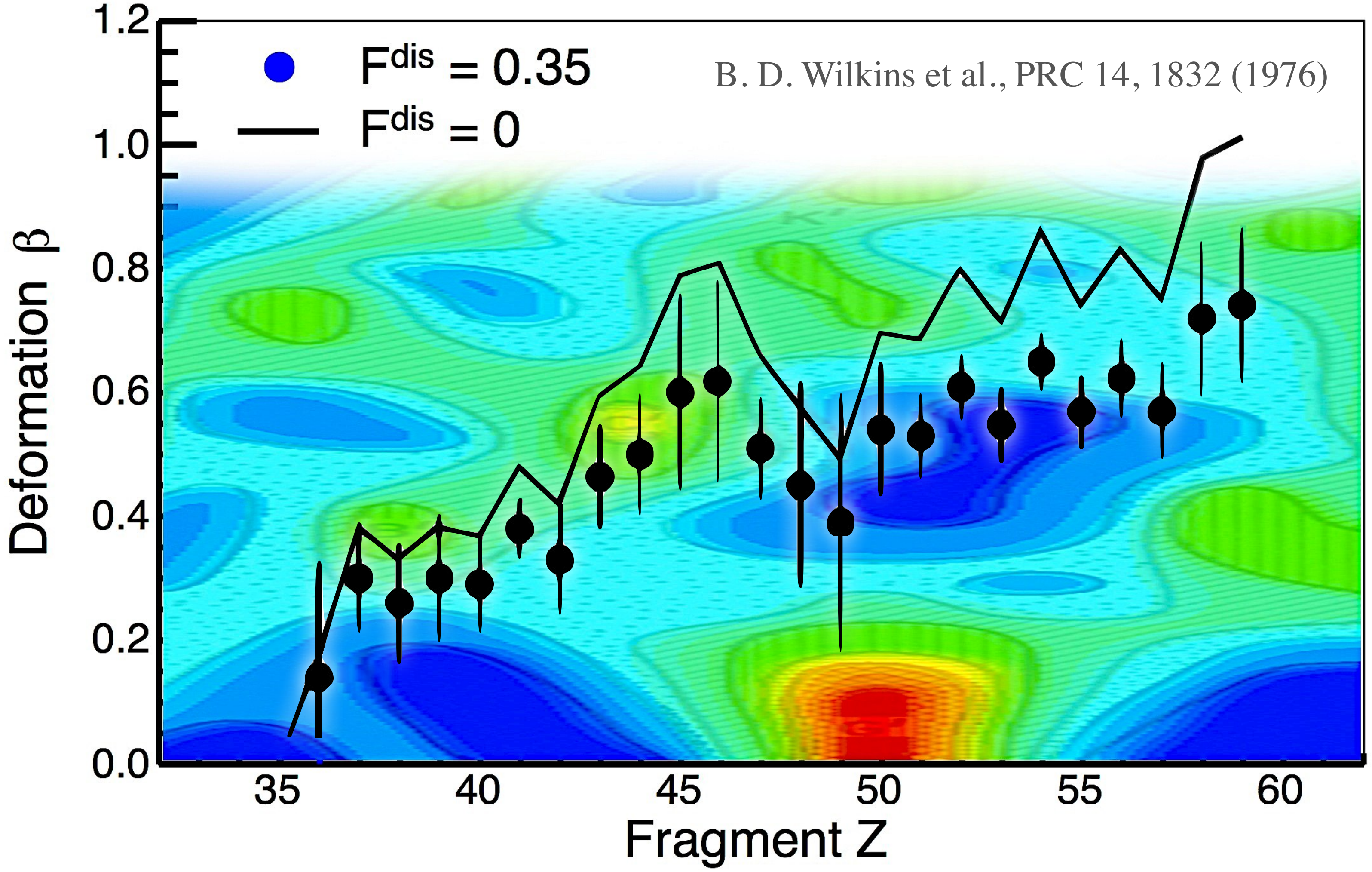
A. Bulgac et al., PRL 116, 122504 (2016)

Deformation



- The overall deformation is around 0.5
- The deformation grows with the size of the fragment, except between Z=45 - 50, reproducing the saw-tooth behaviour of the neutron multiplicity
- A minimum is formed around Z=50, but relatively far from spherical

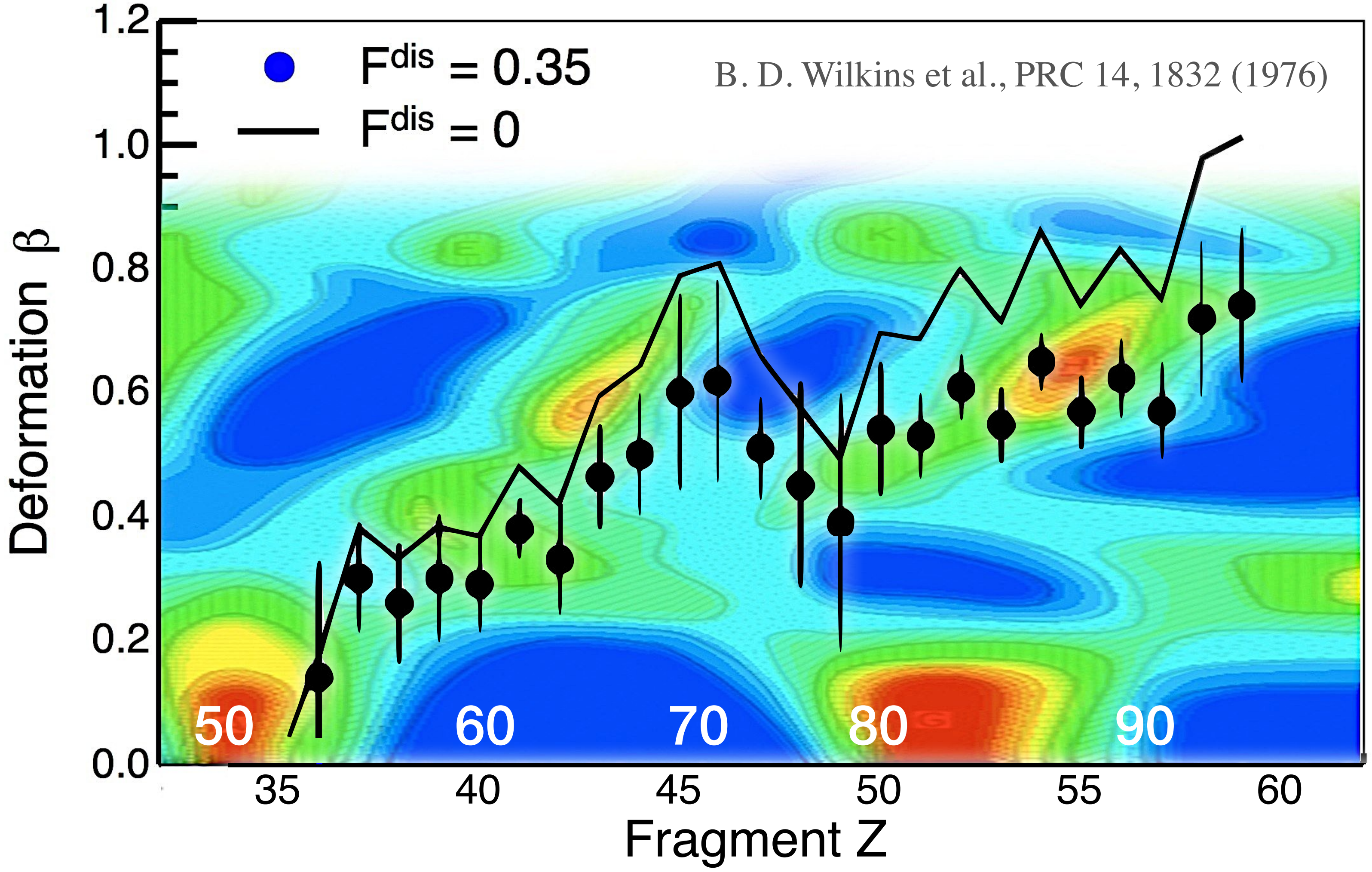
Deformation. Comparing with Wilkins et al.



Energy correction to deformed proton shells

- Light fragments go through a weak minimum around Z=44
- Around Z=50, the deformation seems to be dragged to the spherical configuration, but blocked by a “wall”.

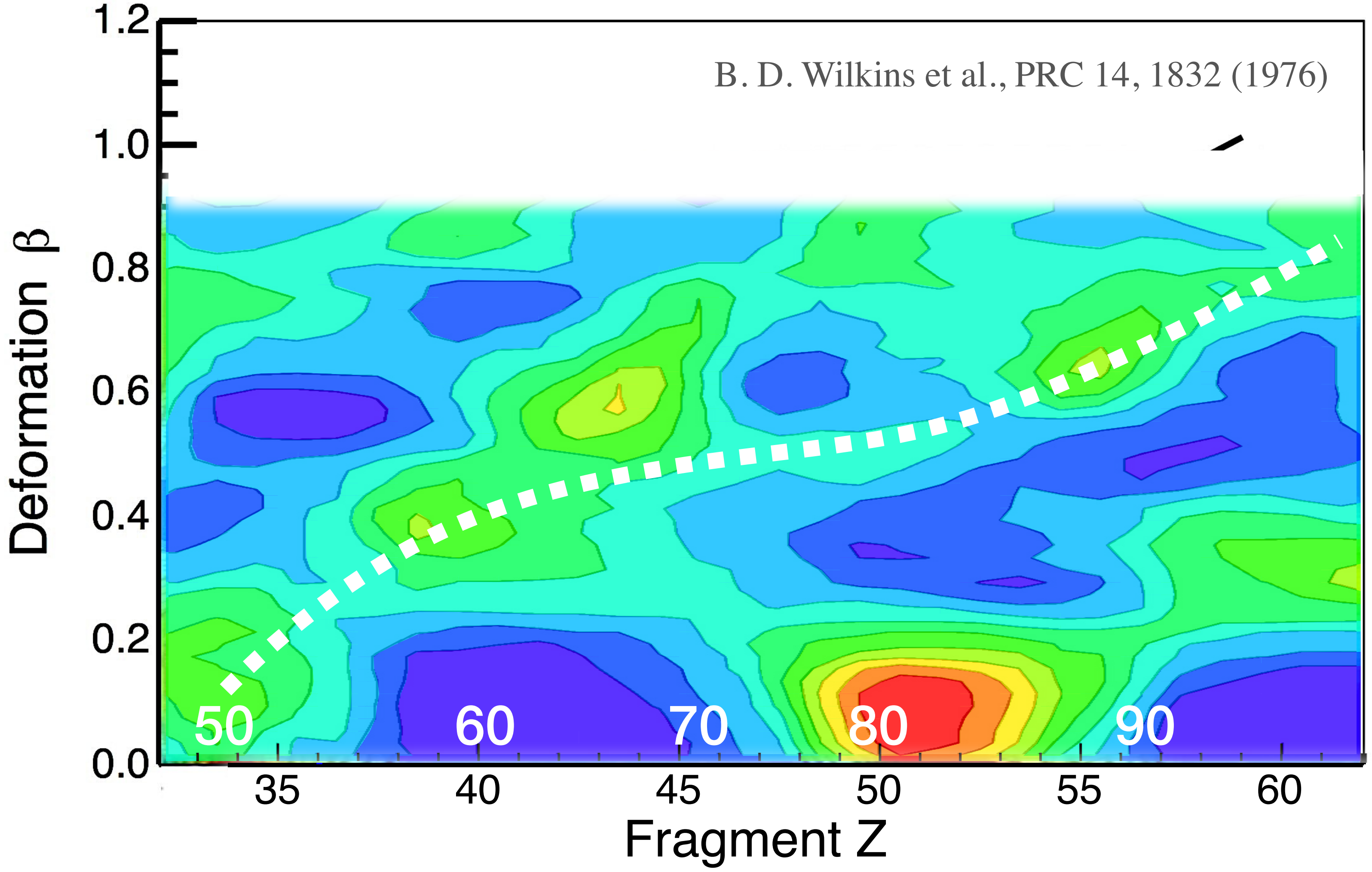
Deformation. Comparing with Wilkins et al.



Energy correction to deformed neutron shells

- Light fragments run through a corridor with local minima at N=50 and 64
- Heavy fragments also run through a corridor with a minimum at N=88
- The deformation hardly approaches spherical configurations and the effect of N=82 seems weak, in this case.

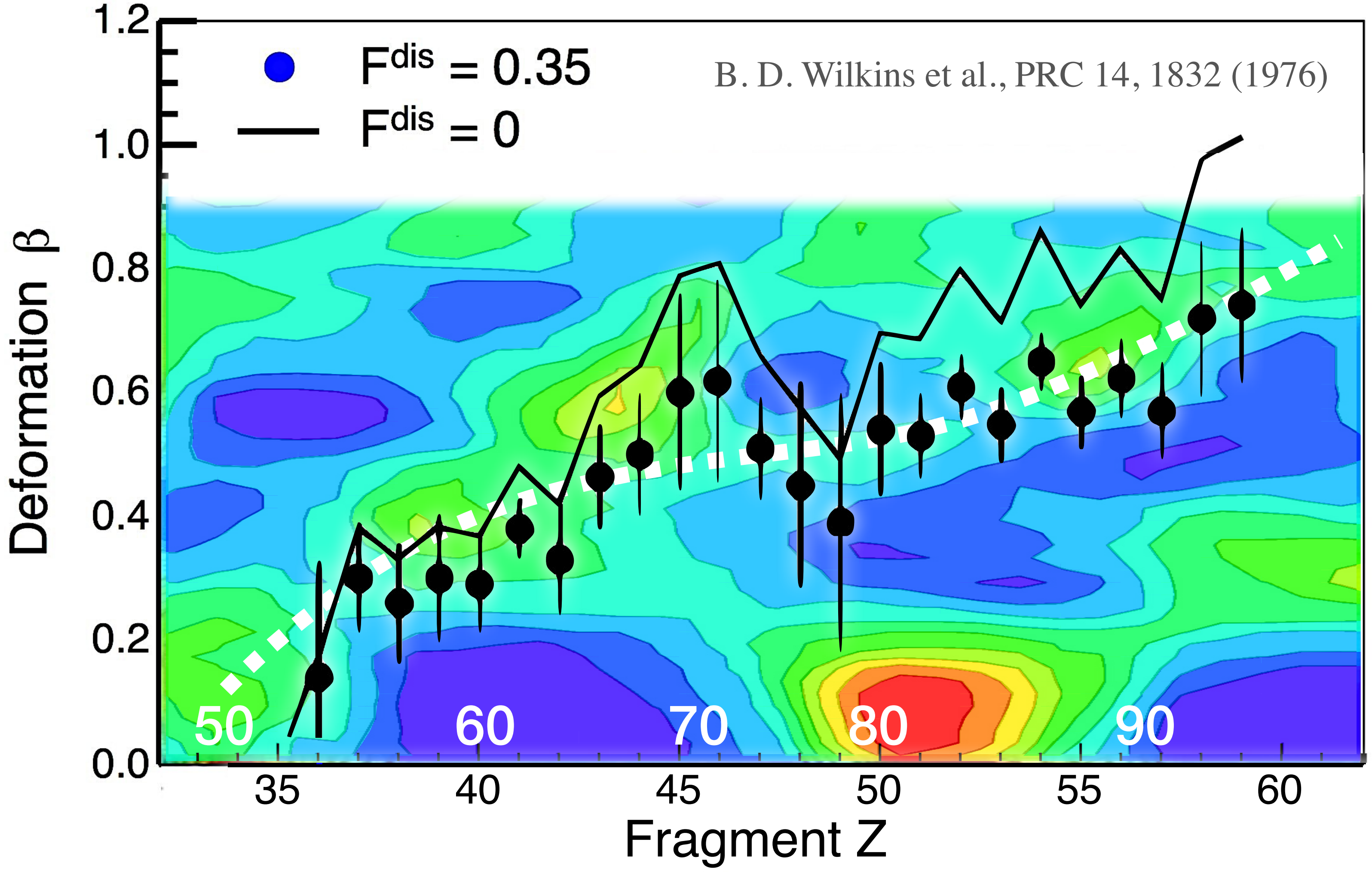
Deformation. Comparing with Wilkins et al.



Energy correction to deformed proton and neutron shells

- When considered together, the corrections to n and p shells weakens the effect of N~88 and some of N~64.
- The N~64 remains as an accessible minimum out of what is seems a long corridor.

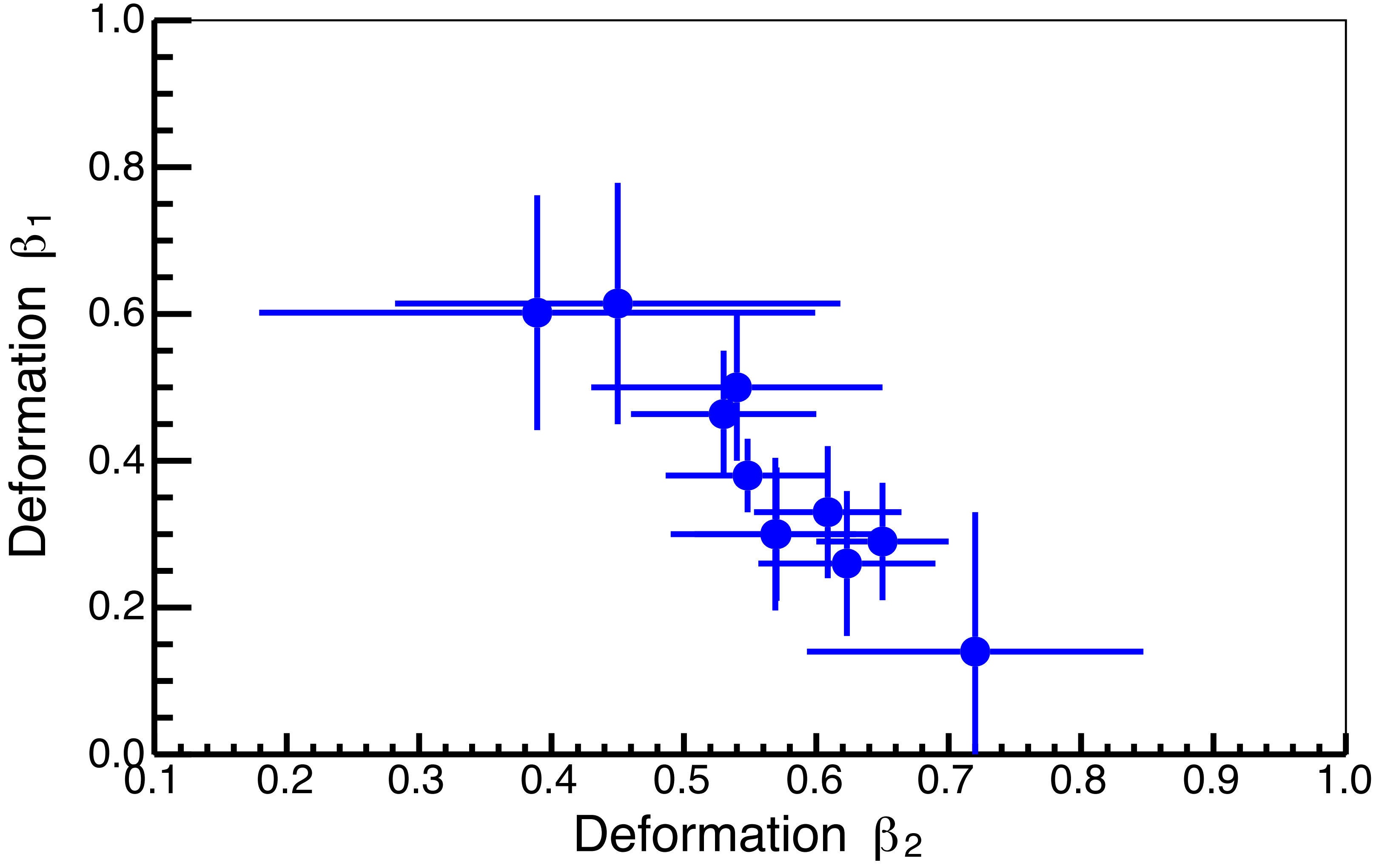
Deformation. Comparing with Wilkins et al.



Energy correction to deformed proton and neutron shells

- The experimental deformations mostly run through this corridor except around N~64, where the approaching of the light fragment competes with the potential wall that its heavy partner finds at N~80.

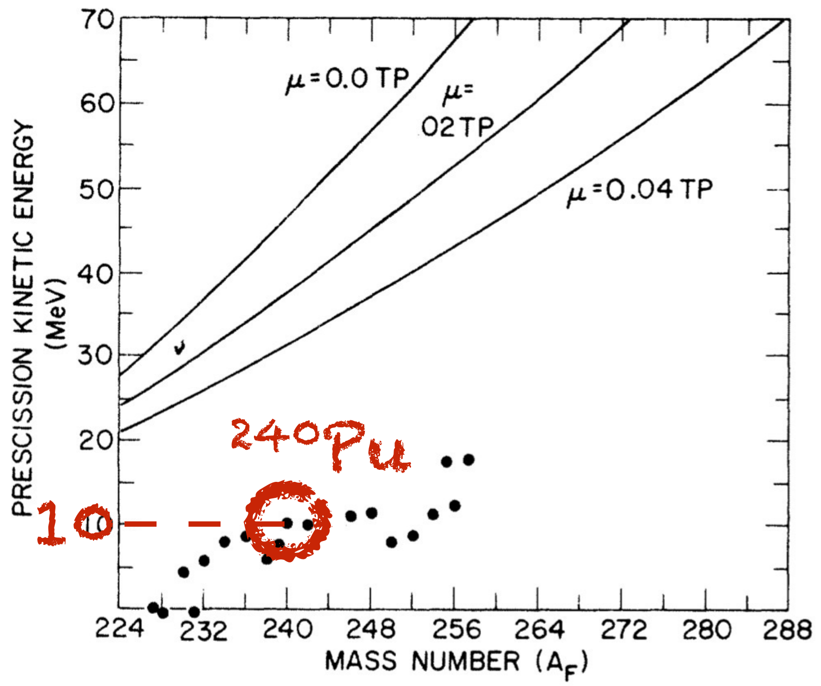
Deformation. Correlation



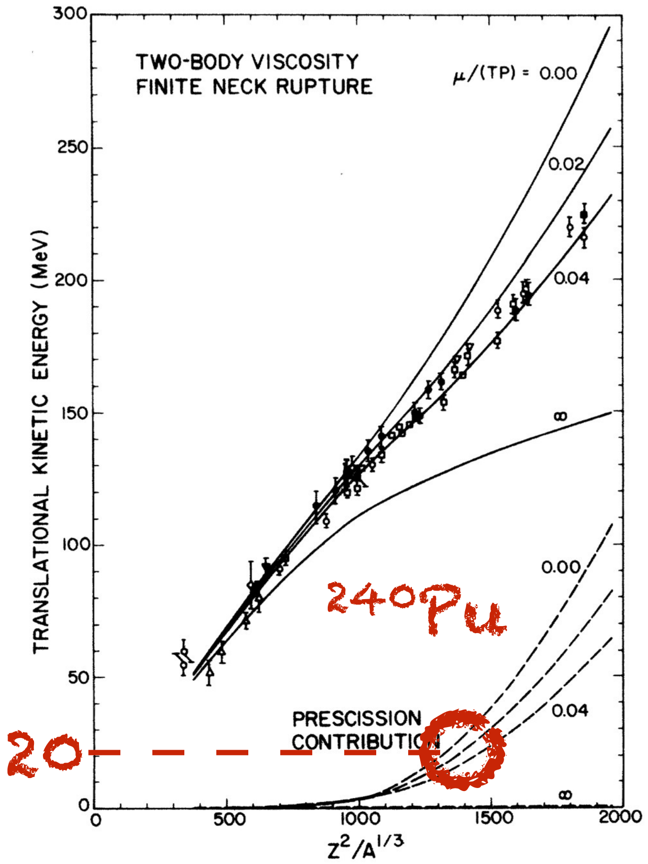
- We also realised there is a strong correlation between the deformation of split partners.

Energy balance at scission. Kinetic energy

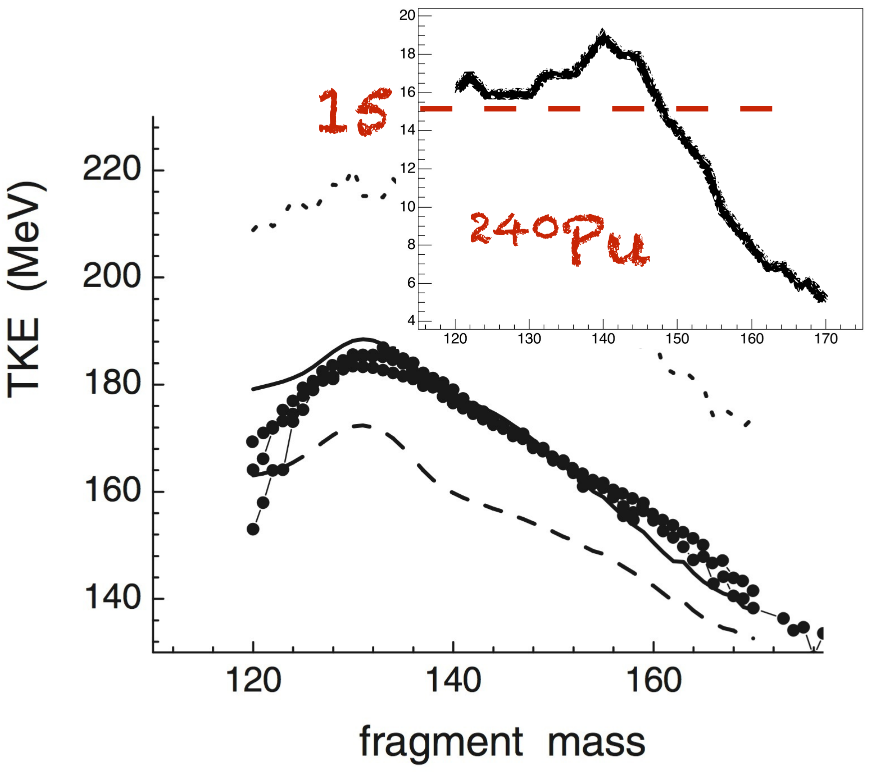
$$\overset{\text{measured}}{TKE} = E^{\text{k,C}}(Z_1, Z_2, \beta_1, \beta_2, d) + \underset{\text{pre-scission energy}}{E^{\text{k,pre}}}$$



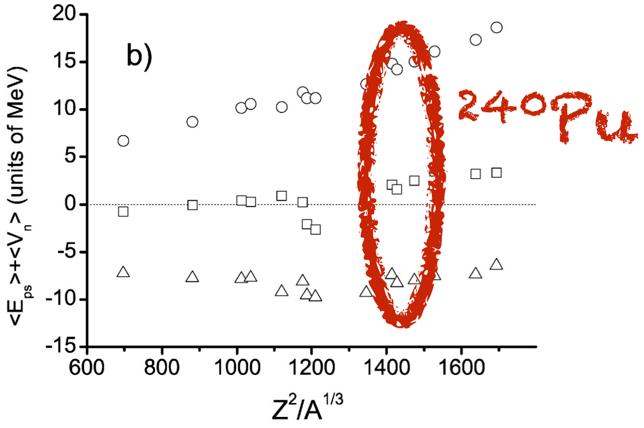
B. D. Wilkins et al., PRC 14, 1832 (1976)



K. Davies et al., PRC 16, 1890 (1977)



F. Ivanyuk et al., PRC 90, 054607 (2014)

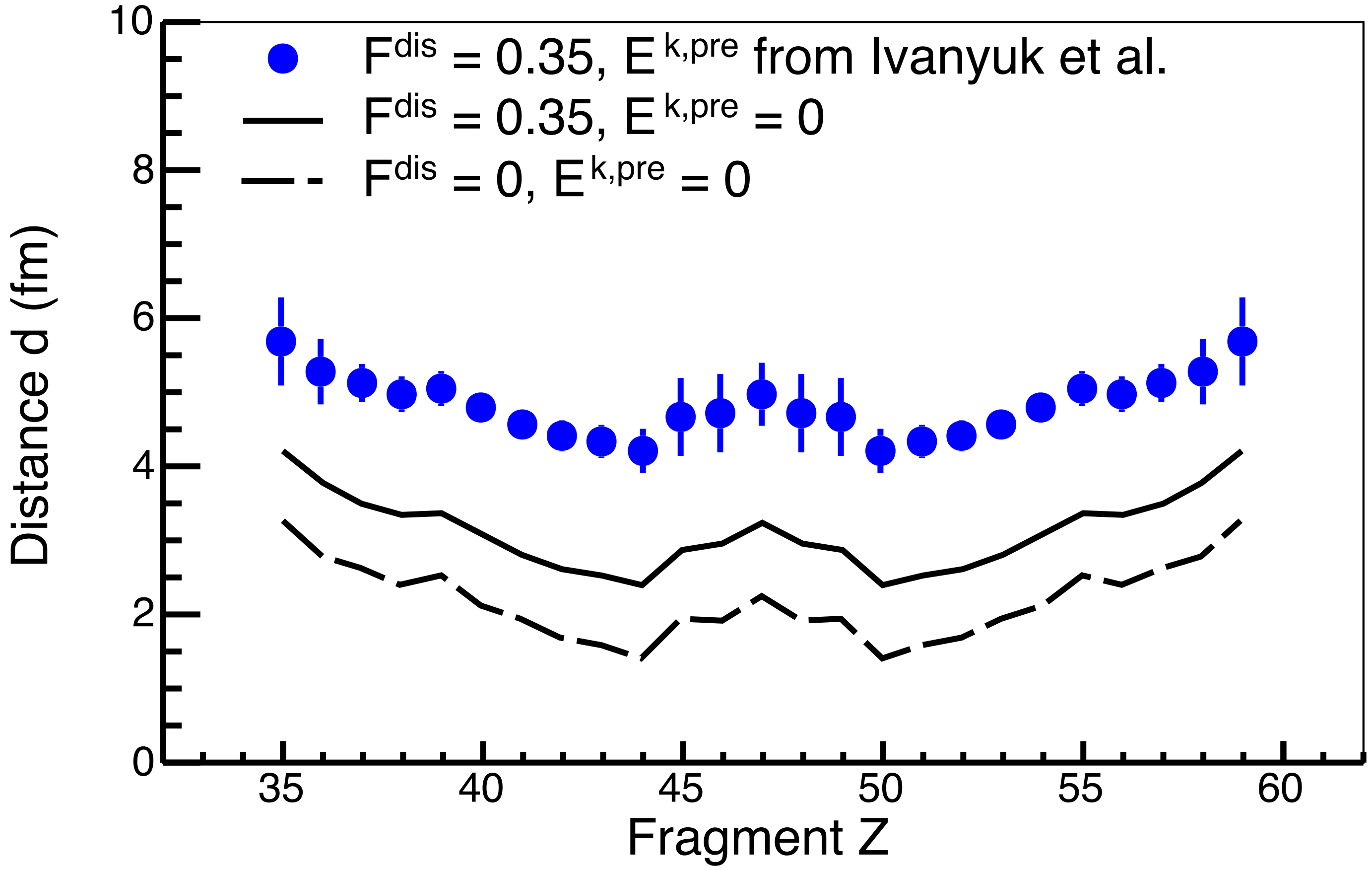


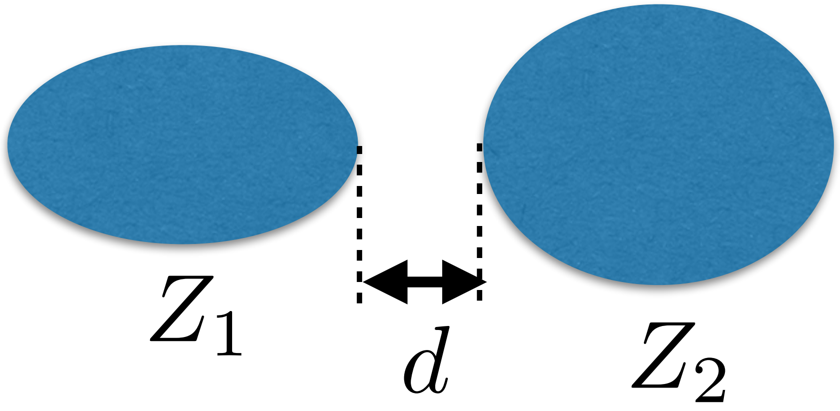
M. Borunov et al., NPA 799, 56 (2008)

Different models estimate it between 10-20 MeV.
We will use the calculations of Ivanyuk et al.

Energy balance at scission. Tip distance

$$\boxed{E^{\mathrm{k,C}}(Z_1, Z_2, \beta_1, \beta_2, d) = \overset{\text{measured}}{TKE} - E^{\mathrm{k,pre}}_{\mathrm{Ivanyuk}}}$$



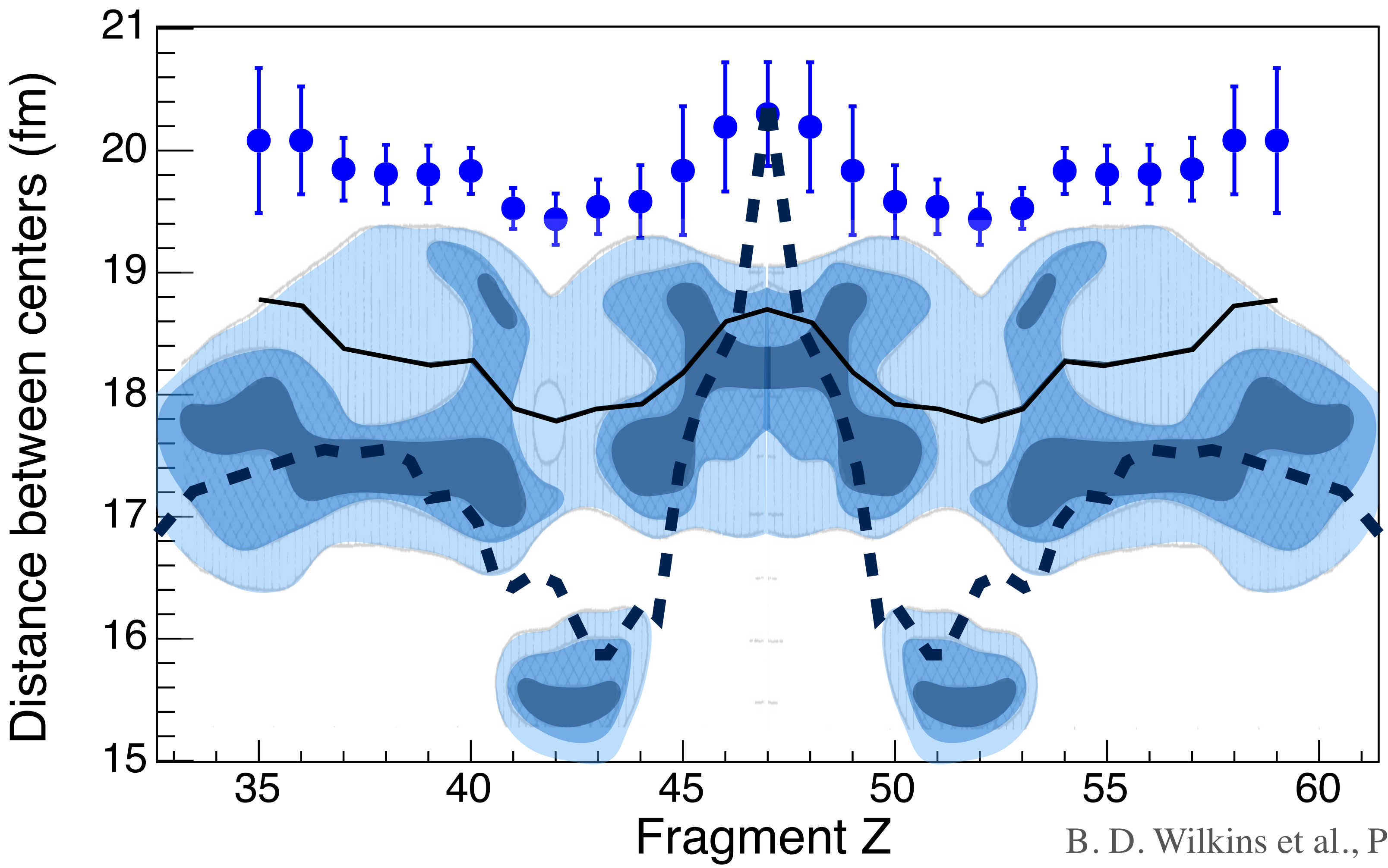


We use the formula of Cohen-Swiatecki to calculate the repulsion between two coaxial homogeneously charged ellipsoids

S. Cohen and W. Swiatecki, Annals of Physics 19, 67 (1962).

- The overall value is ~5 fm, which is much larger than the “standard” (below 3 fm). Only at the lower limit reaches ~2 fm.
- A distinctive minimum appears at Z=50.

Distance. Comparing with…



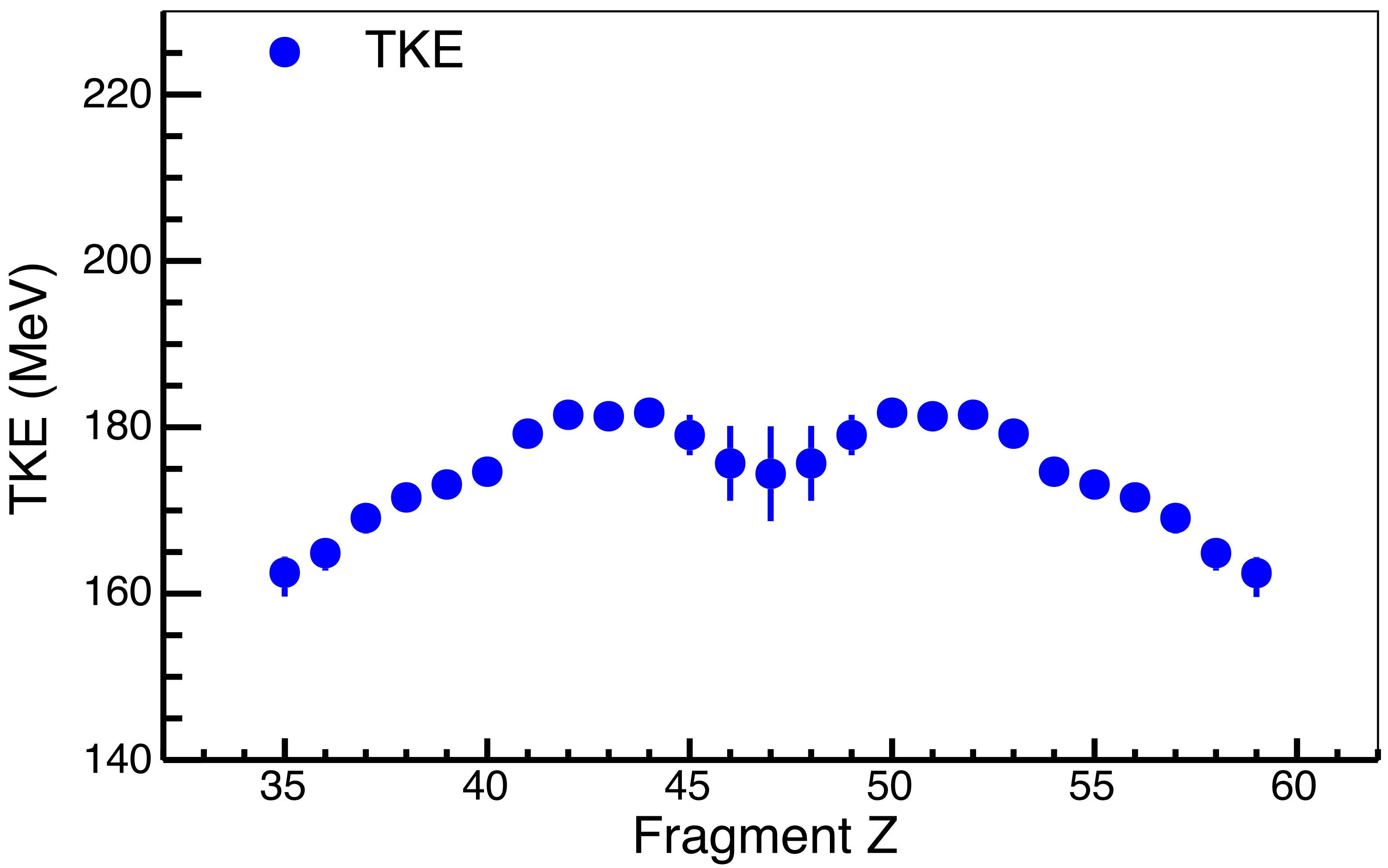
B. D. Wilkins et al., PRC 14, 1832 (1976)

H. Goutte et al., PRC 71, 024316 (2005)

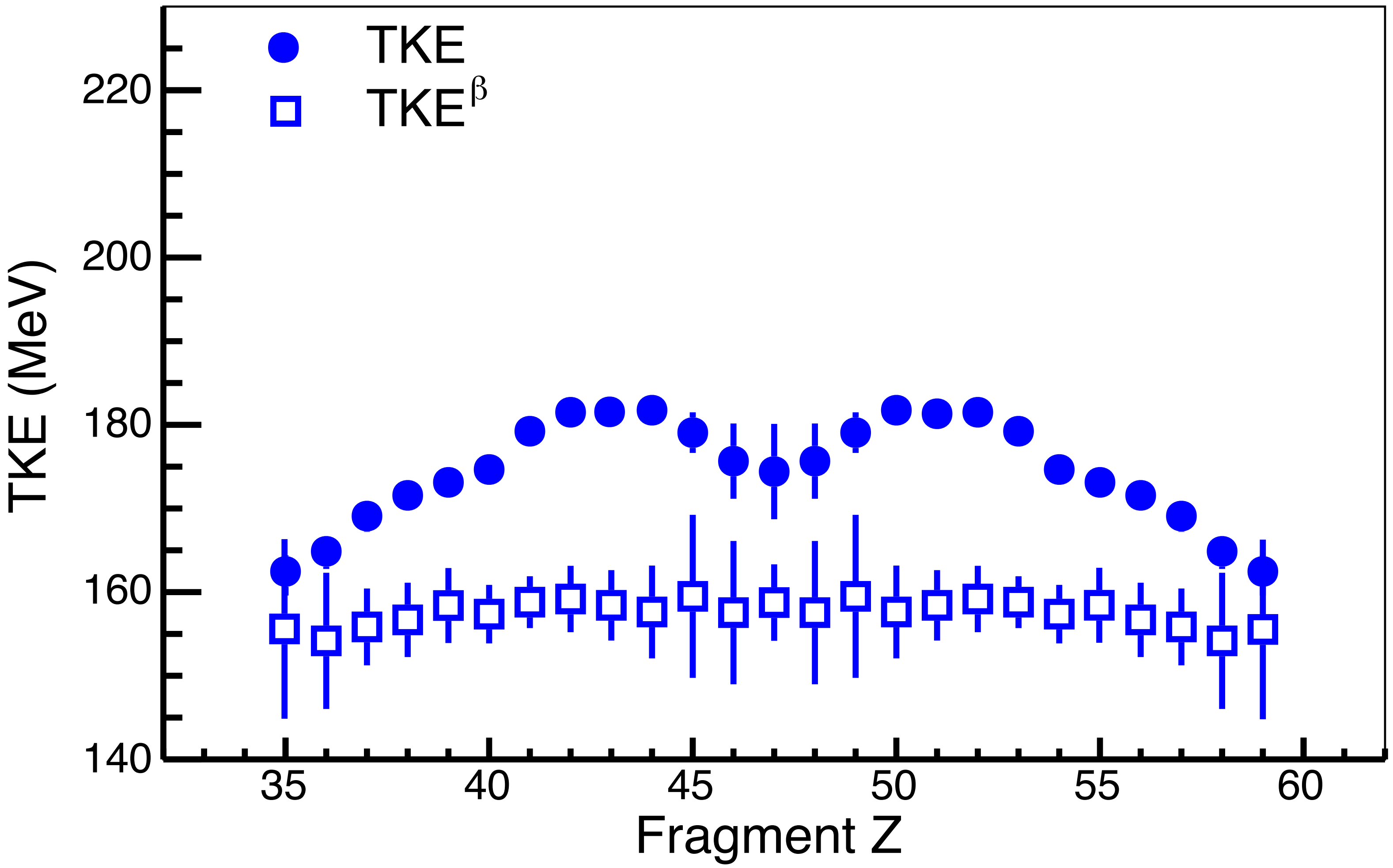
Distance between fragments

- SPM calculations for ^{236}U also predict a minimum around Z~52. Although more pronounced. HFB calculations also calculate a deeper minimum around Z~52 for ^{238}U

TKE, who decides its shape?

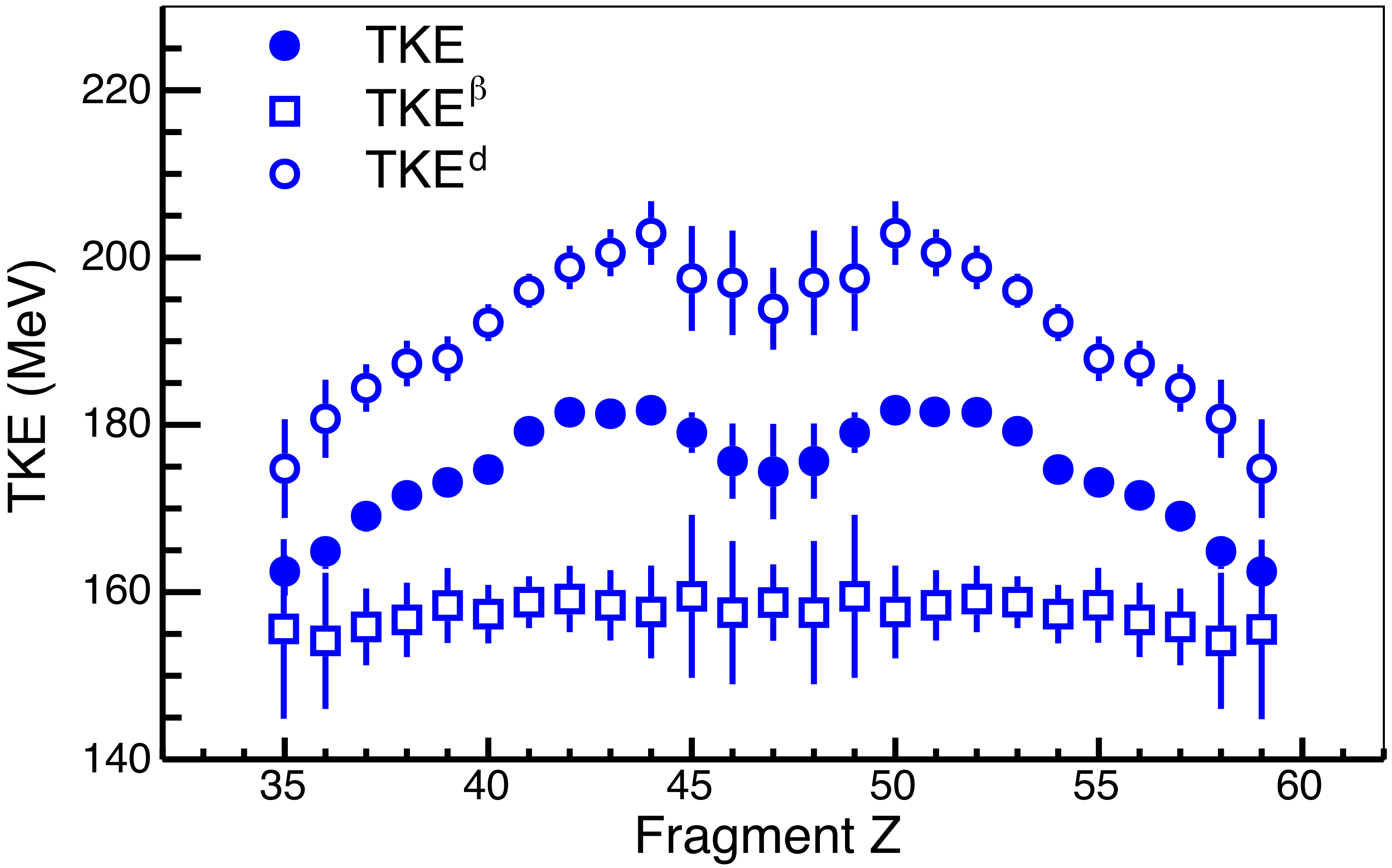


TKE, who decides its shape?



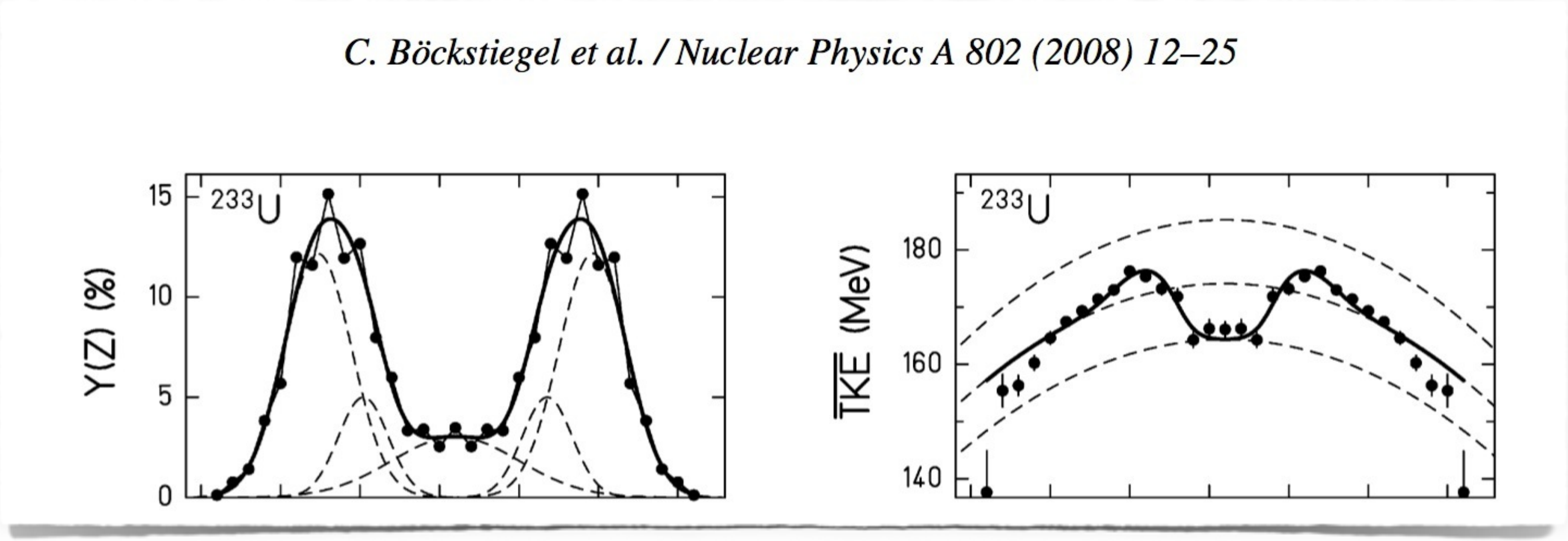
- Fixing the tip distance, the effect of the deformation alone does not reproduce the features of the observed TKE.

TKE, who decides its shape?



- Fixing the tip distance, the effect of the deformation alone does not reproduce the features of the observed TKE.
- The effect of the neck distribution applied to spherical fragments mimics the same behaviour of the TKE.
- There must be a mechanism that links the structure effects to the length of the neck.

Fission modes



C. Böckstiegel et al. / Nuclear Physics A 802 (2008) 12–25

Assuming ß and d are unique for each mode, we fit simultaneously the isotopic yield distribution and the TKE

$$Y_Z = \sum_j \frac{I_j}{\sigma_j \sqrt{2\pi}} \exp\left(\frac{-(Z - Z_{0,j})^2}{2\sigma_j^2} \right)$$

$$TKE_Z = \frac{\sum_j Y_Z(Z_{0,j}, \sigma_j, I_j) \cdot E^{\mathrm{k,C}}(\beta_{1,j}, \beta_{2,j}, d_j)}{\sum_j Y_Z(Z_{0,j}, \sigma_j, I_j)} + E^{\mathrm{k,pre}}$$

Fission modes

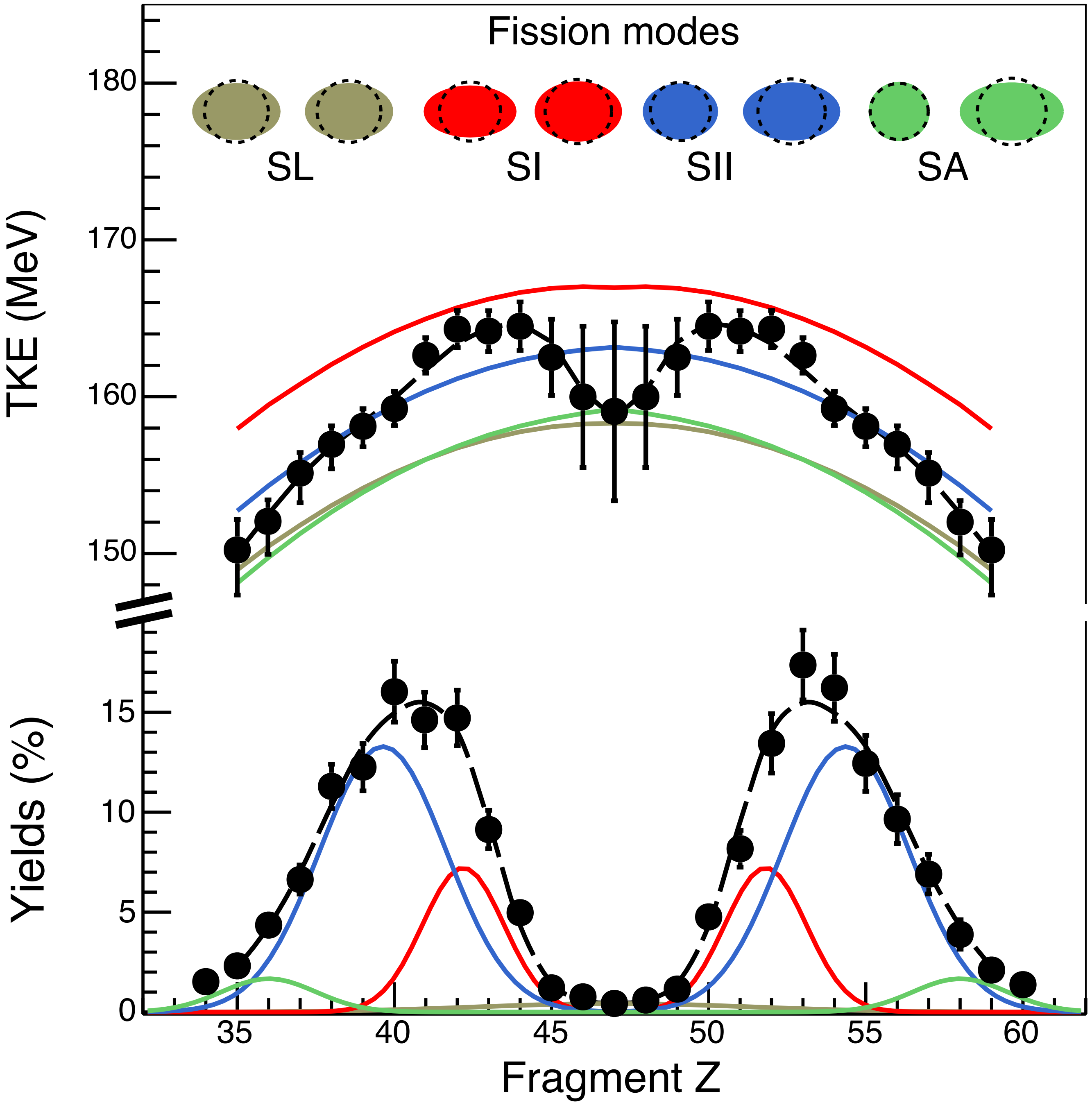


TABLE I. Fission channel parameters.

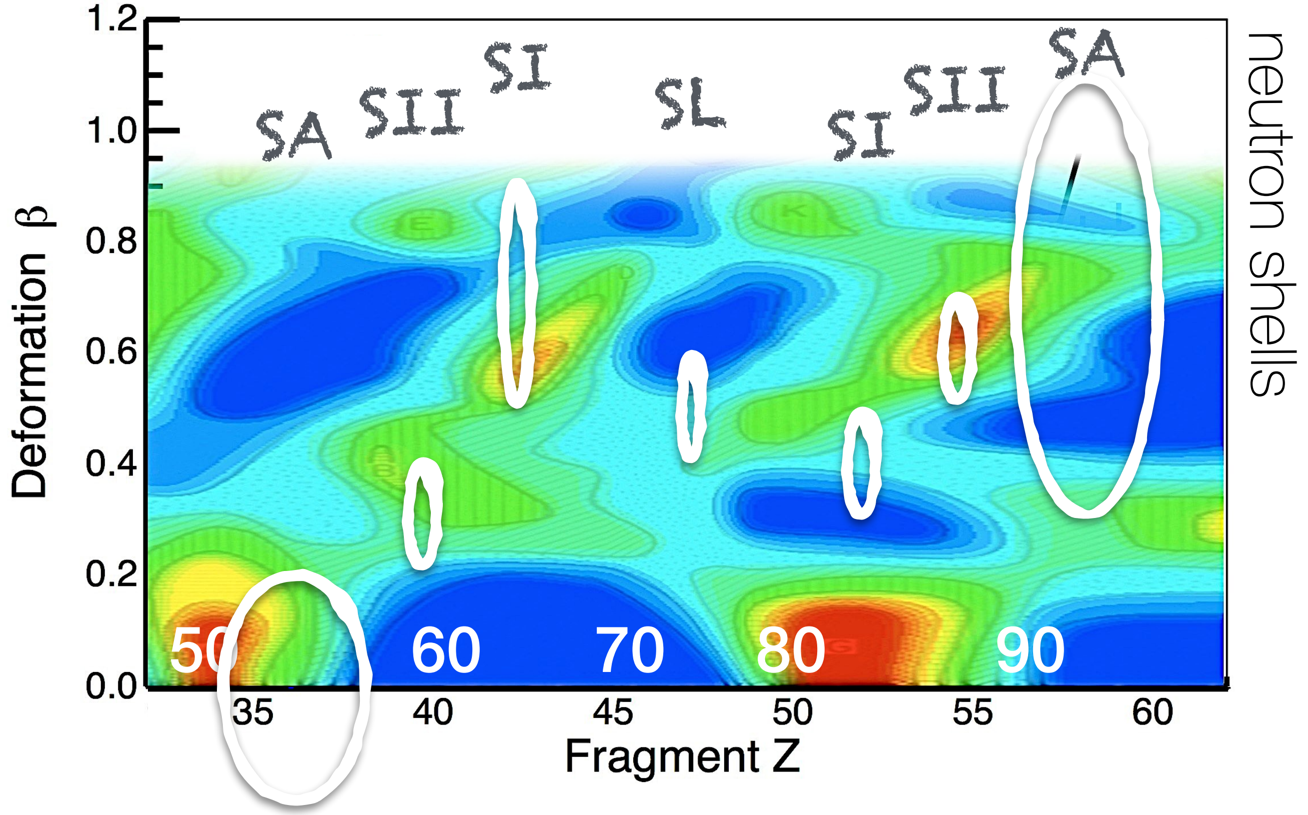
	SL	SI	SII	SA
Z_0	47	51.8(4)	54.4(4)	58(2)
σ	4.4(4)	1.3(2)	2.0(1)	1.5(2)
Yield (%)	5(1)	23(8)	66(9)	6(3)
β_1	0.5(1)	0.7(2)	0.3(1)	0.0(2)
β_2	0.5(1)	0.4(1)	0.6(1)	0.7(4)
d (fm)	4.9(3)	3.8(4)	4.9(2)	5.9(7)
$R_{c.m.}$ (fm)	20.4(6)	19.3(6)	19.8(6)	20(1)

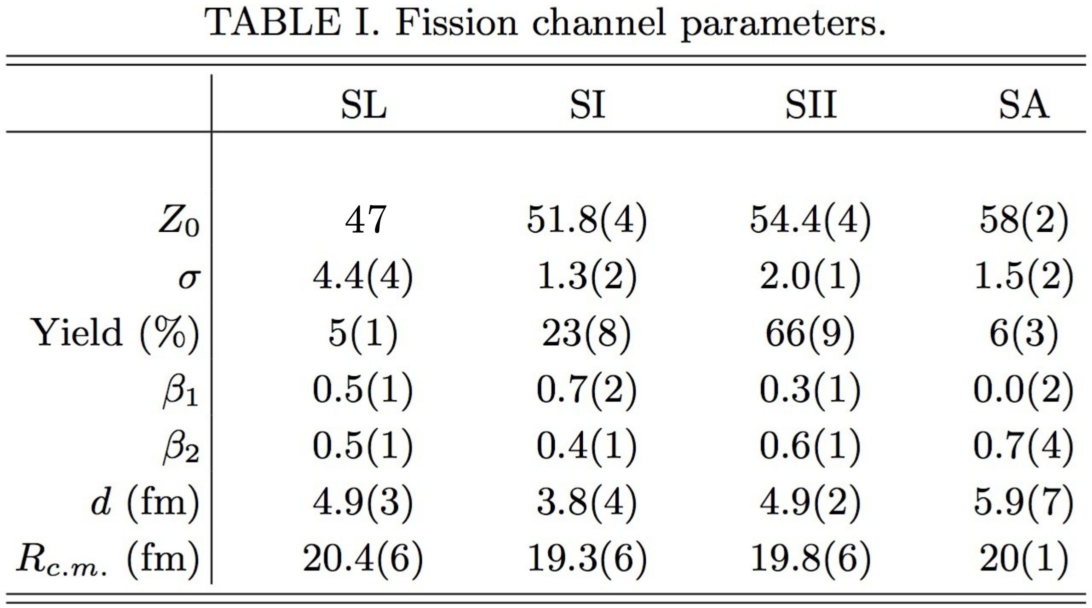
- The modes on the yield distribution are pretty much in agreement with previous measurements

- We find a super-asymmetric component with similar contribution as that of the super-long mode.

Fission modes

B. D. Wilkins et al., PRC 14, 1832 (1976)





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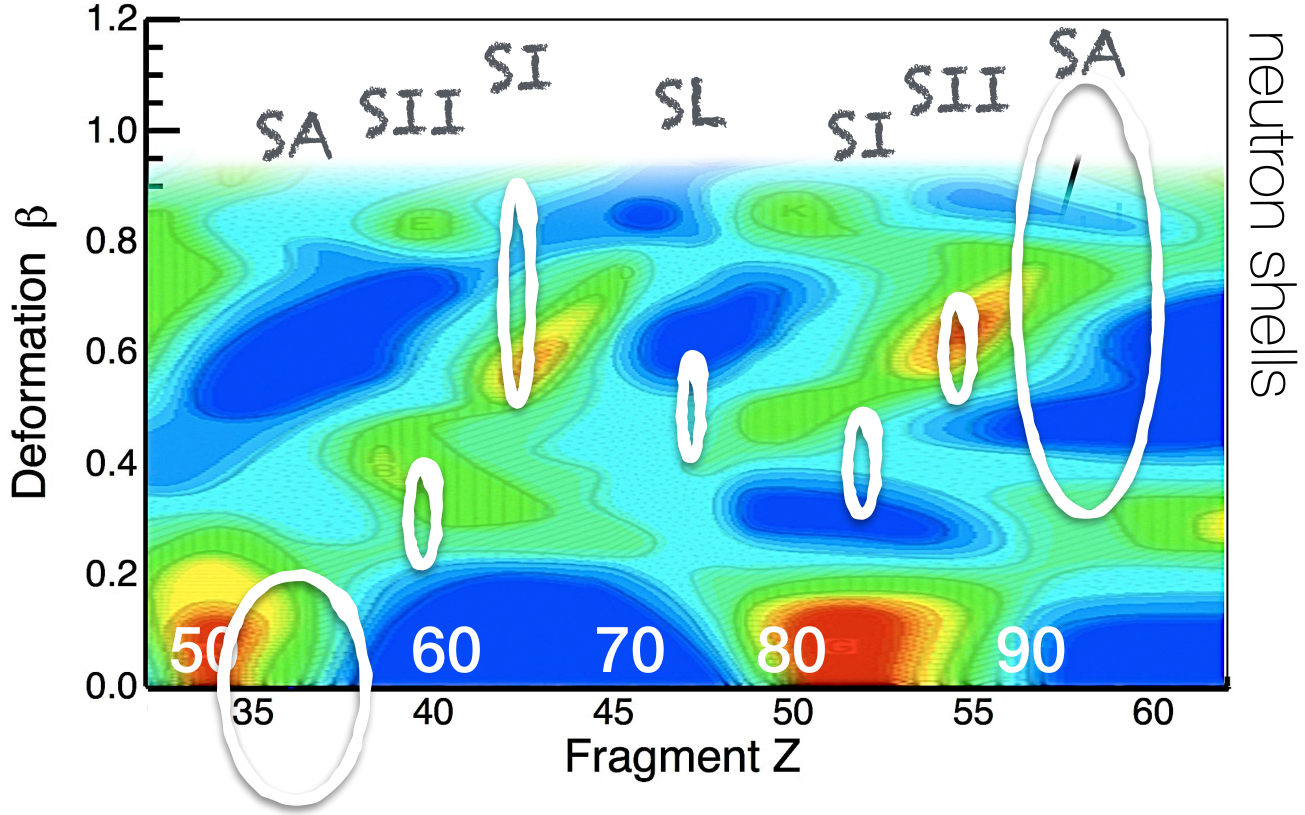
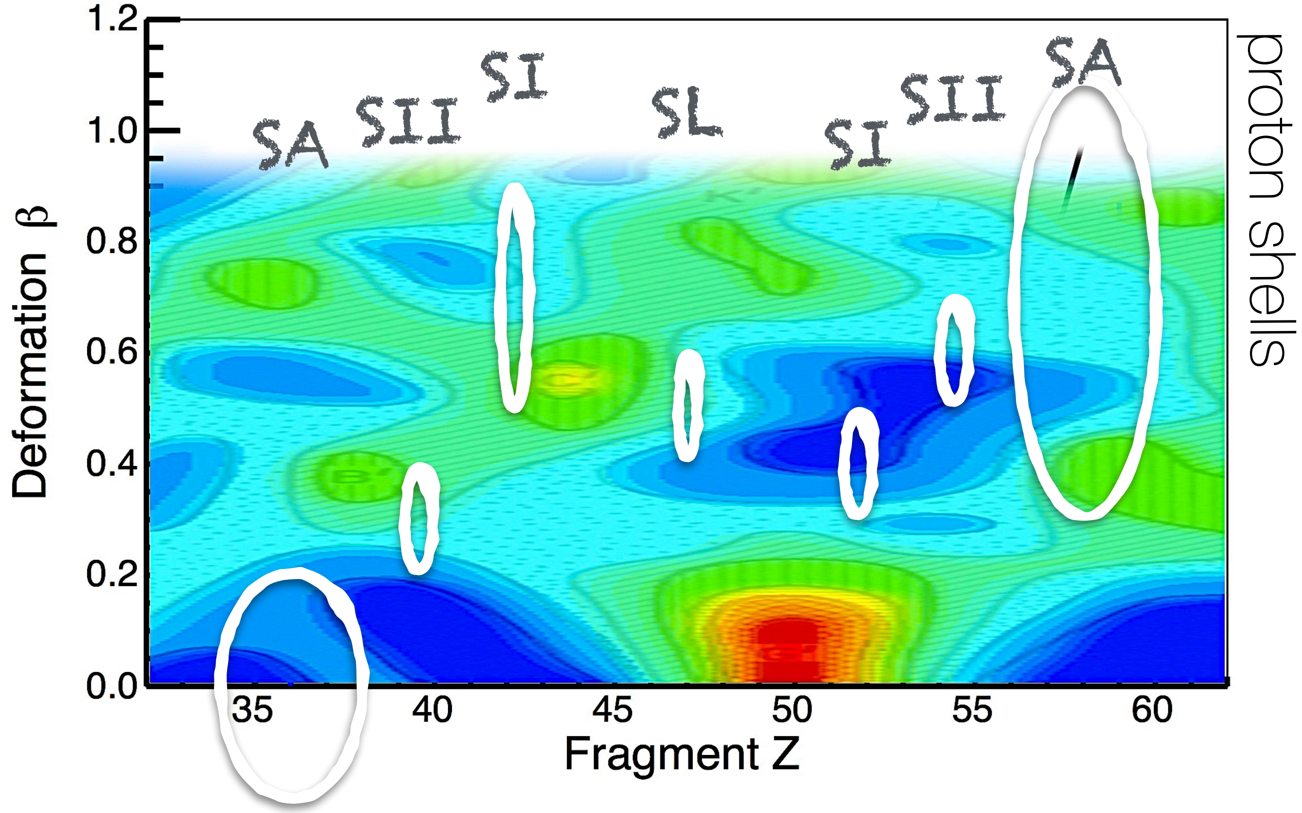
d (fm)	4.9(3)	3.8(4)	4.9(2)	5.9(7)
$R_{c.m.}$ (fm)	20.4(6)	19.3(6)	19.8(6)	20(1)

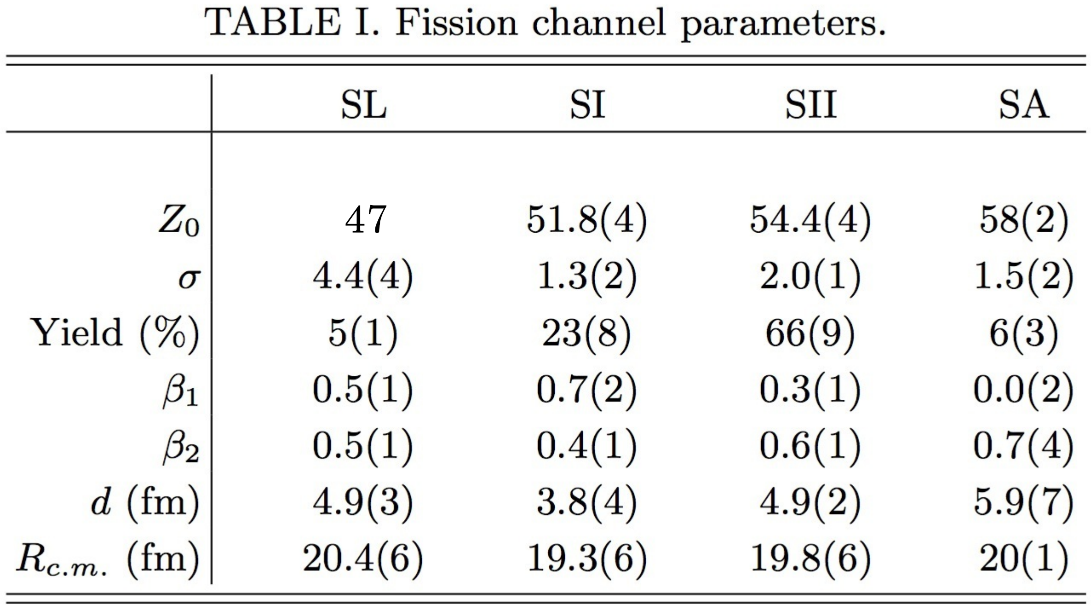
- SL: Is “stuck” between two walls
- SI: N~64 decides the deformation on the light fragment
- SII: N~88 decides the deformation on the heavy fragment
- SA: Might be dragging its light fragment towards N=50

Fission modes

B. D. Wilkins et al., PRC 14, 1832 (1976)







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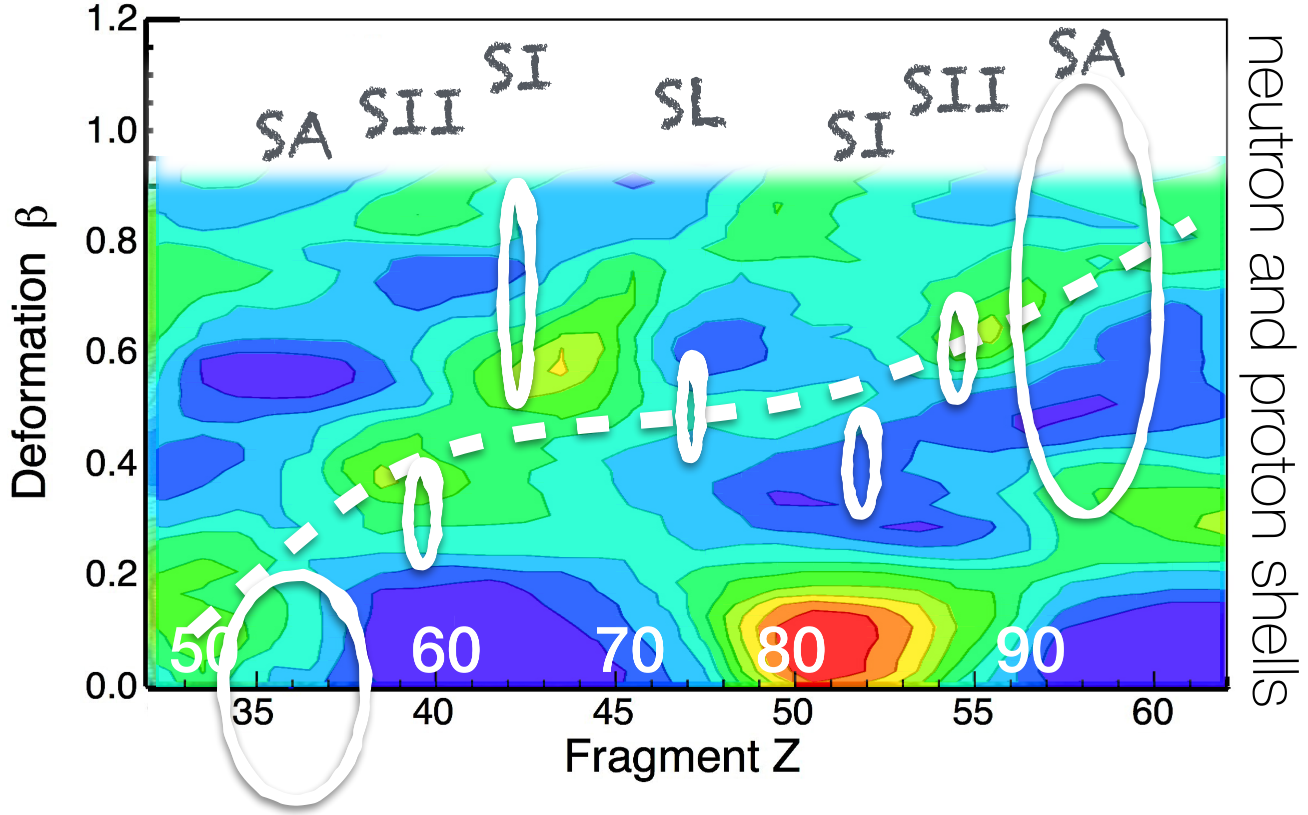
Yield (%)	5(1)	23(8)	66(9)	6(3)
β_1	0.5(1)	0.7(2)	0.3(1)	0.0(2)
β_2	0.5(1)	0.4(1)	0.6(1)	0.7(4)
d (fm)	4.9(3)	3.8(4)	4.9(2)	5.9(7)
$R_{c.m.}$ (fm)	20.4(6)	19.3(6)	19.8(6)	20(1)

- SL: Is “stuck” between two walls
- SI: N~64 decides the deformation on the light fragment
- SII: N~88 decides the deformation on the heavy fragment
- SA: Might be dragging its light fragment towards N=50
- Proton shells seem to have little influence, except, maybe, at SI (Z~44)

Fission modes

B. D. Wilkins et al., PRC 14, 1832 (1976)



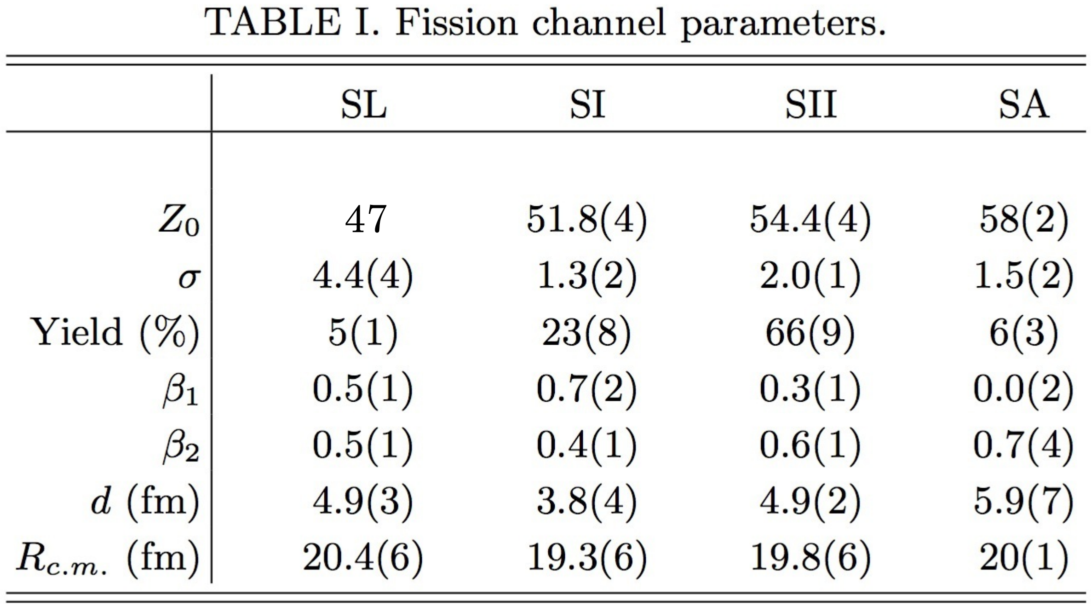
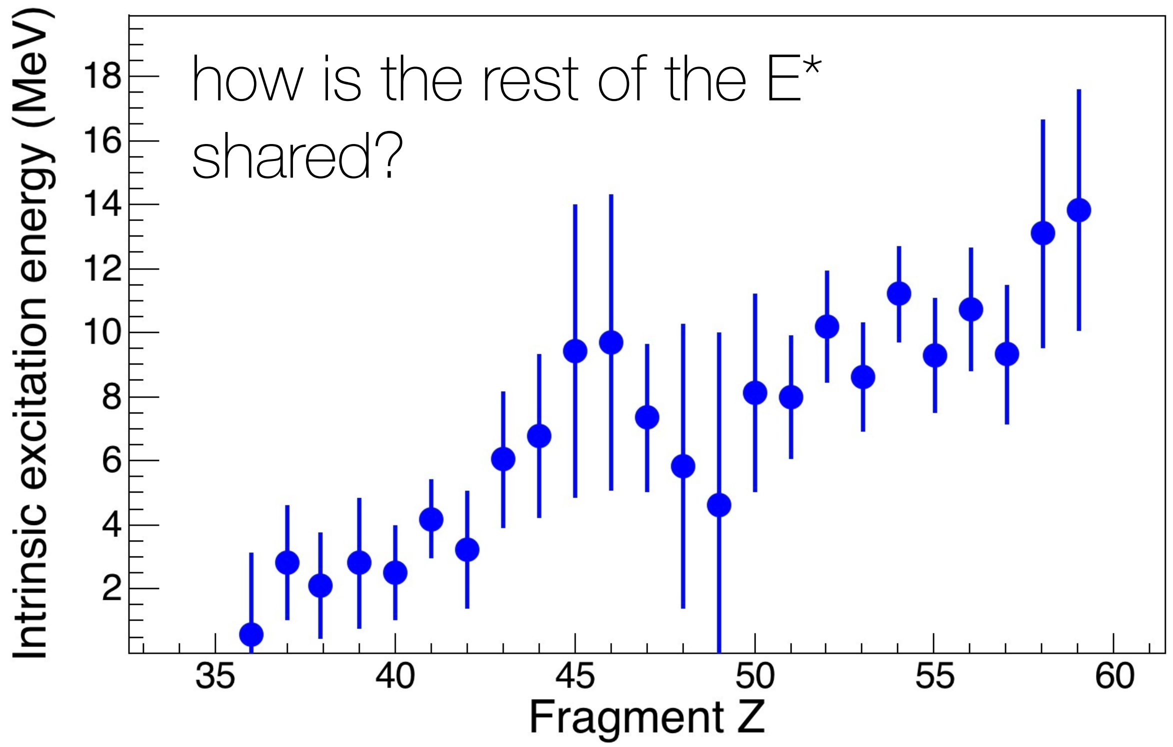


TABLE I. Fission channel parameters.

	SL	SI	SII	SA
Z_0	47	51.8(4)	54.4(4)	58(2)
σ	4.4(4)	1.3(2)	2.0(1)	1.5(2)
Yield (%)	5(1)	23(8)	66(9)	6(3)
β_1	0.5(1)	0.7(2)	0.3(1)	0.0(2)
β_2	0.5(1)	0.4(1)	0.6(1)	0.7(4)
d (fm)	4.9(3)	3.8(4)	4.9(2)	5.9(7)
$R_{c.m.}$ (fm)	20.4(6)	19.3(6)	19.8(6)	20(1)



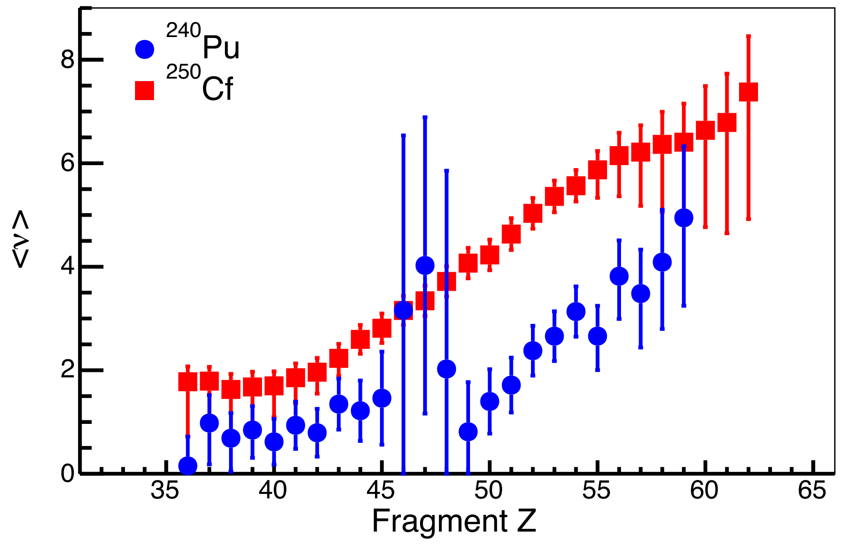
how is the rest of the E* shared?

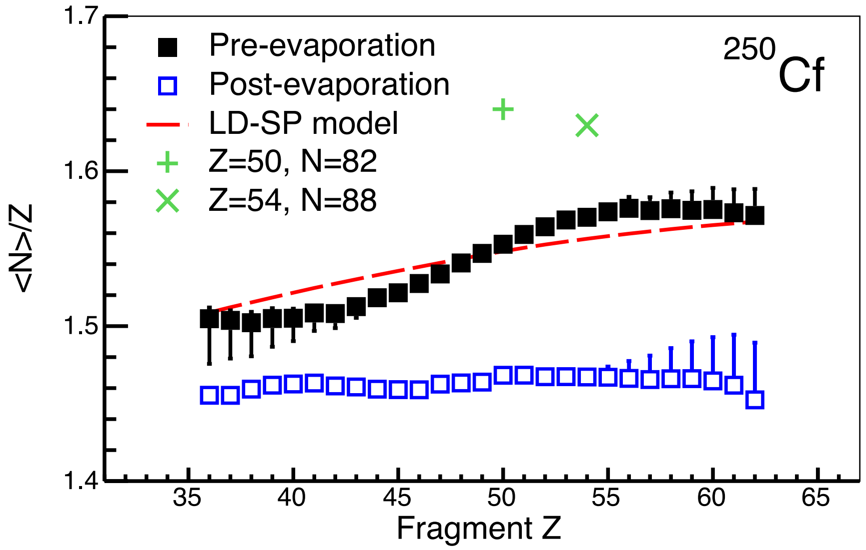
- Mostly, all the modes have configurations around 20 fm, except the SI.
- As we saw previously, the SI mode: is the only deviation from a long corridor.
- Also, more nucleons “blocked” in shells and less on the neck, making it “brittle”? Is this the connection between shells and TKE?

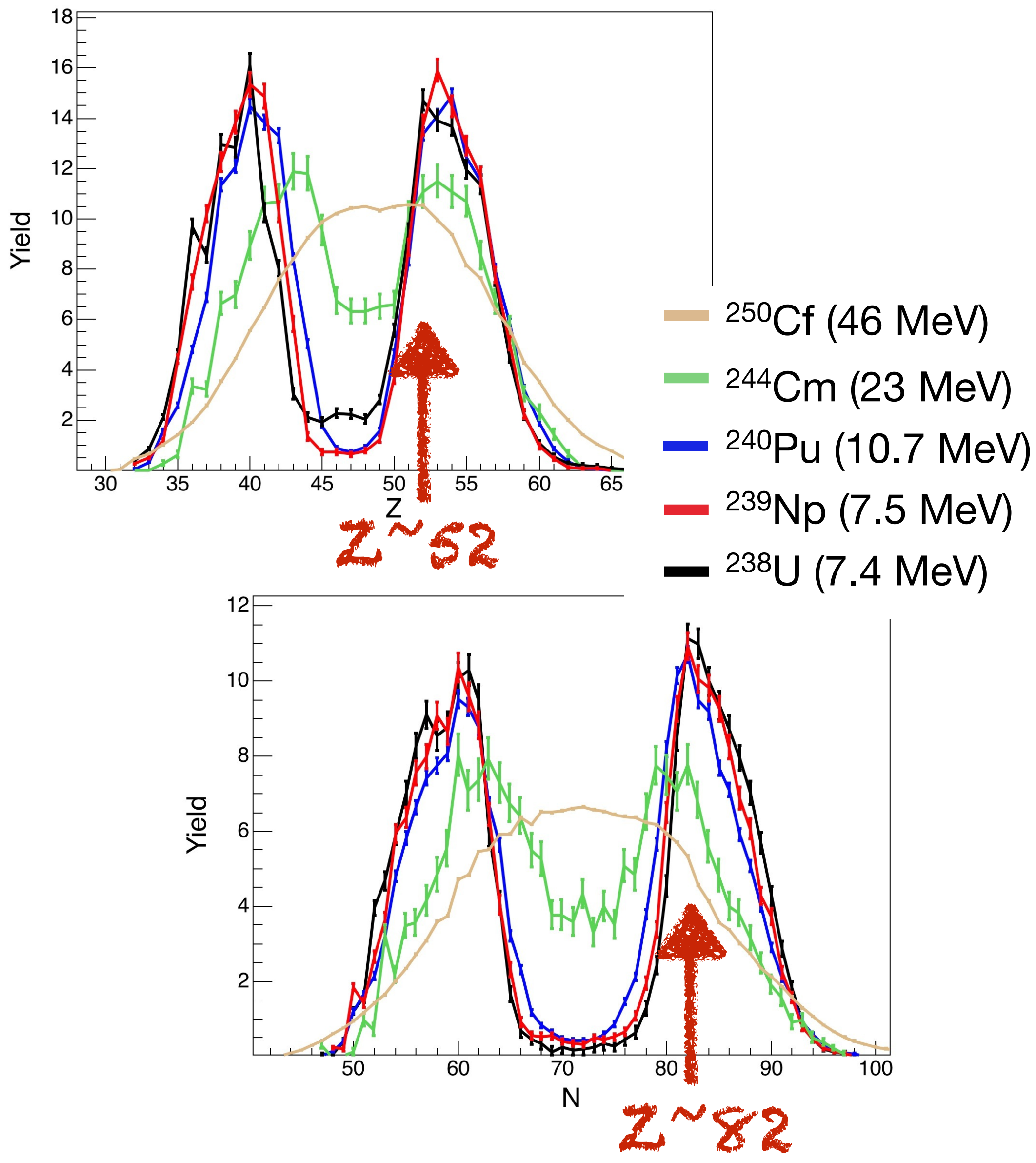
Wrap up

- The calculation of TKE, TXE, neutron multiplicity, and neutron excess at scission was possible with the measurement of the fragments yield, velocity, and as a function of the fragment identification in (Z,A).
- A detailed energy balance at scission with these observables allowed us to estimate the deformation and separation of the emerging fragments.
- The results show that mostly deformed neutron shells are responsible for the fragment deformation.
- The link between these shell effects and the measured TKE is done through the tip distance, hinting at a direct link between structure and the length of the neck.

Next

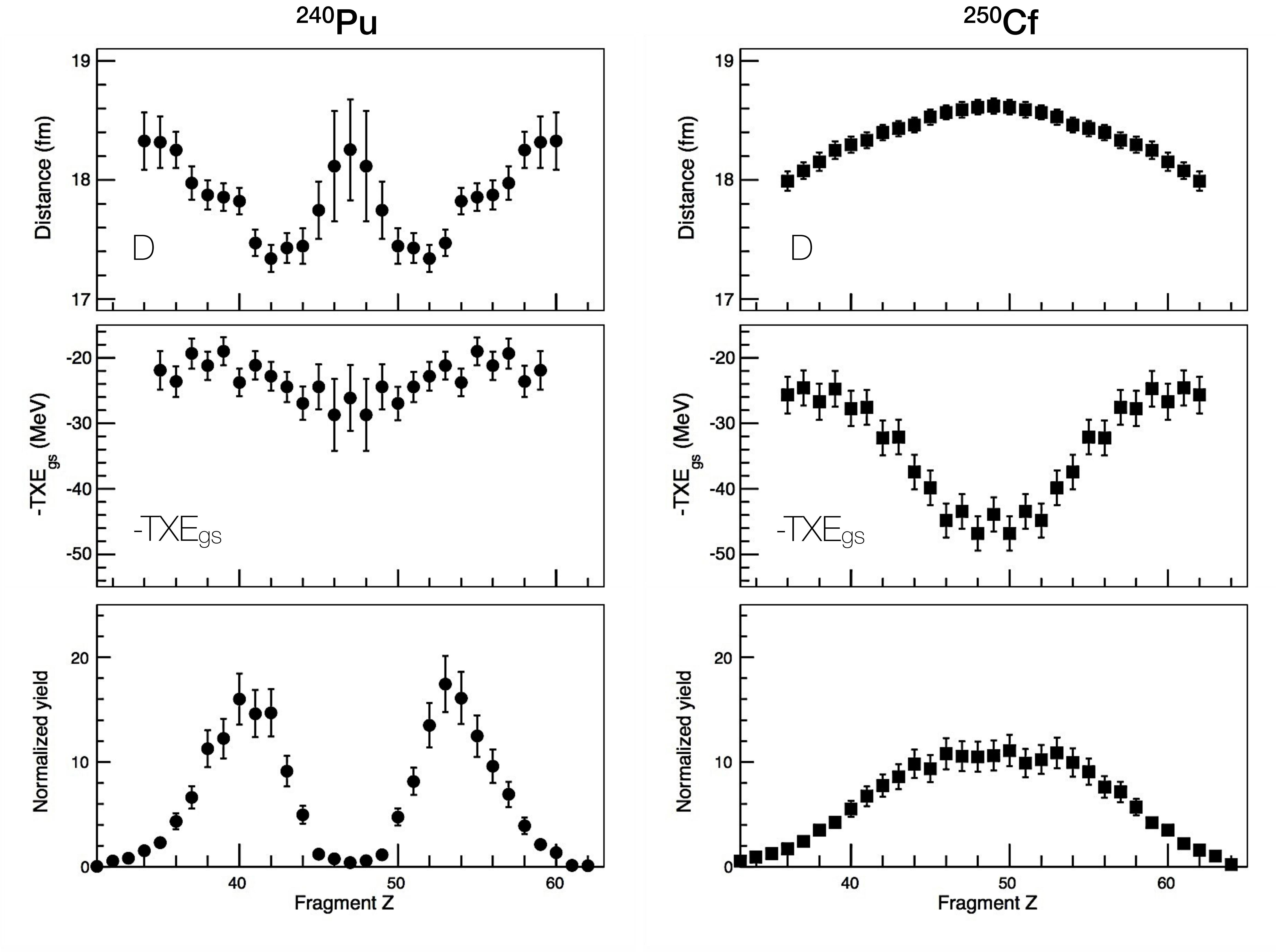






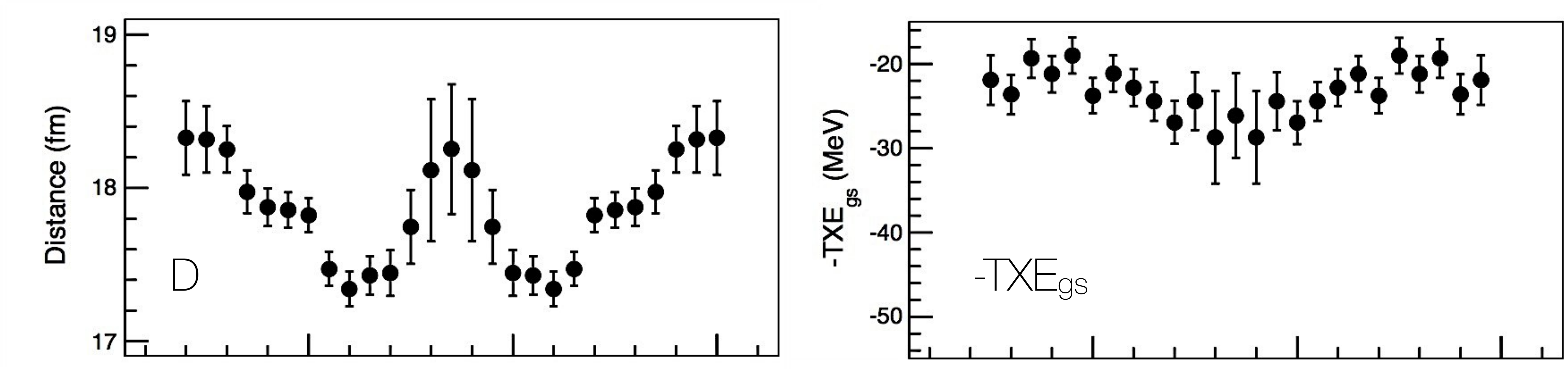
- Investigate the deformation of ^{250}Cf at 42 MeV and its mysterious N/Z
- Study the systematics of Diego's data as a function of E*

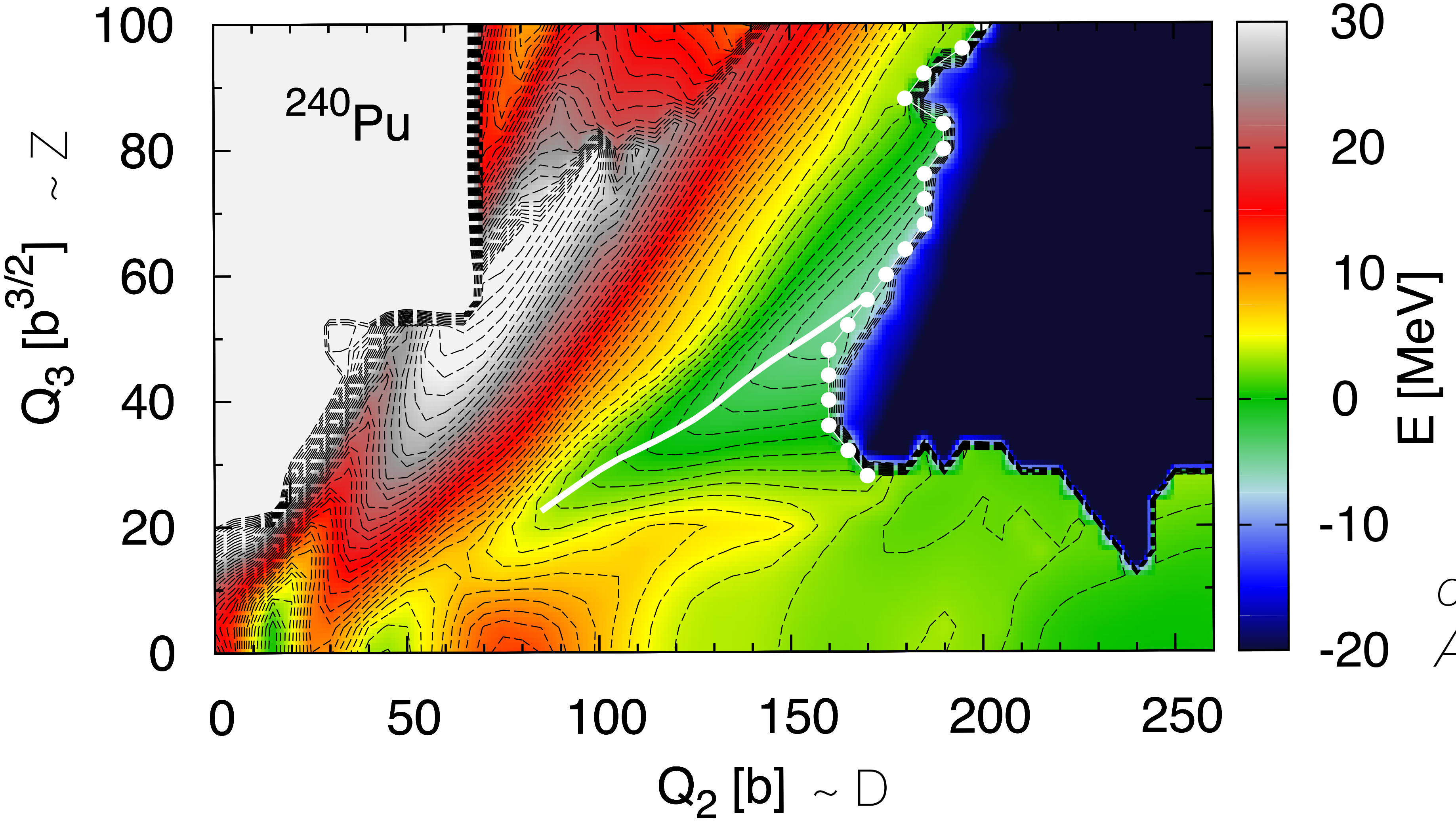
Scission landscape



Scission landscape

^{240}Pu

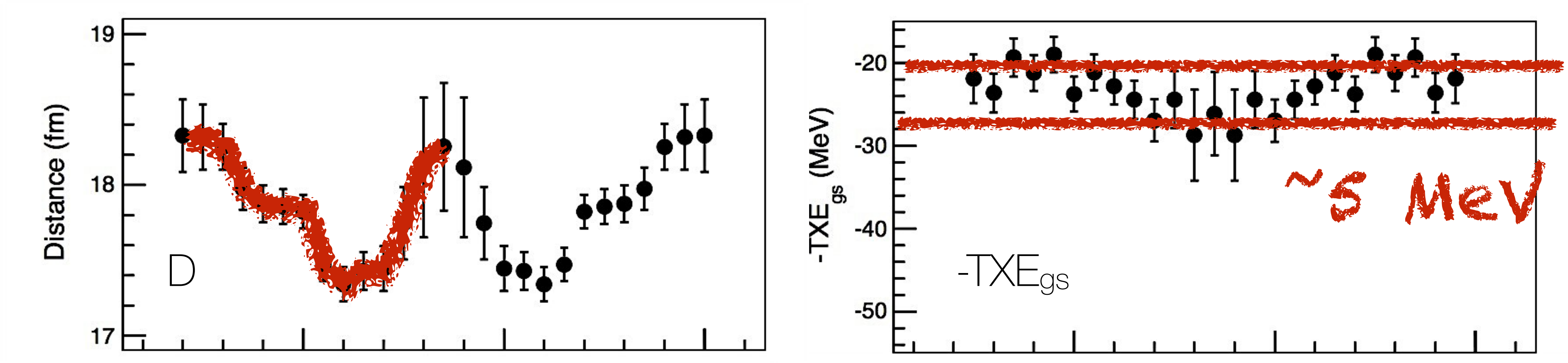


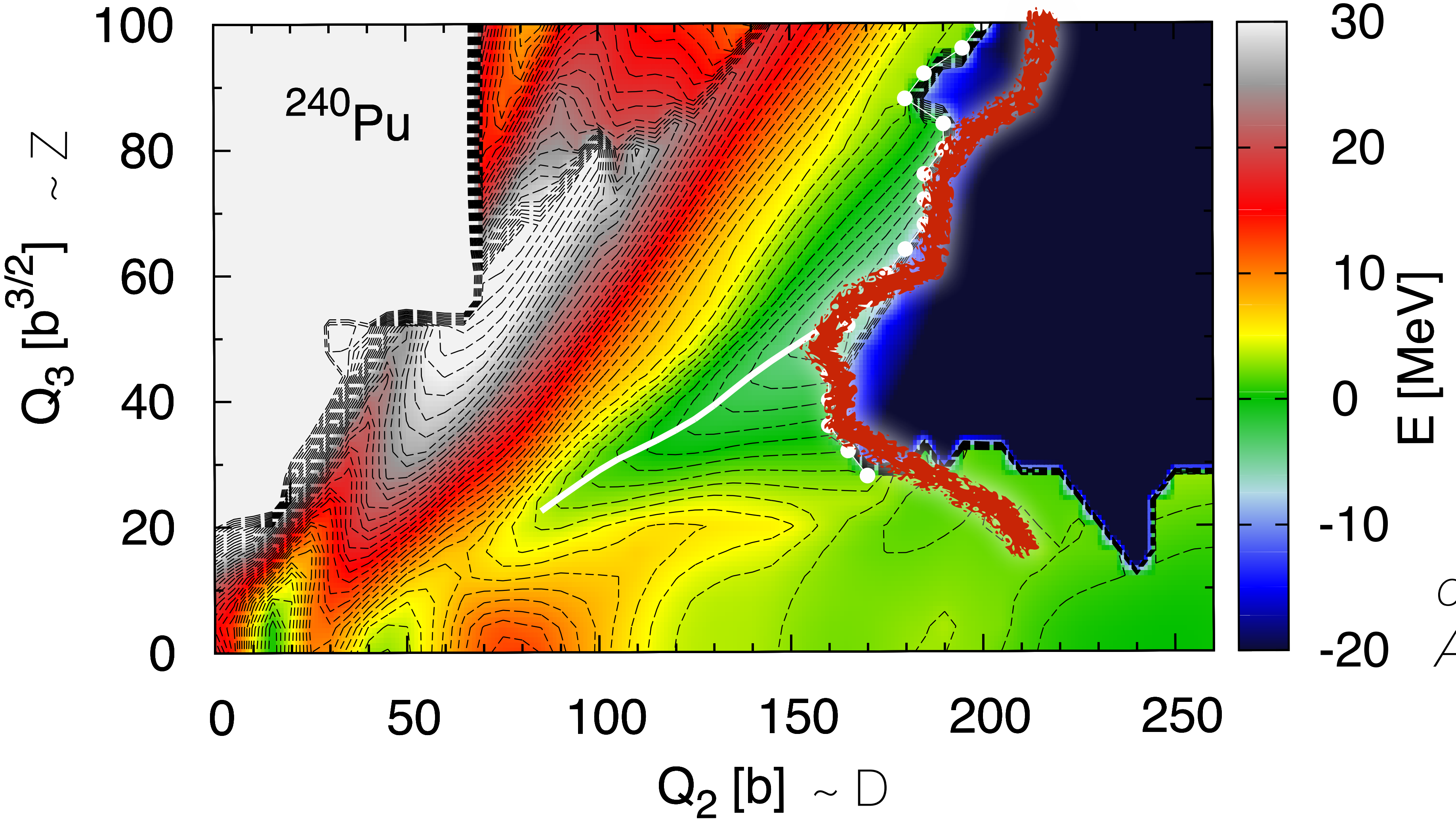


courtesy of Anna Zdeb

Scission landscape

^{240}Pu





courtesy of Anna Zdeb