

Fragment and Cluster Formation in Heavy Ion Collisions



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**FUSTIPEN Topical Meeting on
Dynamical cluster formation and correlations in heavy-ion collisions,
within transport models and in experiments
May 17-19, 2016, GANIL, Caen, France**

Fragment and Cluster Formation in Heavy Ion Collisions

very wide subject: Essentially the subject of the meeting
 introductory remarks and overview,
 subjects discussed in much more detail in later talks

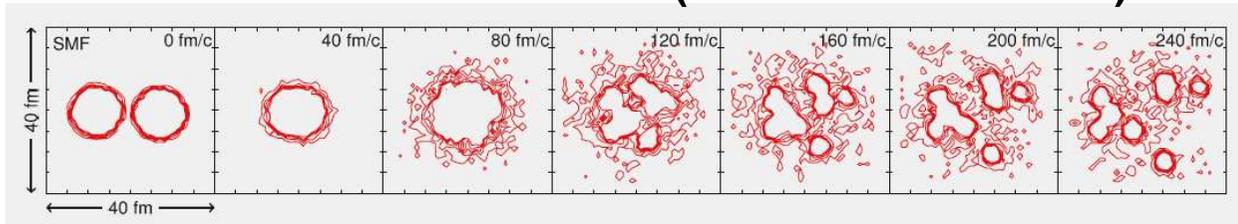
P. Danielewicz, J. Natowitz, P. Napolitani, G. Verde, R. Bougault, et al.

Aim of this talk:

- emphasize the importance of correlations and clustering in the study of the EoS in HIC, in particular the symmetry energy
- discuss what is involved in a proper treatment
- stress simple concepts, and open problems

large fractions of particles in clusters, e.g.

Clusters ($A \leq 4$) (correlation stabilized)



Fragments (Int. Mass Fragn. IMF, $A > 4$) (mean field stabilized)

Partitioning of protons		
	Xe + Sn 50 MeV/u	Au + Au 250 MeV/u
p	≈10%	21%
α	≈20%	20%
d, t, ^3He	≈10%	40%
$A > 4$	≈60%	18%

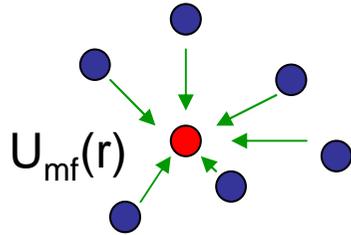
INDRA data, [Hudan et al., PRC67 \(2003\) 064613.](#)
 FOPI data, [Reisdorf et al.](#) from A. Ono

My Collaborators:

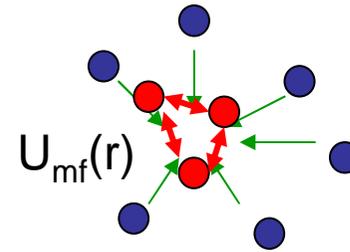
M. Colonna, M. Di Toro, J. Rizzo (Catania), M. Zielinska-Pfabe (Smith College, USA)
 Theo Gaitanos (Univ. Thessaloniki) ; T. Mikhailova (JINR, Dubna)
 Stefan Typel (GSI), Gerd Röpke (Rostock), David Blaschke (Wroclav)
 - but use material from many other workers in the field

the two
main players:

mean field (mf) ↔ clustering



particle in a mean field,
one-body approach



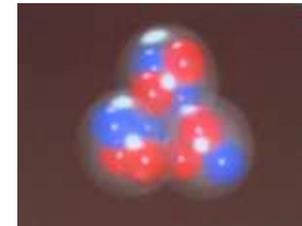
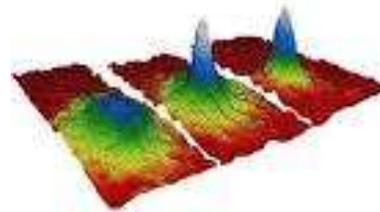
few-body correlation
in a medium
(medium-modification of cluster)

Examples:

1.) e.g. pairing. can be converted into a one-quasi-particle picture.
→ well studied

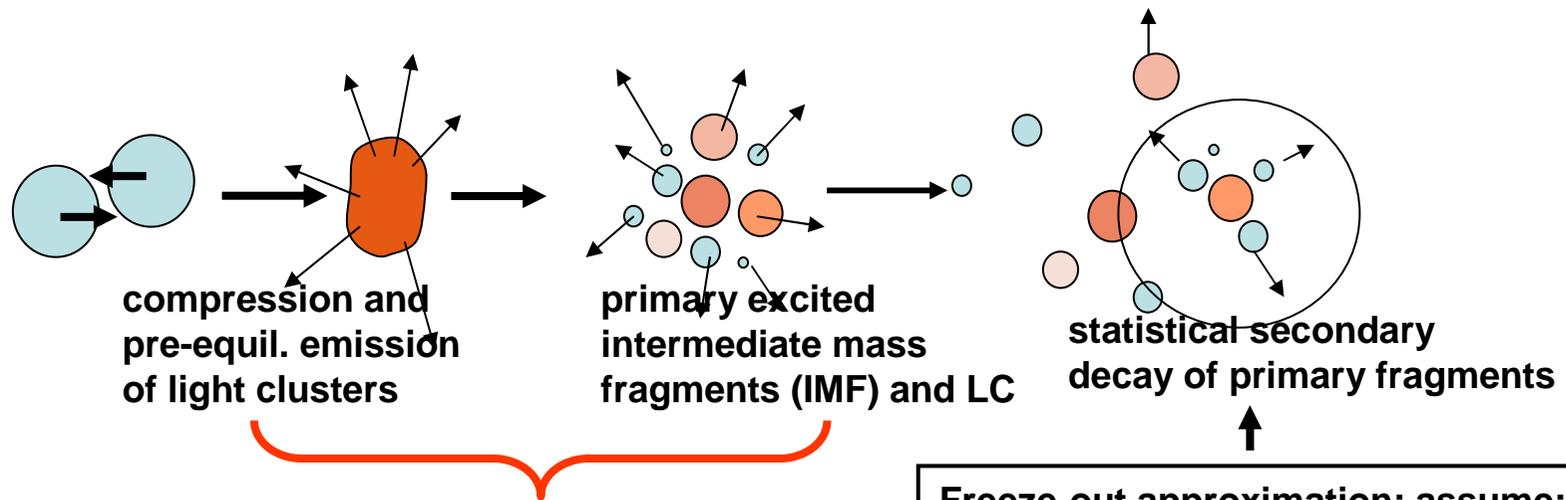
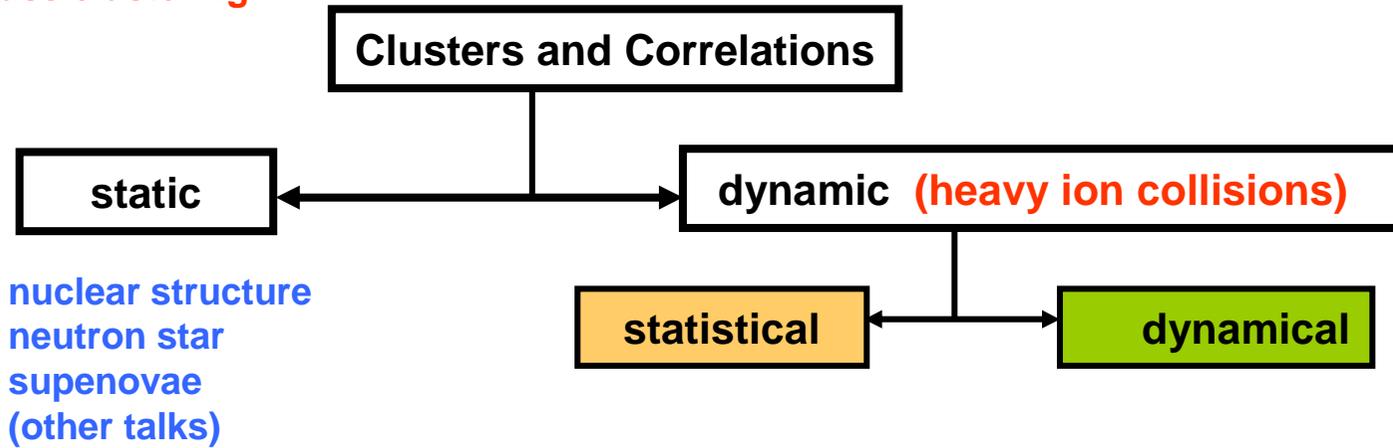
2.) quartetting, α -correlations

3.) BEC, other fields,...



... correlations: the seeds to clustering and cluster production

scheme to discuss clustering

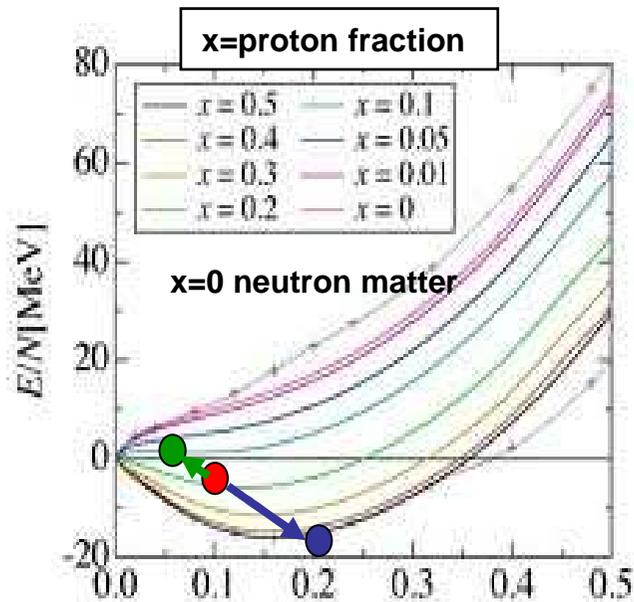


statistical or dynamical?

in statistical treatment often use grand canonical ensemble: T, μ
„unreasonable successfull“, examples

Freeze-out approximation: assume: $A, Z, \text{ volume, excitation energy } E^*/A$
statistical decay code: SMM, Gemini++, SIMON, etc. („afterburner“)
canonical ensemble: E, ρ

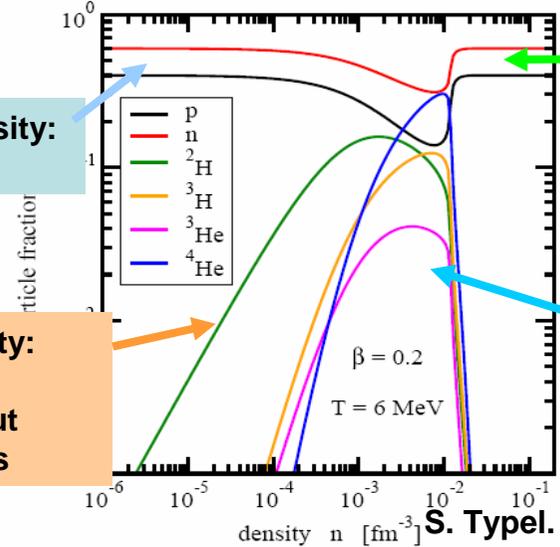
Remark about statistical application: Clustering of very dilute nuclear matter



composition as fct of density; $x=0.2, T=6$ MeV

very low density: p,n

Increasing density: clusters arise: deuteron first, but then α dominates



Mott density: clusters melt, homogeneous p,n matter;

here heavier nuclei (embedded into a gas) become important here

S. Typel. et al., PRC 81(2010)

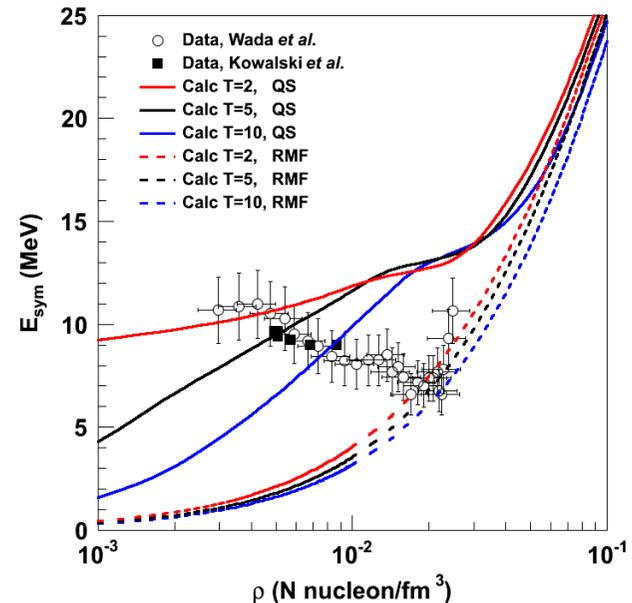
decrease energy by inhomogeneity
 \rightarrow fractionation into clusters of higher density and neutron gas

Can be checked in heavy ion collisions
under the assumption of statistical decay

Semi-central heavy ion collisions,
 ($^{64}\text{Zn} + ^{92}\text{Mo}, ^{197}\text{Au}$ at 35MeV/A)
 and time-resolved measurement of light fragments from decay of fireball:

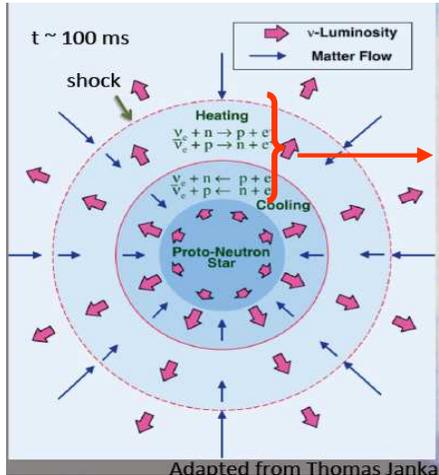
S. Kowalski, J. Natowitz, et al., PRC75 014601 (2007)
 J. Natowitz, G. Röpke, ..., PRL 104, 202501 (2010)

extract symmetry energy and compare with quantumstat. calculation of clustered matter



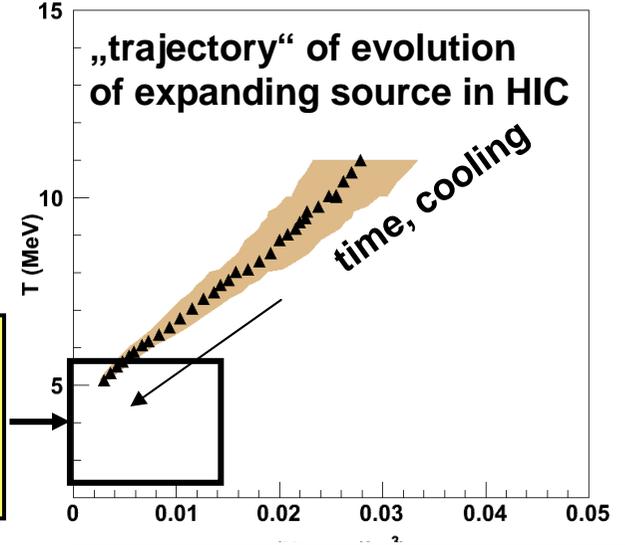
Relevance for Supernovae physics?

Results relevant for neutrino opacity
in ν -sphere in Core Collapse Supernovae
→ workshop in ECT*, Trento, 2014



ν -sphere

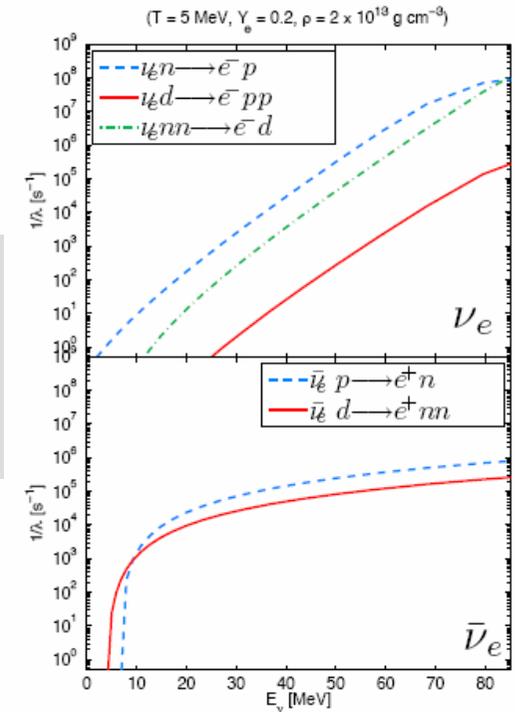
conditions of neutrinosphere:
densities 1/1000 to 1/10 ρ_0
temperature $T=1-5$ MeV
asymmetry $Y_e=0.1 - 0.25$



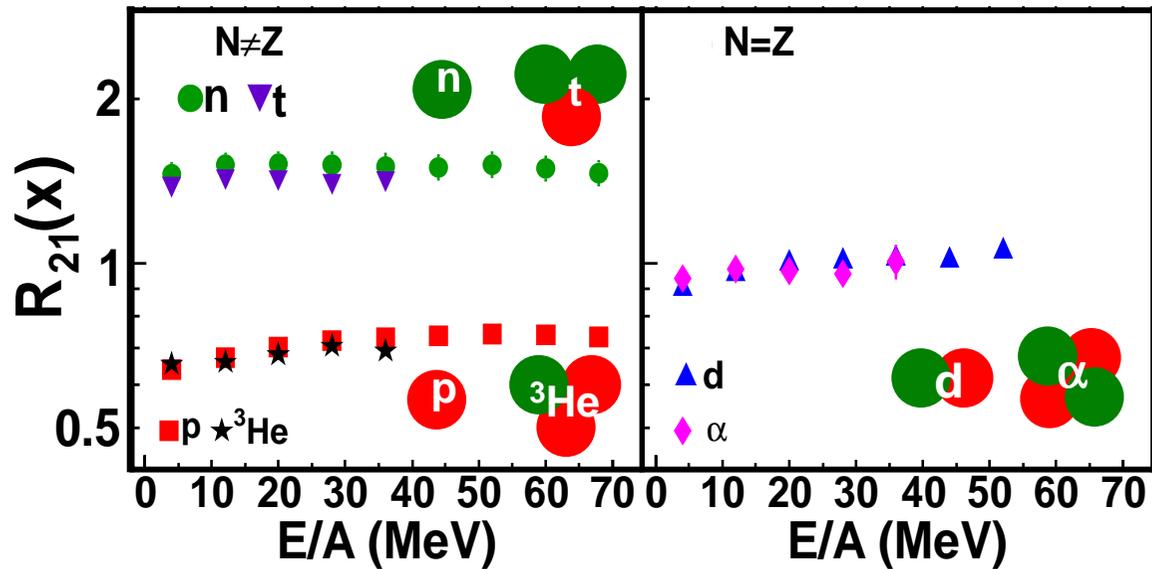
Super-Nova ↔ Femto-Nova (heavy ion collision)

calculation by T. Fischer (Wroclaw),
 ν and anti- ν absorption on nucleons
and light clusters,
effect of clustering does not seem to
be large

but see also talk of S. Burrello at IWM-EC15 on influence of
pairing correlations



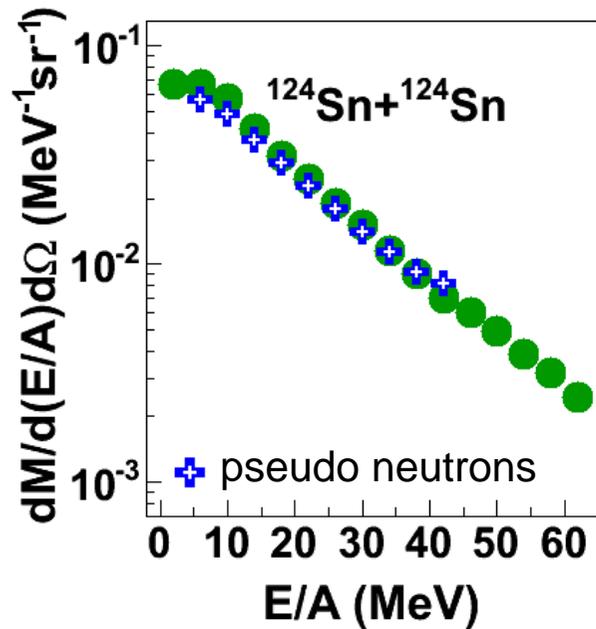
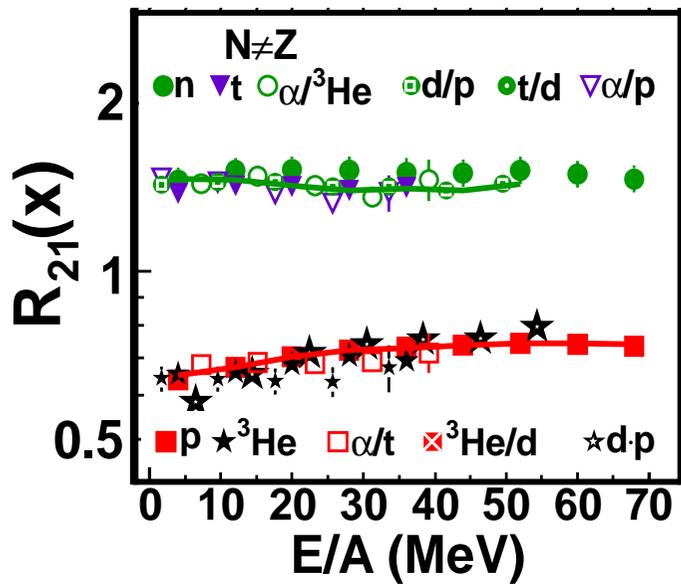
„Chemical potential scaling“ Z.Chajecski, et al., arXiv 1402.5216



$$R_{21}(N, Z) = \frac{dM_2(N, Z)}{dM_1(N, Z)}$$

expect from chemical potential dependence without correlation contributions

$$R_{21}(N, Z) = \exp\left[\frac{(N\Delta\mu_n + Z\Delta\mu_p)}{T}\right]$$



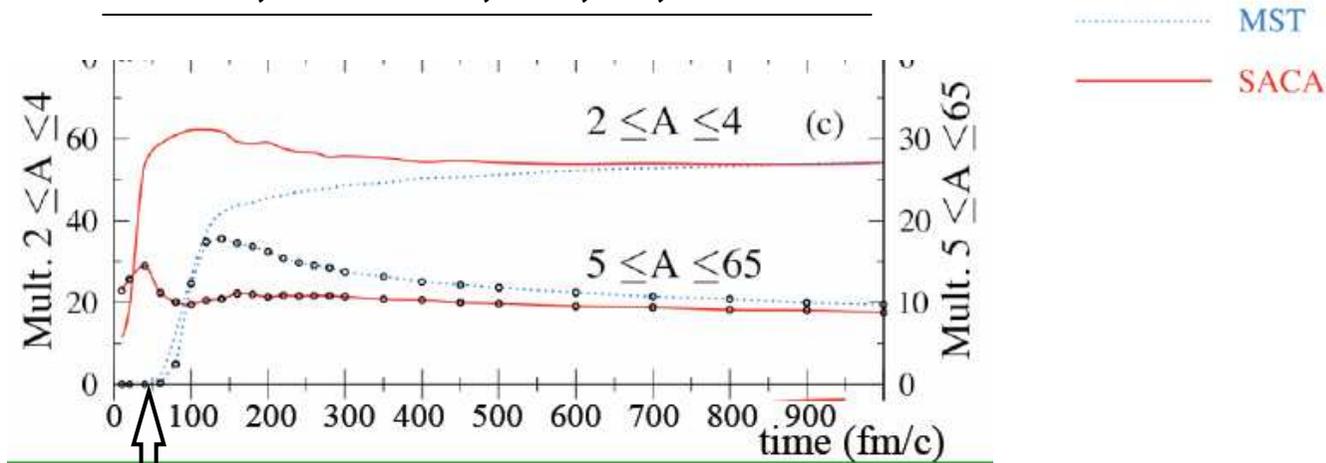
$$\Rightarrow Y(n) = \frac{Y(t)}{Y(^3\text{He})} Y(p)$$

“Pseudo neutron yields”

correlation effects seem to be small for light cluster yields (?)

Dynamical cluster and fragment formation in HIC

Formation history of clusters and fragments in a HIC
Au+Au, 150 AMeV, $b=3$, fm,



SACA (simulat. Annealing and Cooling Algorithm):
clusters identified earlier than in coalescence methods (MST minimal spanning tree algorithm) (Vermani,..Aichelin, J.Phys.G 37 (2010)),

→ early identification in SACA method does not influence dynamics,

→ **but indicates that fragments are dynamical**

→ **what determines the formation of clusters and fragments?**

correlations



fluctuations

The crucial issue in Fragment and Cluster formation in HIC collisions:



How correlations got lost:
BBGKY hierarchy of coupled
Green fcts. is truncated (formally)
by introduction of self energy Σ

$$D(1)G^{(1)}(1,2) = \delta(1-1') + (12 | V | 1'2')G^{(2)}(12,1'2')$$
$$=: \delta(1-1') + \Sigma(1,1'')G^{(1)}(1'',2)$$

This neglects higher order correlation effects

They have to reintroduced

- in the form of fluctuations (for fragments, IMF)
- explicitly (for light clusters, LC)

discuss next , how this is handled in BUU and QMD approaches

1. Boltzmann-Uehling-Uhlenbeck (BUU)

$$f(\vec{r}, \vec{p}; t)$$

1-body phase space distribution fct.

$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{m} \vec{\nabla}^{(r)} f - \vec{\nabla} U(r) \vec{\nabla}^{(p)} f(\vec{r}, \vec{p}; t) = \int d\vec{v}_2 d\vec{v}_1 d\vec{v}_2' v_{21} \sigma_{12}(\Omega) (2\pi)^3 \delta(\vec{p}_1 + \vec{p}_2 - \vec{p}_1' - \vec{p}_2') [f_1' f_2' (1-f_1)(1-f_2) - f_1 f_2 (1-f_1')(1-f_2')]$$

- Derived: → From non-equilibrium theory (Kadanoff-Baym); collision term included,
 → quasi-particle approx., i.e. spectral function on-shell
 → deterministic, no fluctuations
 → test particle method, exact for NTP → ∞

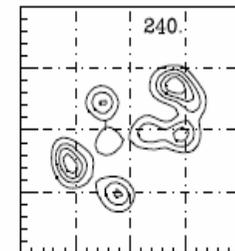
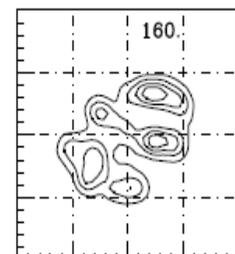
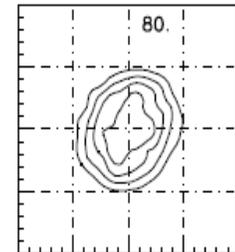
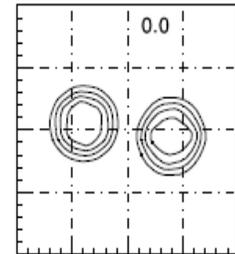
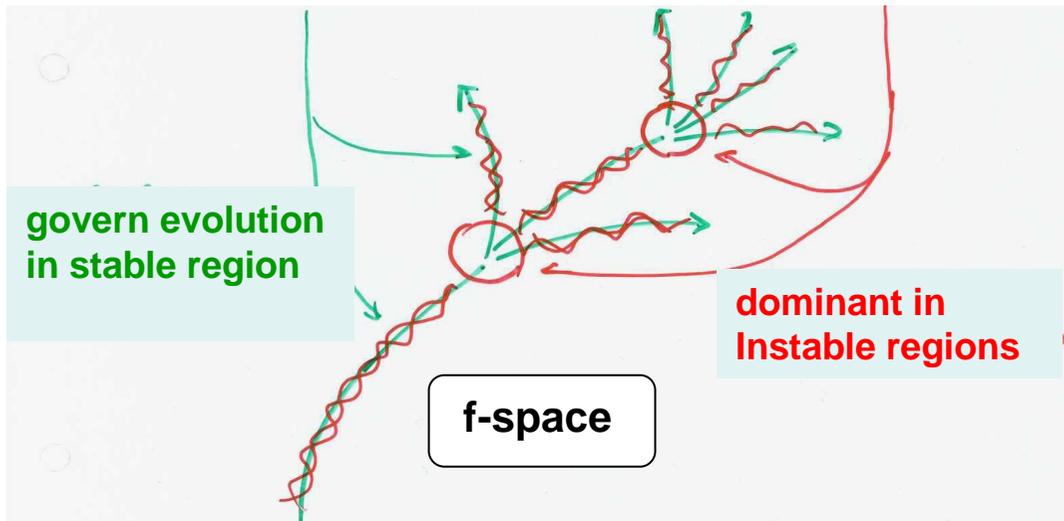
include fuctuations around dissipative solution

$$f(r, p, t) = \bar{f}(r, p, t) + \delta f(r, p, t)$$

Mean field evolution (dissipative) fluctuations

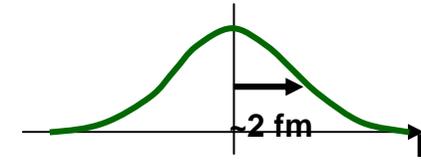
$$\frac{df}{dt} = I_{\text{coll}} + I_{\text{fluc}}$$

Boltzmann-Langevin eq.



2. Molecular Dynamics (QMD)

can be derived in two ways (two parents)



a) Classical Molecular Dynamics with Gaussian particles to reduce fluctuations

b) from Time-Dependent Hartree, using a product wave function

$$\Psi(\vec{r}_1, \dots, \vec{r}_N; t) = \prod_i \phi_i(\vec{r}_i; t)$$

$$\phi_i(\vec{r}_i; t) = \frac{1}{(2\pi L)^{3/4}} \exp\left(-\frac{(\vec{r}_i - \vec{R}_i(t))^2}{4L} + i\vec{r}_i \vec{P}_i(t)\right)$$

both lead to the same equations of motion of wave packets centers

$$\frac{dr_i}{dt} = \{r_i, \mathcal{H}\}; \quad \frac{dp_i}{dt} = \{p_i, \mathcal{H}\}; \quad H = \sum_i t_i + \sum_{i<j} V_{ij}$$

c) Antisymmetrized MD (AMD, FMD):

Slater determinant wave function, antisymmetrization, eom are more complicated also approximatively CoMD (constrained MD)

d) in all cases add a phenomenological collision term

**Thus QMD has no quantum correlations,
but classical N-body correlations, damped by the smoothing.**

However, more fluctuations, since dof are nucleons and **not** test particles:

→ more fluctuations in representation of phase space distribution

→ more fluctuation gained from collision term

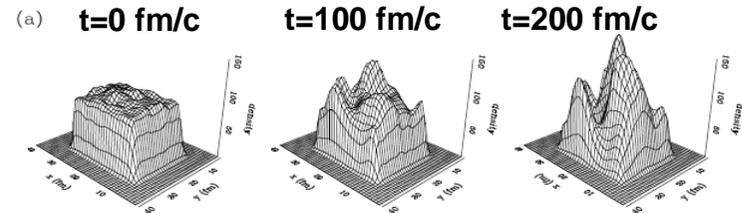
→ amount controlled by width of single particle packet L

**Intermediate conclusion:
Both BUU and QMD do not naturally have the correct fluctuations (except AMD)**

way out???
answer is different
for LCs ($A \leq 4$) and IMFs ($5 \leq A \leq \approx 30$)

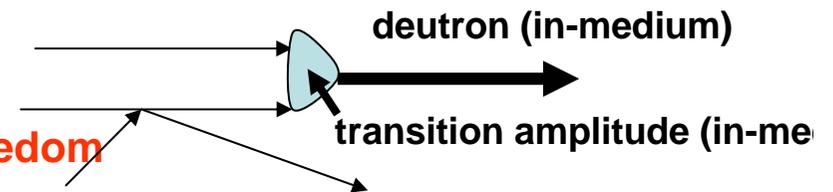
⇒ IMF: develop from fluctuation as seeds
which are amplified by the mean field
**issue: correct amplitude and spectrum of
fluctuations**

BUU calculation in a box (i.e. periodic boundary conditions) with initial conditions inside the instability region: $\rho = \rho_0/3$, $T = 5$ MeV, $\delta = 0$



→ Formation of „clusters (fragments)“, from small (physical) fluctuations in the density. (V. Baran, et al., Phys.Rep.410,335(05))

⇒ LC: correlation dominated
(common density functionals are not sophisticated enough to describe LC properly)
Issue: Introduce LCs as explicit degrees of freedom formed in 3-body collisions
(P. Danielewicz and Q. Pan, PRC 46 (1992)) (d,t,3He, **but no α !**)



discuss next, how this is done in BUU and QMD

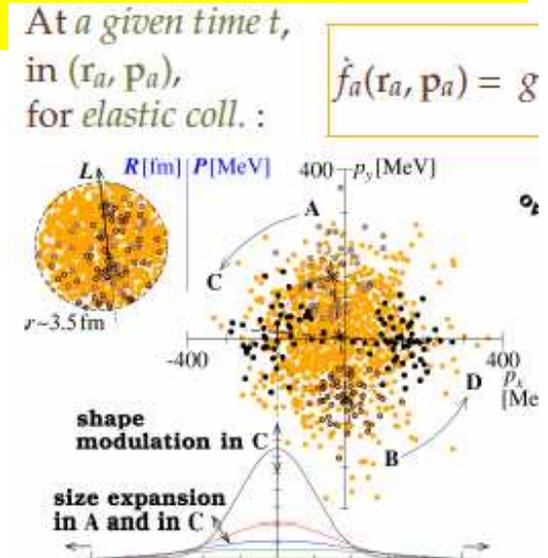
	LC	IMF
BUU	pBUU	BLOB
QMD	clustAMD	wp width

Methods to introduce fluctuations

BUU: statistical fluctuation of the mean field distribution function f in a Fermi system is $\sigma_f^2(r,p) = \bar{f}(r,p)(1-f(r,p))$

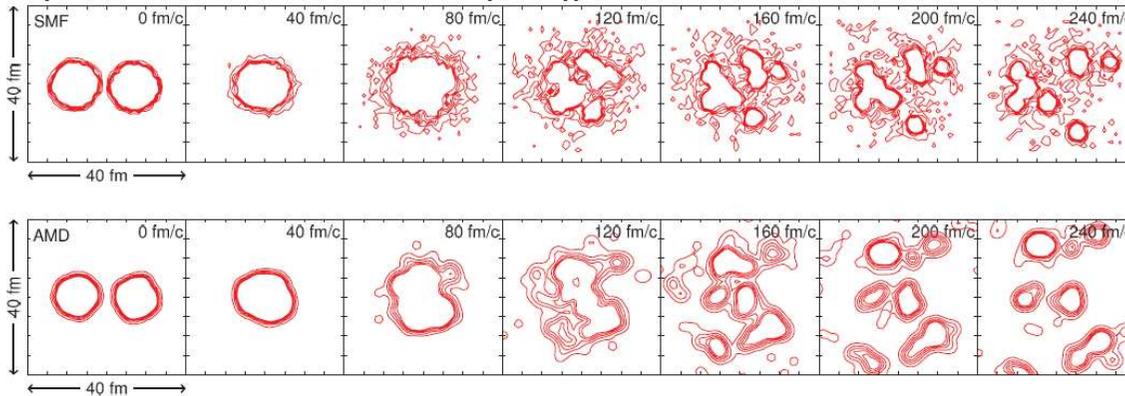
SMF (stochastic mean field): project on density fluctuations and introduce these „by hand“
BLOB (Boltzmann-Langevin One-Body dynamics) Move N_{TP} testparticles simultaneously (in p-space) to simulate fluctuation connected to NN collisions

QMD: fluctuations controlled by wave packet width L :
 $L \rightarrow 0$ classical point partucles, nuclei not bound
 $L \rightarrow \infty$ complete smoothing, no fluctuations

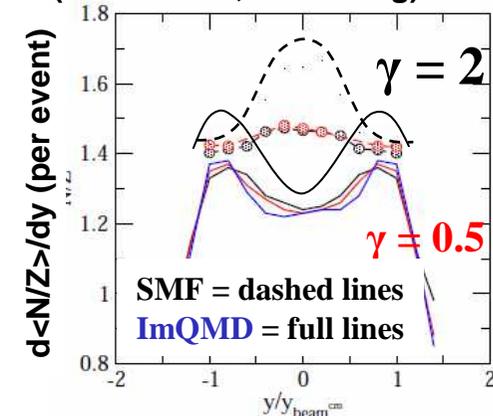


Comparison of fluctuations:

Comparison of simulations: BUU(SMF)-AMD:
 (Rizzo, Colonna, Ono, PRC82 (2010))



Comparison, SMF-ImQMD:
 more transparency in QMD
 (M. Colonna, X.Y.Zhang)



Check fluctuations in the context of the Code Comparison Project see talk at IWM-EC 2016

check consistency of transport codes in calculations with same system (Au+Au), $E=100,400$ AMeV, and identical (simple) physical input (mean field (EOS) and cross sections)

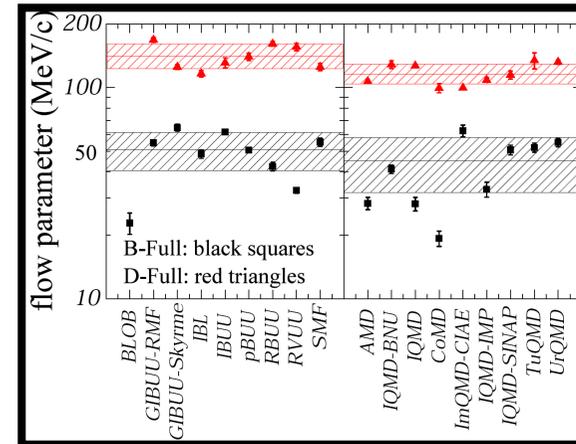
idea: establish sort of **theoretical systematic error of transport predictions** (and hopefully to reduce it)

quantify spread of simulations by value of „flow“=slope at midrapidity

BUU and QMD approx. consistent

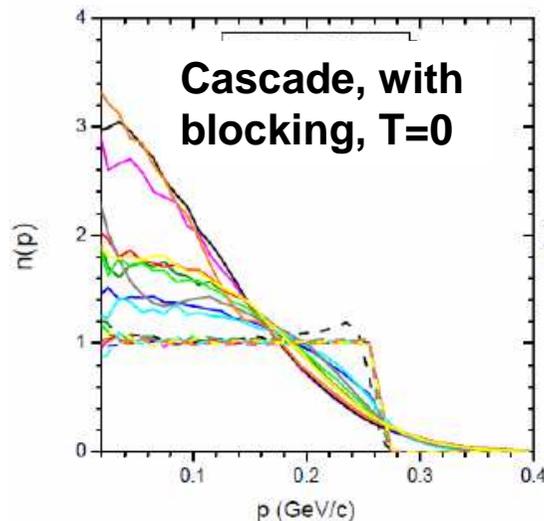
uncertainty

100 AMeV: ~30%
400 AMeV: ~13%

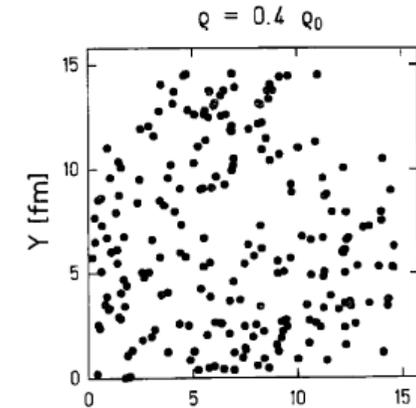
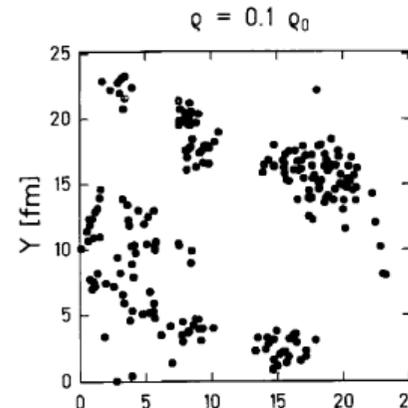


further development: box calculations

test collision routine and Pauli blocking under controlled conditions;



test also fluctuations and fragmentation



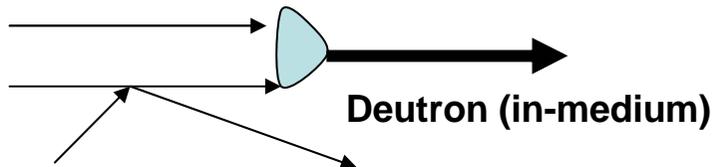
Treatment of Light Cluster dynamics in HIC: BUU

circumvent: compare to „coalescent invariant“ cross sections
only justified if clusters play no dynamical role

solutions: different in BUU and Mol.Dyn.(MD) models:

Solution for BUU models:

LC distribution functions as explicit
degrees of freedom of type $NNN \rightarrow N\Delta$
(P. Danielewicz and Q. Pan, PRC 46 (1992))
(d,t,3He, **but no α !**)
→ coupled transport equations



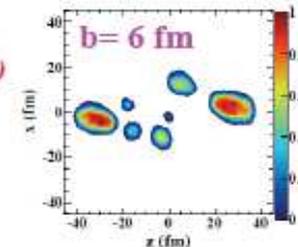
Caveat: Medium properties of LC:

Medium modification of properties and transition
amplitudes of light clusters in heavy ion reactions
C. Kuhrt, Beyer, Danielewicz,..PRC63 (2001) 034605

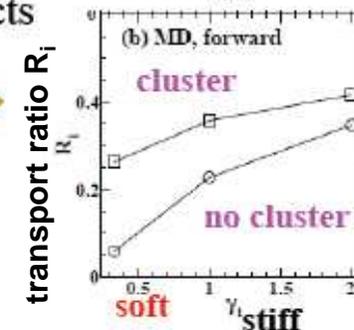
Calculated in nuclear matter and static nuclei in Generalized
RMF approach by Typel, Röpke, et al., PRC81 (2010)
→ see talk by S. Typel

pBUU calculations (Danielewicz)

Coupland et al.,
PRC 84, 054603 (2011)



Cluster effects
(d,t,He³)

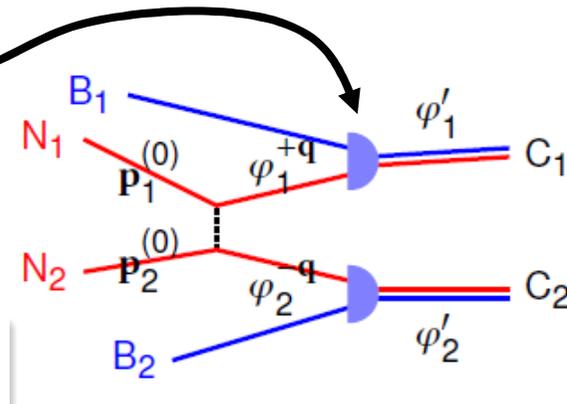


R_i : isospin transport ratio for charge
equilibration in HIC between nuclei
with different isospin content
e.g. $^{112,124}\text{Sn} + ^{112,124}\text{Sn}$
(MSU experiment)

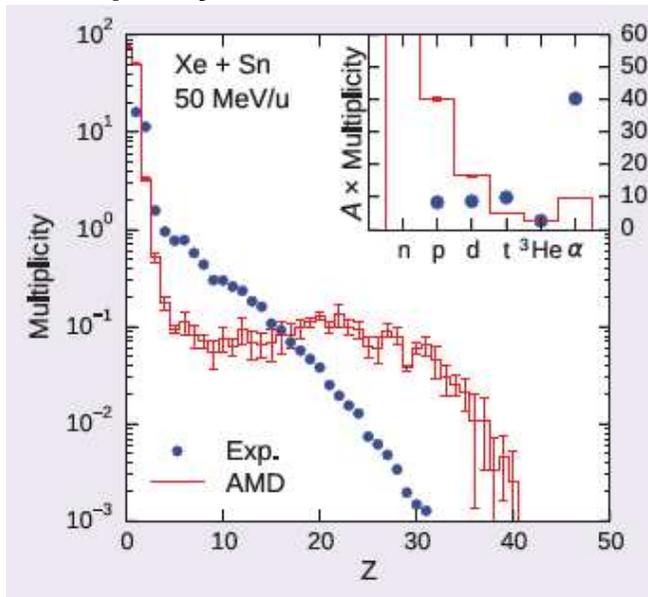
Treatment of Light Cluster dynamics in HIC: QMD

Solution für AMD

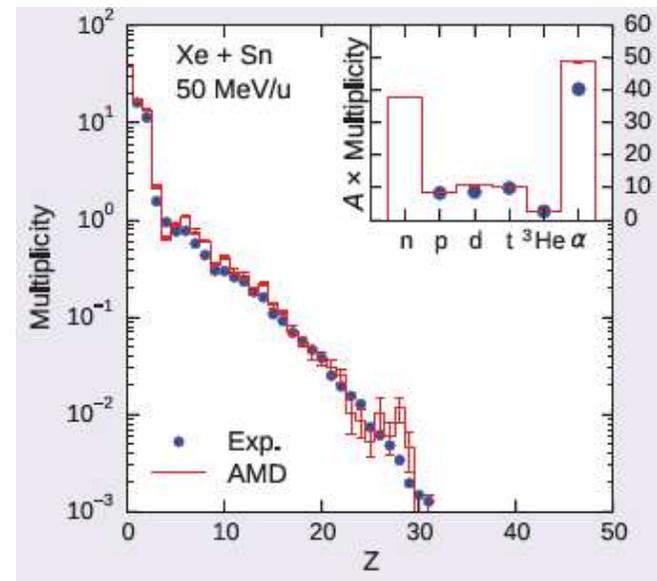
1. in collision term consider formation of clusters in terms of overlap with cluster wave function
2. manipulate phase space: put wave packets of cluster constituent in one place (conservation laws?)
3. consider Pauli principle fully
4. include also cluster-cluster collisions to form bigger clusters



multiplicity distribution w/o clusters



with clusters and cluster-cluster collisions



(A. Ono, NuSYM2015)

Clustering \longleftrightarrow Symmetry Energy. Relevance to workshop?

nucleons come in two flavors: n, p

→ nn,pp interactions different from pn interaction (stronger)

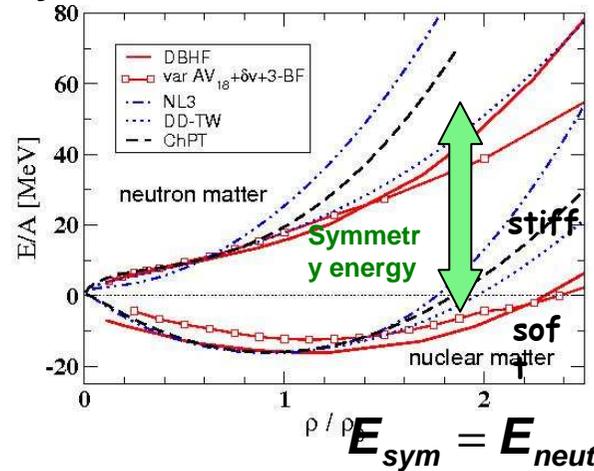
→ in asymmetric system: → U_p and U_n different → symmetry energy

density-
asymmetry dep.
of nucl.matt.

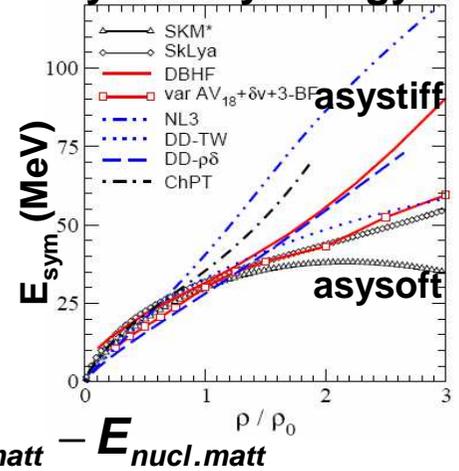
$$\varepsilon(\rho_B, \delta) = \varepsilon_{nm}(\rho_B) + \mathbf{E_{sym}(\rho_B)} \delta^2 + \mathbf{O(\delta^4)} + \dots \quad \delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

$$U(\rho, k; \delta) = \frac{\partial \varepsilon(\rho, \delta)}{\partial f(\rho, k)} = \underbrace{U_0(\rho, k)}_{U_\tau(\rho, k)} + \mathbf{U_{sym}(\rho, k)} (\tau \delta) + \dots$$

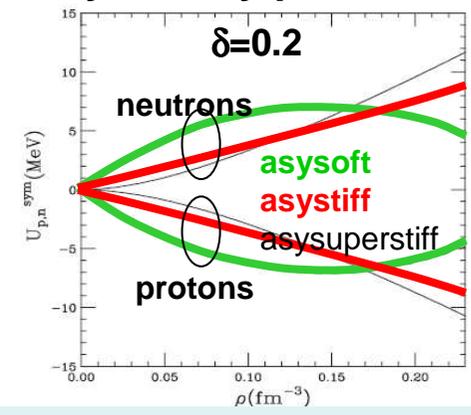
symmetric and neutron matter



symmetry energy



symmetry potential



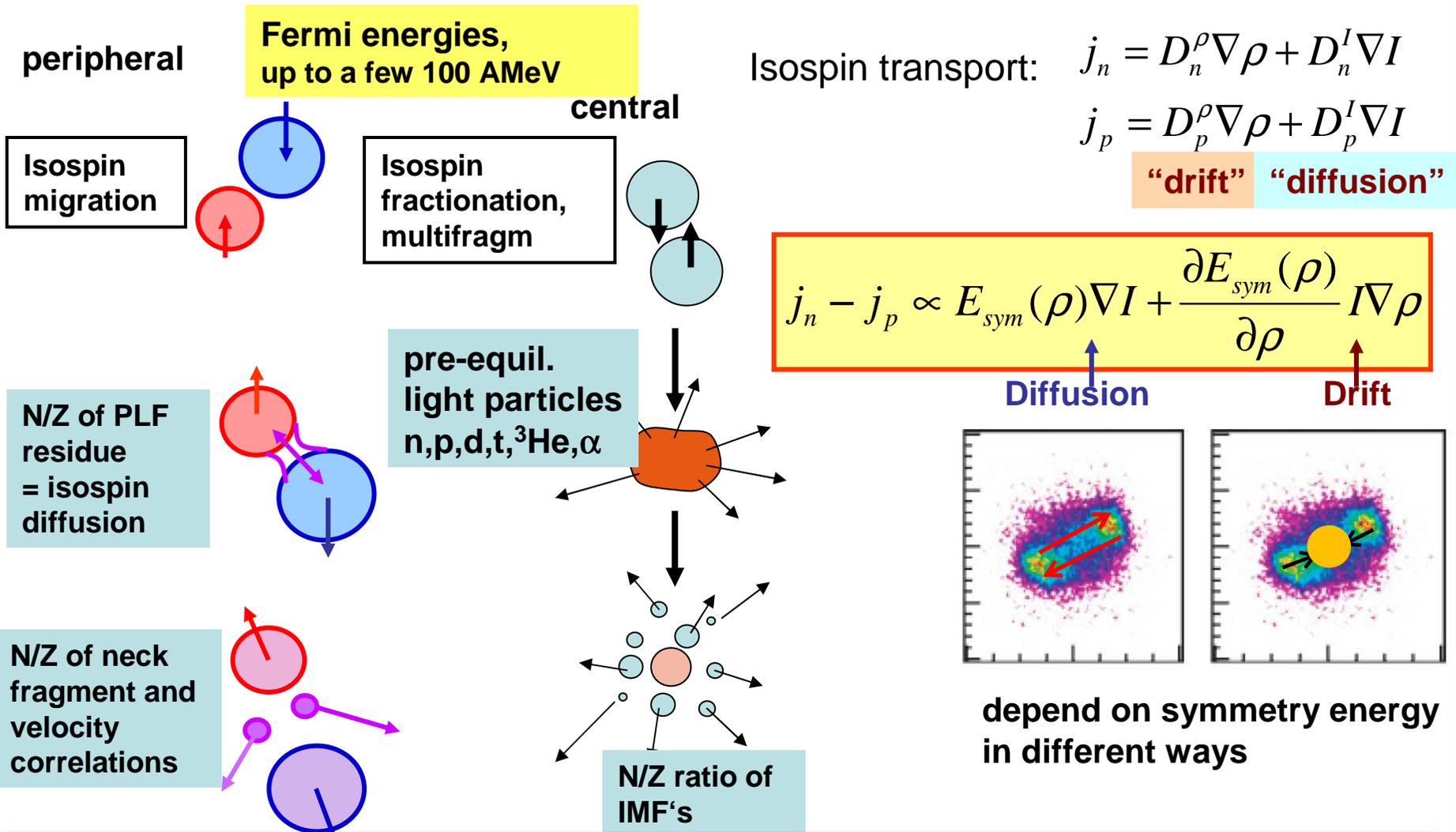
C. Fuchs, H.H. Wolter, EPJA 30(2006)5

..also momentum dependence
→ effective mass splitting

where does the symmetry energy come in?

1. clusters properties are driven by the symmetry energy, i.e. the N/Z ratio
2. isospin fractionation between clusters and gas
3. clusterization gives a direct contribution to the symmetry energy:
correlation depends on asymmetry of system; stronger in symmetric system

Isospin sensitive Observables in Heavy Ion Collisions



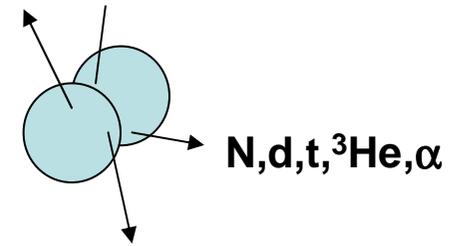
Symmetry energy effects are small absolutely (about 10%)

→ use **differences or ratios**, to eliminate uncertainties in the isoscalar sector.

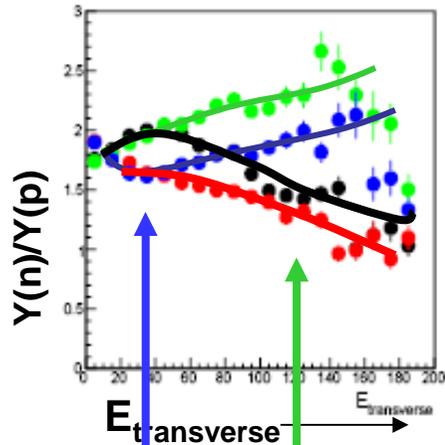
→ successful applications for many observables:

e.g. isospin transport and diffusion, liquid-gas phase transition, etc.

Pre-Equilibrium Emission of Nucleons or Light Clusters

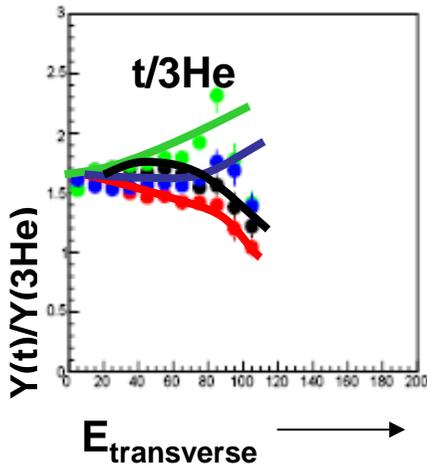


136Xe+124Sn, 150 MeV



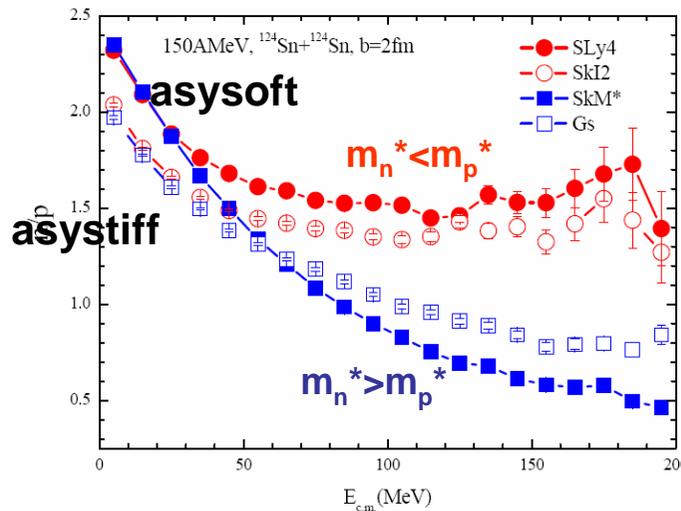
son: asysoft, $m_n^* > m_p^*$
 stn: asystiff, $m_n^* > m_p^*$
 sop: asysoft, $m_n^* < m_p^*$
 stp: asystiff, $m_n^* < m_p^*$

density dep. dominates for slow particles;
 mom.dep. (effective mass) for fast particles,
 → separate density and momentum dependence



similar findings for Sn+Sn collisions (MSU)

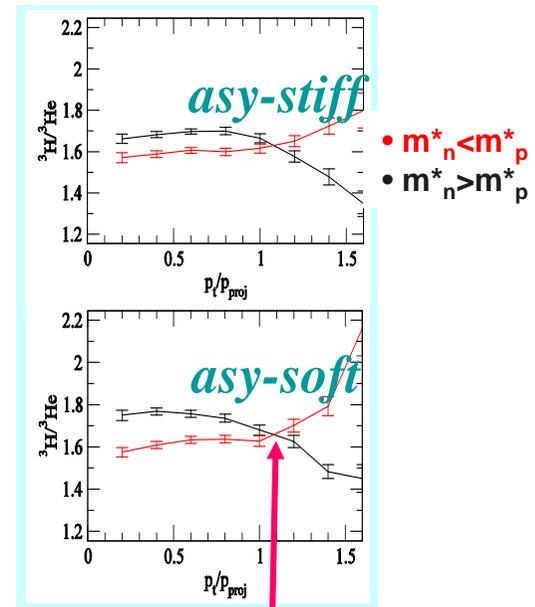
124Sn+124Sn, 150 MeV



Y. Zhang, et al., PLB 732, 186 (2014)

197Au+197Au

600 A MeV b=5 fm, $|y_0| \leq 0.3$



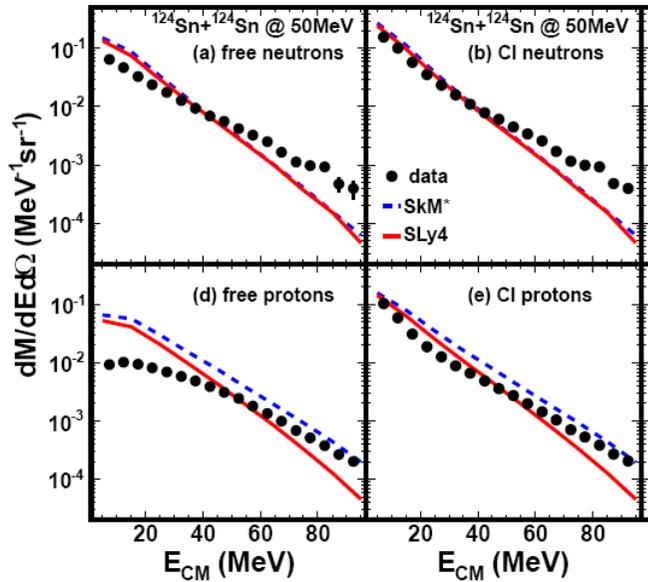
crossing connected to crossing of Lane potentials

effect of effective mass more prominent than that of asystiffness

(V.Giordano, et al., PRC 81(2010))

Comparison with data: problem of light cluster description in transport approaches

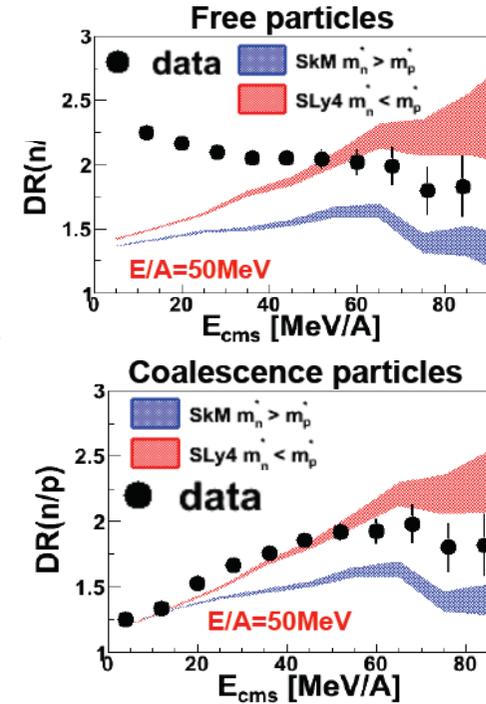
p,n spectra: free coalescence invariant (CI)



Double Ratios

$$\frac{^{124}\text{Sn} + ^{124}\text{Sn}}{^{112}\text{Sn} + ^{112}\text{Sn}}$$

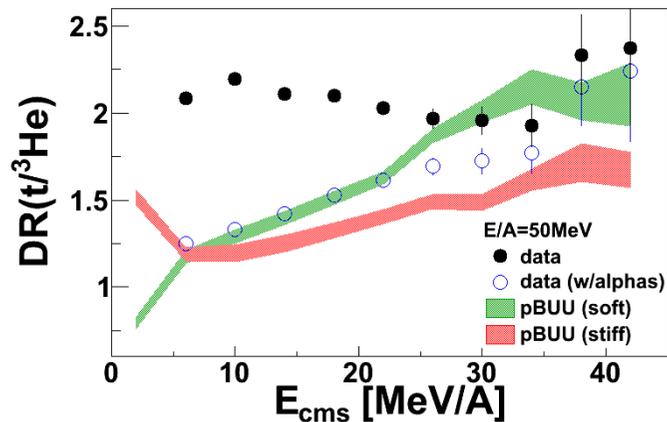
agree only for
CI spectra



..or with calculation, where clusters are included explicitly

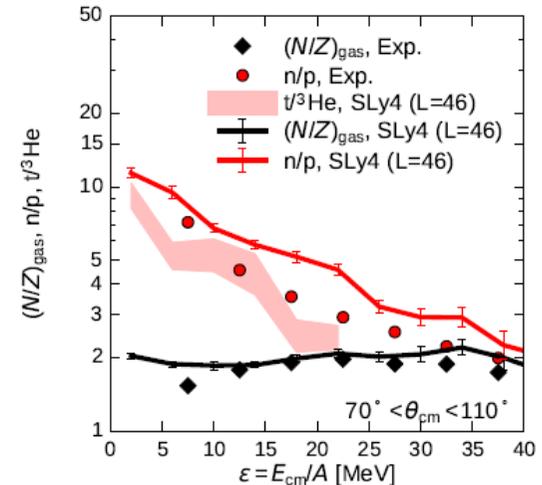
1. pBUU, when exp. α -particles are counted as t and ^3He

(Z. Chajecski, NuSYM 13)



2. AMD with clusters
(A. Ono, NuSYM2015):
n/p (and t/h) ratios only
reproduced if α -clusters
included

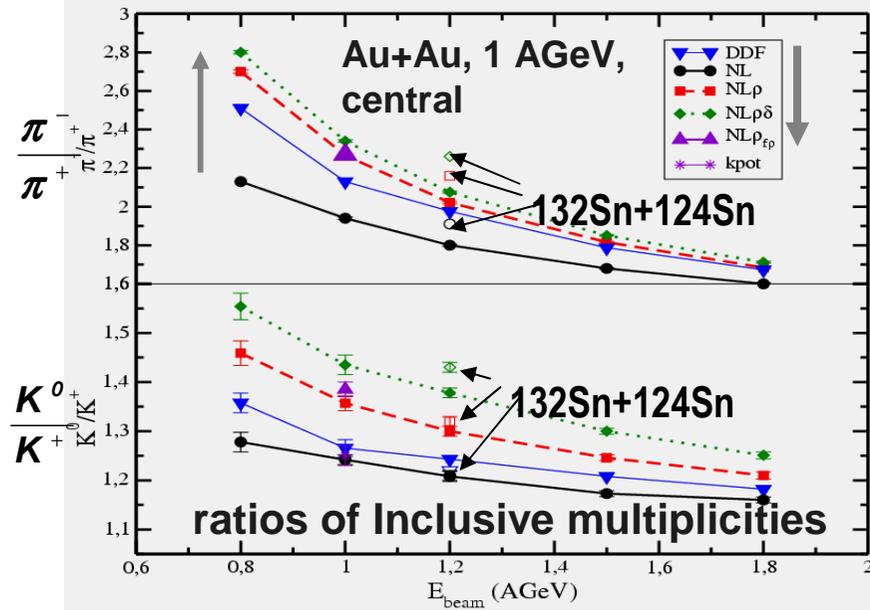
Ratios: n/p , $t/{}^3\text{He}$, $(N/Z)_{gas}$



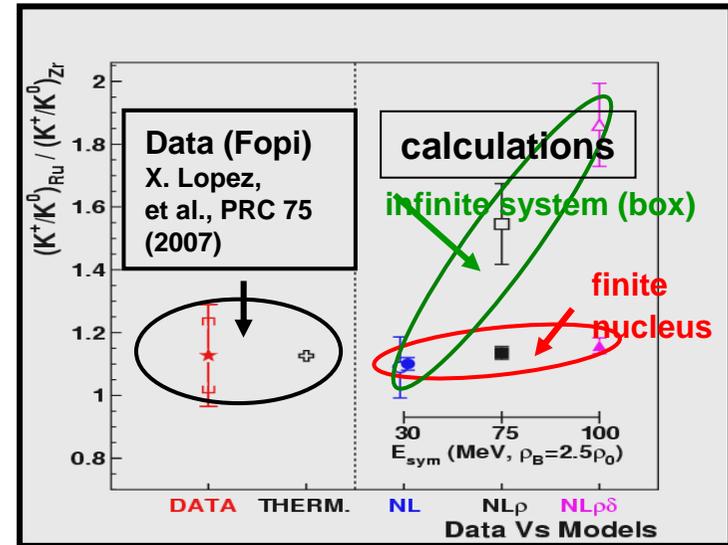
Strangeness in HIC: Kaons

Kaons ratios are (perhaps more) sensitive:
closer to threshold, come from **high density**,
 K^0 and K^+ have **large mean free path**

Comp. to FOPI data: Double ratios (Ru+Ru)/(Zr+Zr)
 Single ratios are more sensitive!



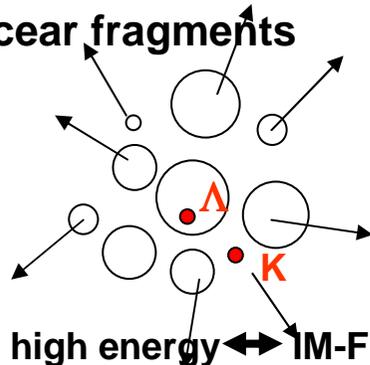
G.Ferini et al., PRL 97 (2006) 202301



G. Ferini, et al., NPA762(2005) 147

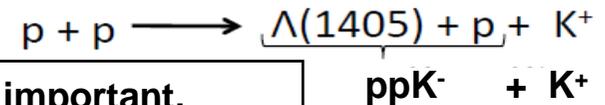
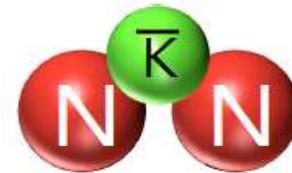
Strange clusters in HIC?

Hypernuclear fragments



competition: high energy \leftrightarrow IM-Fragments

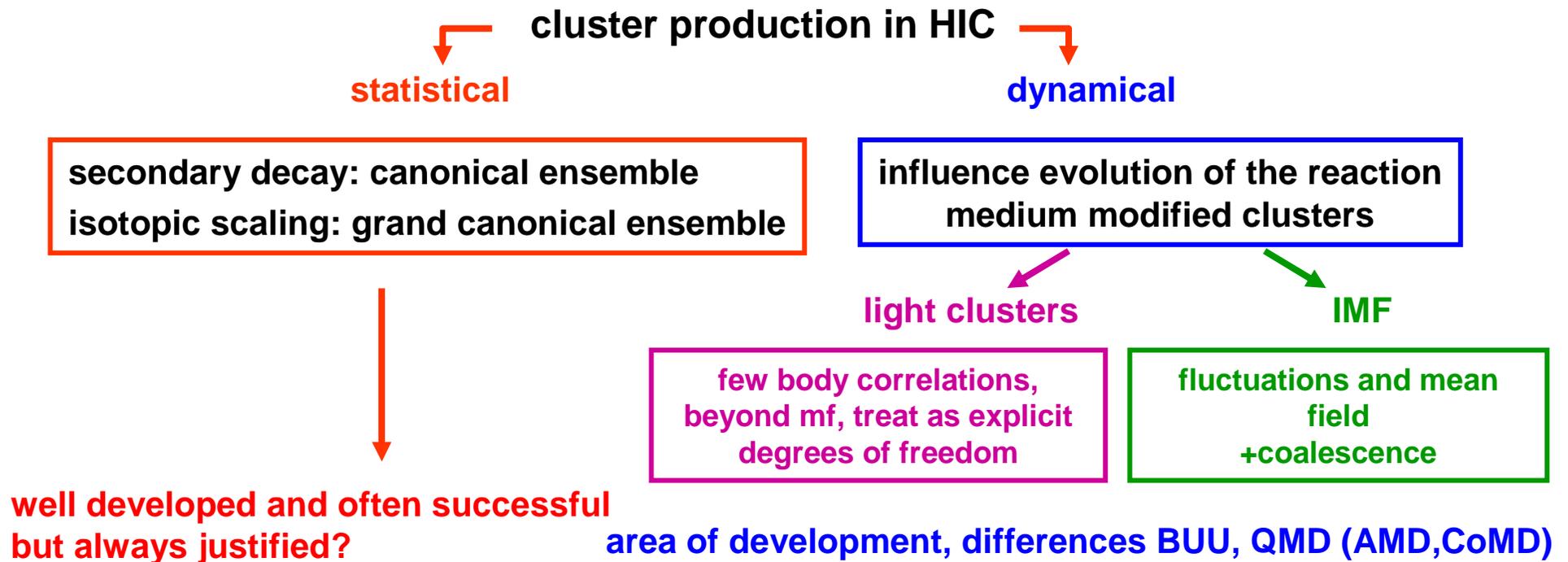
Kaonic clusters?



correlations important,
 like for normal light clusters

Summary: Clusters and Fragments in Heavy Ion Collisions:

- Clusters are ubiquitous in HIC (at low and intermediate energies)
important for analysis (observables depend on treatment of clustering)
- contain important information on the state of the system
(e.g. equilibration, temperature, density, symmetry energy, etc)





**Thank you for attention
and fruitful discussion on the workshop**

**basse Normandie
our excursion on the weekend**