EFT of ^{3}H and ^{3}He

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Outline

- ☐ Why Pionless EFT
- ☐ What Pionless EFT
- ☐ A spin on Pionless EFT
- ☐ Next for Pionless EFT

Why

The 60s schism: nuclear and particle physics part ways

- o “too many divergences”
- o “too many interactions”

➡ fairwell field theory

data fitting eclipses consistency

The 90s re-unification: nuclear again connected to particle physics

- ✓ renormalization
- ✓ power counting

➡ welcome effective field theory

consistency before fitting

simplest in Pionless EFT

- ▪ two-body system analytical with separable regulators
- ▪ three-body system numerically clean after two-body regulator removed
- ▪ more generally, few-nucleon physics relatively transparent
- ▪ close connection with cold-atom physics and other long-distance physics

- Renormalization and power counting playground for nuclear EFTs

naïve dimensional analysis too naïve
for non-perturbative renormalization

Cohen *et al.* '96, '97
…

but message not yet fully digested in Chiral EFT

- A theory of (real) light nuclei

emphasis today

$A \le 6$ nuclei described up to 30% in LO,
$A \le 3$ much better to N^2LO

Bedaque + vK '97
…

bound states and low-energy reactions, including symmetry violation

- A theory of nuclei at larger quark masses ("lattice nuclei")

only EFT for pion masses
in most current LQCD calculations

Barnea *et al.* '13
…

extrapolation of LQCD to heavier nuclei and to reactions

Effective Field Theory ©

$$\begin{cases} T = T^{(\infty)}(Q \sim m \ll M) \propto \displaystyle\sum_{\nu=\nu_{\min}}^{\infty} \left[\frac{Q}{M}\right]^{\nu} \sum_{i} \underbrace{\tilde{c}_{\nu,i}(M,\Lambda)}_{\text{“low-energy constants”}} \, \underbrace{F_{\nu,i}\left(\frac{Q}{m};\frac{Q}{\Lambda}\right)}_{\text{non-analytic, from loops}} \\[2ex] \dfrac{\partial T}{\partial \Lambda} = 0 \end{cases}$$

light scales

hard scales

counting index

“power counting”

arbitrary UV regulator

For $Q \sim m$, truncate …

$$T = T^{(\bar{\nu})}\left[1 + \mathcal{O}\left(\frac{Q}{M}, \frac{Q}{\Lambda}\right)\right]$$

controlled

⇨

… consistently with RG invariance

$$\frac{\Lambda}{T^{(\bar{\nu})}} \frac{\partial T^{(\bar{\nu})}}{\partial \Lambda} = \mathcal{O}\left(\frac{Q}{\Lambda}\right) \ll 1$$

model independent

If so $\begin{cases} \text{to minimize cutoff errors, } \Lambda \gtrsim M \\ \text{for realistic error estimate, } \Lambda \in [M, \infty) \end{cases}$

OTHERWISE THERE IS NO ERROR ESTIMATE

Pionless EFT

$$Q \ll M_{hi} \sim m_\pi$$

- d.o.f.: nucleons
- symmetries: Lorentz, $\cancel{P}$, $\cancel{T}$, $\cancel{B}$

$$\mathcal{L}_{EFT} = N^+\left(i\mathsf{D}_0 + \frac{\mathsf{D}^2}{2m_N}\right)N + \sum_{I=0,1} C_{0I}\, N^+ N^+ P_I\, N\, N$$

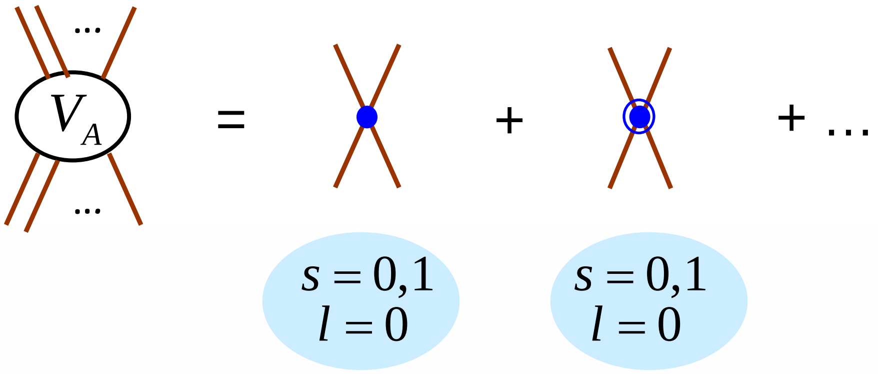
projector on isospin *I*

$$+ D_0\, N^+ N^+ N^+ N\, N\, N$$

$$+ \Delta C_{0I_3=1}\, N^+ N^+ P_{I_3=1} N\, N + \sum_{I=0,1} C_{2I}\left(N^+ N^+ P_I\, \mathsf{D}^2 N\, N + \ldots\right)$$

$$+ \ldots$$

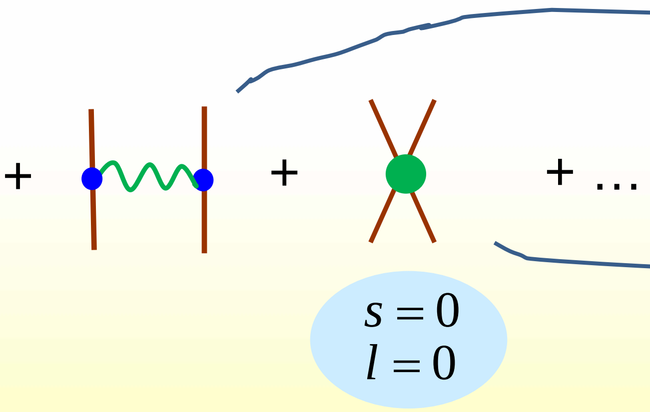
$$\mathsf{D}_\mu = \partial_\mu + ie\frac{1+\tau_3}{2}A_\mu$$



$$\left\{ \begin{aligned} C_{2n}Q^{2n} &\sim \frac{4\pi}{m_N M_{lo}} \frac{Q^{2n}}{M_{lo}^{n} M_{hi}^{n}} \\ &\sim \frac{4\pi}{m_N M_{lo}} \left(\frac{Q}{M_{hi}}\right)^{2n} \left(\frac{M_{lo}}{M_{hi}}\right)^{1-\#} \end{aligned} \right.$$

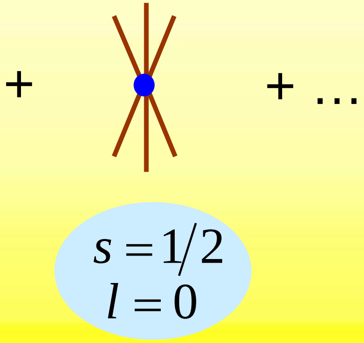
s to *s* waves

#=0,1: *s* waves



$$\equiv \frac{C_{-2}}{Q^2} \sim \frac{4\pi}{m_N M_{lo}} \frac{\alpha m_N M_{lo}}{Q^2}$$

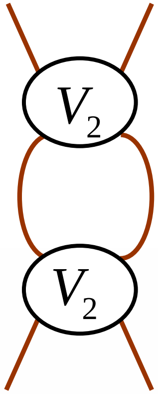
$$\Delta C_{0 I_3=1} \sim \frac{4\pi}{m_N M_{lo}} \frac{\alpha m_N}{M_{lo}}$$



$$\left\{ \begin{aligned} D_0 &\sim \frac{(4\pi)^2}{m_N} \frac{1}{M_{lo}^4} \\ D_2 Q^2 &\sim \frac{(4\pi)^2}{m_N} \frac{1}{M_{lo}^4} \frac{Q^2}{M_{hi}^2} \end{aligned} \right.$$

$A = 2$

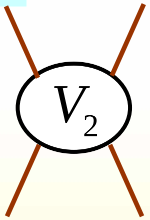
Bedaque + v.K. '97
v.k. '97
Kaplan, Savage + Wise '98
...



$$\sim \frac{m_N}{4\pi} C_{2n} C_{2n'} \left\{ \underbrace{\# \Lambda^{2(n+n')+1} + \ldots + k^{2(n+n')}\Lambda}_{\text{absorbed in same or lower order}} + ik^{2(n+n')+1} + \underbrace{\mathcal{O}\left(\frac{k^{2(n+n'+1)}}{\Lambda}\right)}_{\text{absorbed in higher order}} \right\}$$

non-analytic in *E*

$$\sim \frac{m_N Q}{4\pi} V_2 \times$$



$C_0 \sim \frac{4\pi}{m_N M_{lo}}$: series in Q/M_{lo}

⇒ non-perturbative for $Q \gtrsim M_{lo}$

$C_{2n>0}$: expansion in Q/M_{hi}

poles of T_A at $Q_A \sim M_{lo}$

$\frac{C_{-2}}{Q^2} \sim \frac{4\pi}{m_N M_{lo}} \frac{\alpha m_N M_{lo}}{Q^2}$:

- series in $\alpha m_N / Q$ ⇒ non-perturbative for $Q \lesssim \alpha m_N$
- expansion in $\alpha m_N M_{lo} / Q^2$

$\Delta C_{0 I_3=1} \sim \frac{4\pi}{m_N M_{lo}} \frac{\alpha m_N}{M_{lo}}$: expansion in $\alpha m_N / M_{lo}$

Kong + Ravndal '98

...

importance of E&M hinges on $\alpha m_N / M_{lo}$

N.B. Perturbative treatment of subLOs not (in general) optional

1) Except for regular interactions, iteration can destroy RG invariance

e.g. iterating C_2 ⇒ $r_2 < 0$ Wigner bound Cohen *et al.* '96, '97

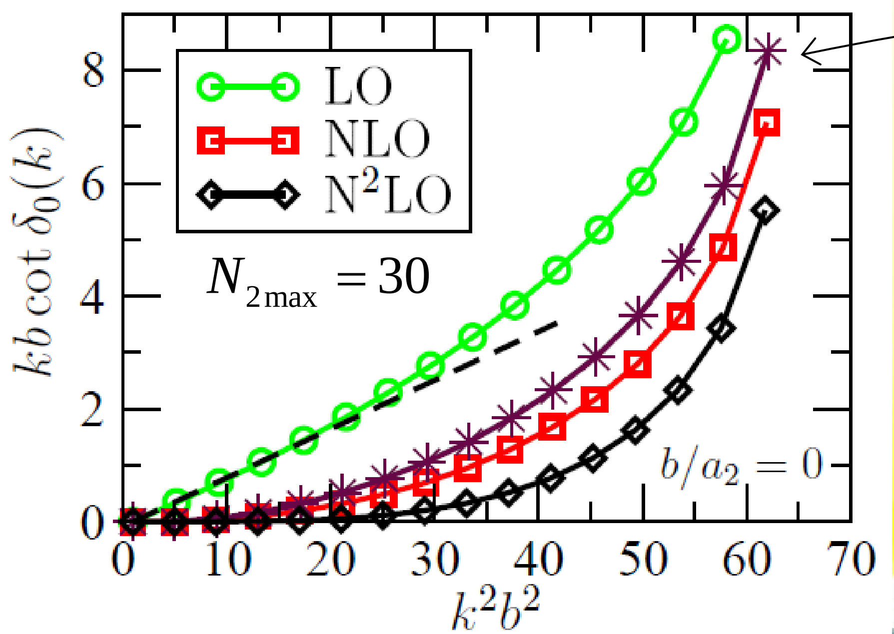
RG invariance

2) Even at fixed cutoff, iteration can *give* worse results

e.g.

Rotureau, Stetcu, Barrett + v.K. '10

two spin-1/2 fermions
at unitarity, in a
harmonic oscillator
of length *b* and
$N_{2\max}$ shells



N²LO Hamiltonian
fully diagonalized:
worse than NLO!

Bedaque, Hammer + v.K. '99 '00

...

$A = 3$

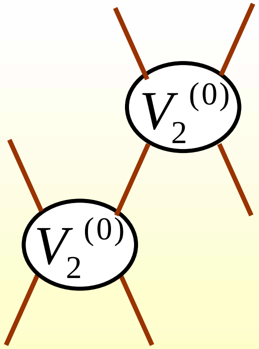
$S_{1/2}$

same for bosons

$$T^{(0)}_{2+1}(\Lambda \gg \underbrace{p \gg M_{lo}}_{\text{approximate scale invariance}}; D_0 = 0) \approx A \cos\left(s_0 \ln\frac{p}{\Lambda} + \delta \right) \Rightarrow \frac{\Lambda_3}{T^{(0)}_{2+1}} \frac{\partial T^{(0)}_{2+1}}{\partial \Lambda_3}(p \sim M_{lo}; D_0 = 0) \sim 1$$

$s_0 = 1.0064\ldots$

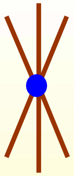
no RG invariance



$$\frac{m_N C_0^2}{Q^2} \sim \frac{(4\pi)^2}{m_N} \frac{1}{M_{lo}^2 Q^2}$$

$$\longleftrightarrow$$

$Q_A \sim M_{lo}$



$$D_0 \sim \frac{(4\pi)^2}{m_N} \frac{1}{M_{lo}^4}$$

Bedaque, Hammer + v.K. '99 '00

LO

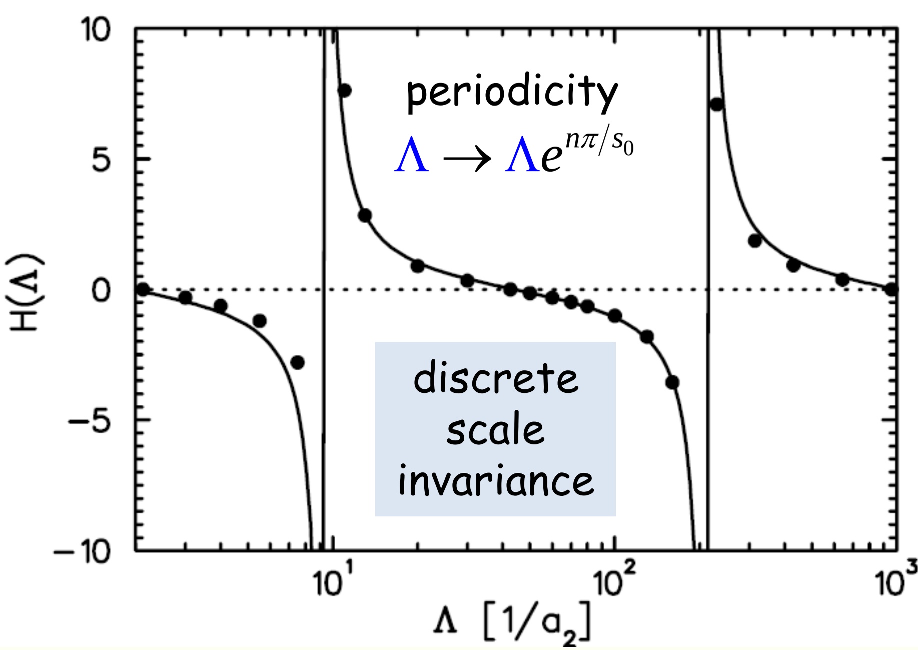
RG invariance:

$$H(\Lambda) \equiv \frac{\Lambda^2 D_0(\Lambda)}{m_N C_0^2(\Lambda)}$$

$$\simeq \frac{\sin\left(\ln(\Lambda/\Lambda_*) + \arctan(1/s_0)\right)}{\sin\left(\ln(\Lambda/\Lambda_*) - \arctan(1/s_0)\right)}$$

dimensionful parameter
(dimensional transmutation)

RG limit cycle!



periodicity
$\Lambda \to \Lambda e^{n\pi/s_0}$

discrete scale invariance

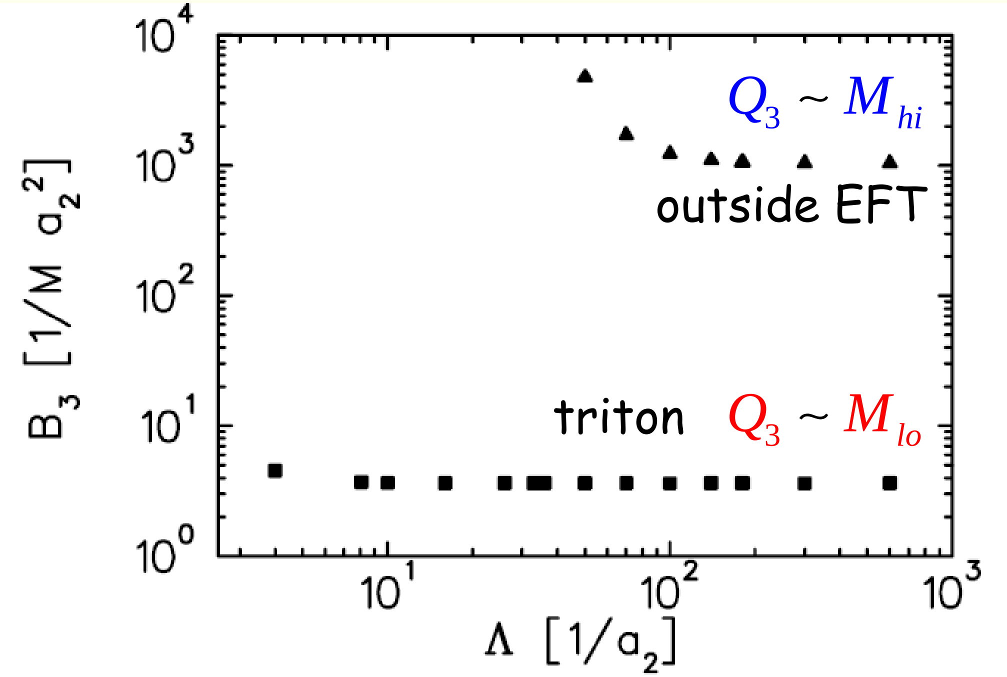
before:

$$B_3 \sim \frac{\Lambda^2}{m_N}$$

Thomas collapse

Thomas '35
...

after:



$Q_3 \sim M_{hi}$
outside EFT

triton $Q_3 \sim M_{lo}$

Efimov '71
...

Efimov state

Bedaque, Hammer + v.K. '99 '00
...
Grießhammer '05
...
Ji, Phillips + Platter '12
Vanasse '13
...

$A=3$

$S_{1/2}$

same for bosons

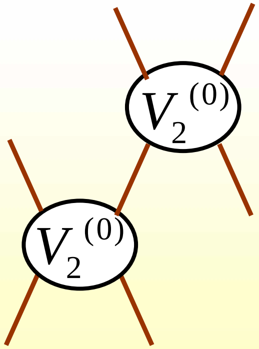
$$T^{(0)}_{2+1}(\Lambda \gg p \gg M_{lo}; D_0=0) \approx A \cos\left(s_0 \ln\frac{p}{\Lambda}+\delta\right)$$

approximate scale invariance

$s_0 = 1.0064...$

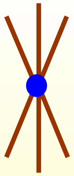
$$\frac{\Lambda_3}{T^{(0)}_{2+1}}\frac{\partial T^{(0)}_{2+1}}{\partial \Lambda_3}(p \sim M_{lo}; D_0=0) \sim 1$$

no RG invariance



$$\frac{m_N C_0^2}{Q^2} \sim \frac{(4\pi)^2}{m_N}\frac{1}{M_{lo}^2 Q^2}$$

$Q_A \sim M_{lo}$



$$D_0 \sim \frac{(4\pi)^2}{m_N}\frac{1}{M_{lo}^4}$$

+ some changes of NDA in other channels

$$D_2 Q^2 \sim \frac{(4\pi)^2}{m_N}\frac{1}{M_{lo}^4}\frac{Q^2}{M_{hi}^2}:$$ expansion in Q^2/M_{hi}^2

Analogous for higher derivatives?

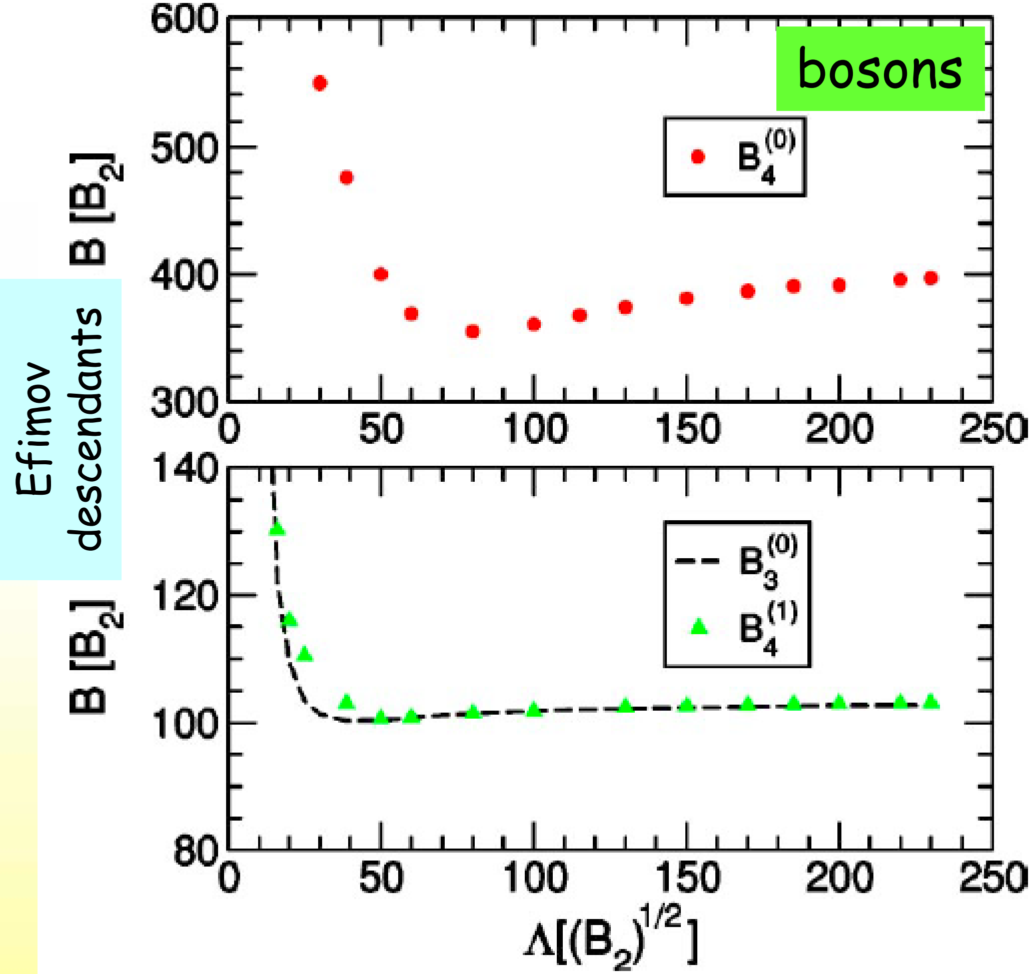
$A = 4$

Hammer, Meißner + Platter '04, '05
Hammer + Platter '07

At what order a four-body force?

Bazak's talk

Not at LO:



Kirscher, Shukla, Grießhammer + Hofmann '09

Not at NLO*

NLO+… iterated
No Coulomb counterterm

$$E_0 \sim \frac{(4\pi)^2}{m_N} \frac{1}{M_{hi}^7}$$ (NDA) ?

Standard Power Counting

$$M_{lo} \sim \frac{1}{\left|a_{2,I=0}\right|} \sim \frac{1}{\left|a_{2,I=1,I_3}\right|} \sim \alpha m_N$$

$$M_{hi} \sim \frac{1}{\left|r_{2,I=0}\right|} \sim \frac{1}{\left|r_{2,I=1,I_3}\right|} \sim \ldots \sim m_\pi$$

- *d* and *d** treated the same way

Chen, Rupak + Savage '99
Bedaque, Hammer + v.K. '99
...

Kong + Ravndal '00
...

- Coulomb LO and thus non-perturbative

Kirscher *et al.* '09
Ando + Birse '10
...

- quark mass splitting effects?

Kirscher + Phillips '11
König + Hammer '14

As a theory of nuclei

ground
states

A	$Q_A \simeq \sqrt{2m_N B_A/A}$ (MeV)
2	45
3	70
4	115
5	100
6	100
…	…
∞	125

$\left\lvert a_{2,I=1,I_3=0} \right\rvert^{-1}$	8
αm_N	7
$\left\lvert a_{2,I=1,I_3=+1} \right\rvert^{-1} - \left\lvert a_{2,I=1,I_3=0} \right\rvert^{-1}$	17
$\left\lvert a_{2,I=1,I_3=-1} \right\rvert^{-1} - \left\lvert a_{2,I=1,I_3=0} \right\rvert^{-1}$	3

König, Grießhammer, Hammer + v.K. '15

$$Q_A \sim M_{lo}$$

$$\left|a_{2,I=1,I_3=0}\right|^{-1} \equiv \aleph_0 \sim \frac{M_{lo}^2}{M_{hi}}$$

$$\left|a_{2,I=1,I_3=+1}\right|^{-1} - \left|a_{2,I=1,I_3=0}\right|^{-1} \sim \alpha m_N \sim \aleph_0$$

$$\left|a_{2,I=1,I_3=-1}\right|^{-1} - \left|a_{2,I=1,I_3=0}\right|^{-1} \sim \frac{M_{lo}^3}{M_{hi}^2}$$

consistent with

$$\left\{\begin{array}{l} \left|a_{2,I=1,I_3=+1}\right|^{-1} - \left|a_{2,I=1,I_3=0}\right|^{-1} = \mathcal{O}\left(\alpha m_N, m_d - m_u\right) \\ \left|a_{2,I=1,I_3=-1}\right|^{-1} - \left|a_{2,I=1,I_3=0}\right|^{-1} = \mathcal{O}\left(m_d - m_u\right) \end{array}\right.$$

while

$$m_n - m_p = \mathcal{O}\left(\frac{\alpha m_N}{4\pi}, m_d - m_u\right)$$

- treat ground states as usual

  but two-nucleon spin-singlet *S* wave is expanded around unitarity

  ⇒ $a_{2,I=1,I_3}^{-1} \neq 0$ at NLO

- Coulomb is perturbative for bound states,

  cf. Hammer, Meißner + Platter '05
  Stetcu, Barrett + v.K. '07

  also Kirscher + Gazit '15

  and needs to be resummed only near scattering thresholds

- smaller quark mass splitting effects

RG invariance

$$\Delta C_{0I_3=1}(\Lambda) \sim 4\pi\alpha \frac{\ln(\alpha m_N/\Lambda)}{\Lambda^2}$$

$$\Downarrow$$

$$\Delta C_{0I_3=1} \sim \frac{4\pi}{m_N M_{lo}} \frac{\alpha m_N}{M_{lo}}$$

$$\left(\frac{m_N}{4\pi}\right)^2 4\pi\alpha\left(\ln\frac{\Lambda}{\alpha m_N}+\mathcal{O}(1)\right)T^{(0)2}P_{I_3=1} \qquad \left(\frac{m_N}{4\pi}\right)^2 \Delta C_{0I_3=1}\left(\Lambda+Q\right)^2 T^{(0)2}P_{I_3=1}$$

NLO

+ no three-body LEC: can predict ^{3}He up to NLO

in contrast to standard power counting where

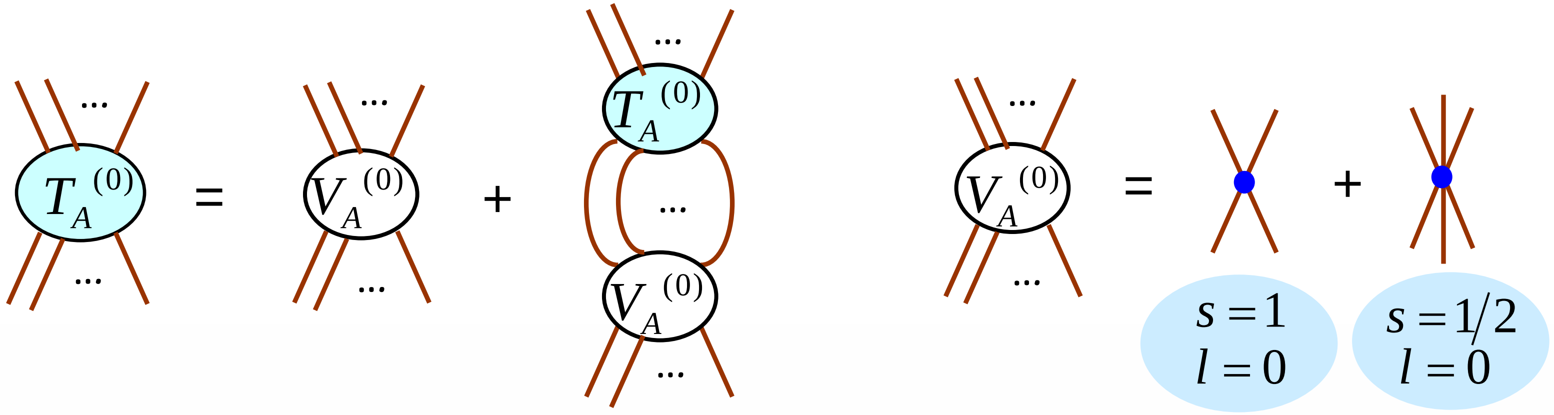
- Coulomb and the associated two-body LEC are LO
- a new three-body LEC appears at NLO

Vanasse *et al.* '14

smaller number of LECs at each order,
more predictive power

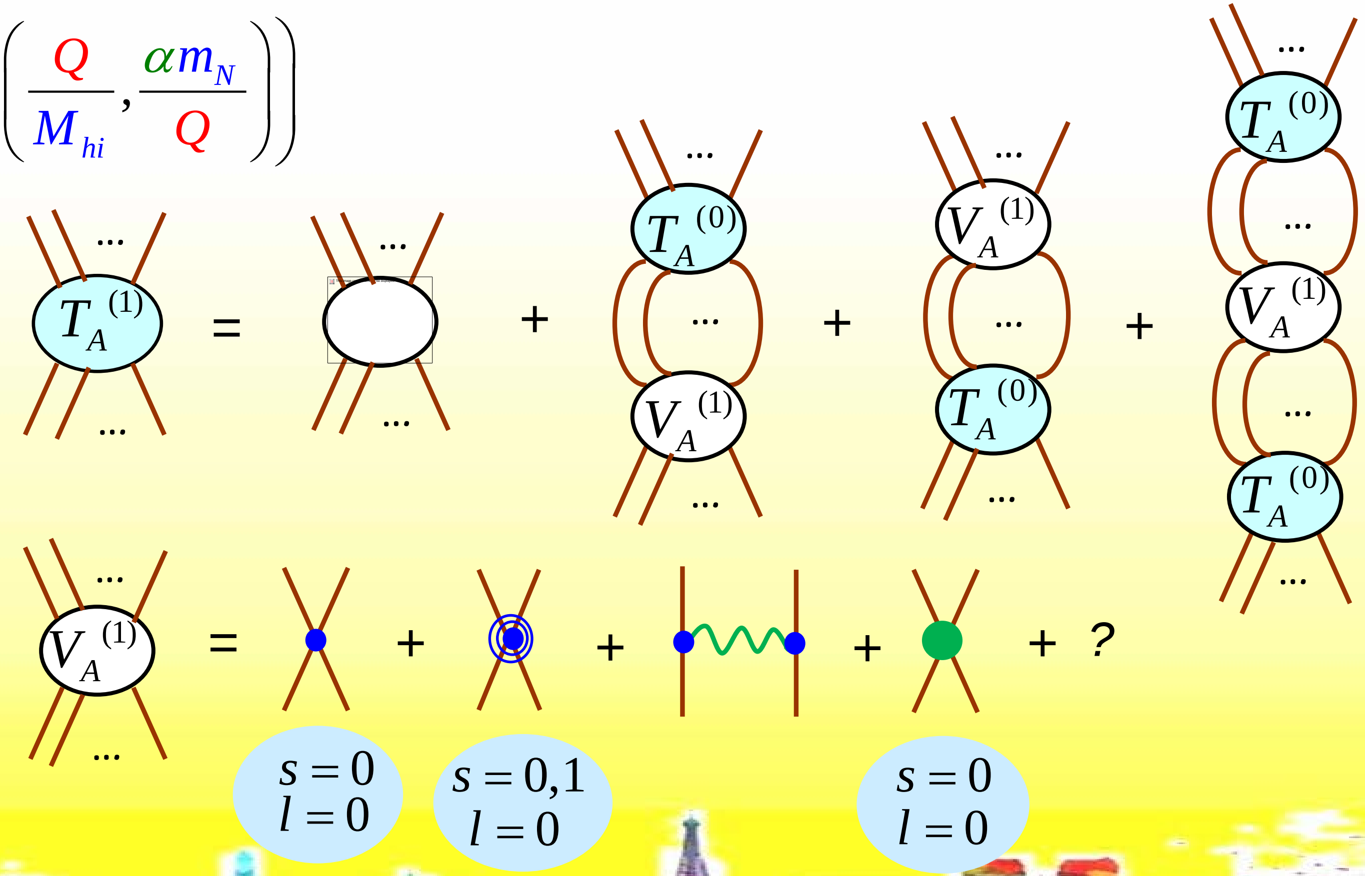
LO

$$\mathcal{O}\left(\frac{4\pi}{m_N M_{lo}}\right)$$



NLO

$$\mathcal{O}\left(\frac{4\pi}{m_N M_{lo}}\times\left(\frac{Q}{M_{hi}},\frac{\alpha m_N}{Q}\right)\right)$$



etc.

$A=2$

König, Grießhammer, Hammer + v.K. '15

$$T_{2,I=0}(k)=\frac{4\pi}{m_N}\frac{1}{\gamma_d+ik}\left\{1+\frac{\rho_d}{2}\frac{k^2+\gamma_d^2}{\gamma_d+ik}+\dots\right\}$$

LO $\gamma_d=45.7\,\mathrm{MeV}\Rightarrow C_{0I=0}$

NLO $\rho_d=1.765\text{ fm}\Rightarrow C_{2I=0}$

$$T_{2,I=1,I_3\neq1}(k)=\frac{4\pi}{m_N}\frac{1}{ik}\left\{1+\frac{1}{ik}\left[-a^{-1}_{2,I=1,I_3=0}+\frac{r_{2,I=1,I_3=0}}{2}k^2\right]+\dots\right\}$$

NLO

$a_{2,I=1,I_3=0}=-23.714\text{ fm}\Rightarrow C_{0I=1}$

$r_{2,I=1,I_3=0}=2.73\text{ fm}\Rightarrow C_{2I=1}$

$$T_{2,I=1,I_3=+1}(k)=T_C(k)+\frac{\Gamma\left(1+i\frac{\alpha m_N}{2k}\right)}{\Gamma\left(1-i\frac{\alpha m_N}{2k}\right)}t_{sC}(k)$$

NLO

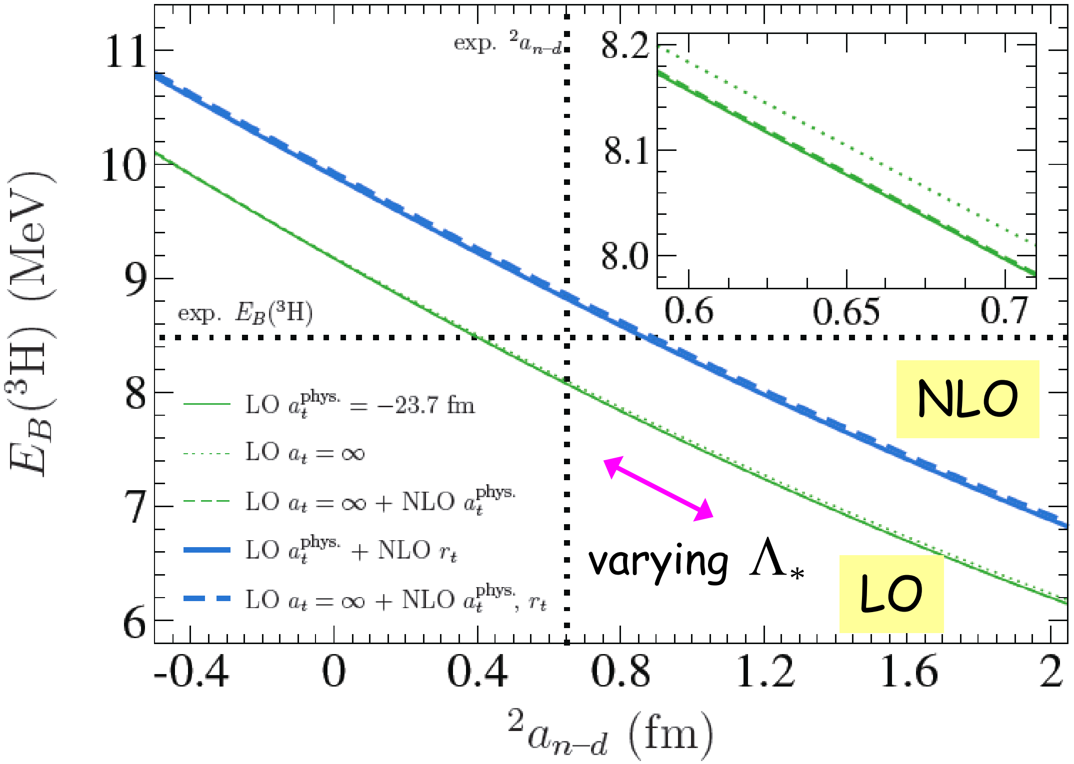
$a_{2,I=1,I_3=+1}=-7.8063\text{ fm}\Rightarrow \Delta C_{0I_3=1}$

$$t_{sC}(k)=\frac{4\pi}{m_N}\frac{1}{ik}\left\{1+\frac{1}{ik}\left[\alpha m_N\left(C_E+\ln\frac{\alpha m_N}{2k}\right)-a^{-1}_{2,I=1,I_3=+1}+\frac{r_{2,I=1,I_3=0}}{2}k^2\right]+\dots\right\}$$

predict
$$\begin{cases} a_{2,I=1,I_3=-1}(\mathrm{NLO})=a_{2,I=1,I_3=0}\\ r_{2,I=1,I_3=-1}(\mathrm{NLO})=r_{2,I=1,I_3=0}\\ r_{2,I=1,I_3=+1}(\mathrm{NLO})=r_{2,I=1,I_3=0}\end{cases}$$
vs.
$$\begin{cases} a_{2,I=1,I_3=-1}=-18.7\text{ fm (?exp)}\\ r_{2,I=1,I_3=-1}=?\ \text{(exp)}\\ r_{2,I=1,I_3=+1}=2.79\text{ fm (exp)}\end{cases}$$

$1/(Qa_{2,I=1,I_3=0}) \sim \aleph_0/M_{lo}$ expansion

$A = 3$

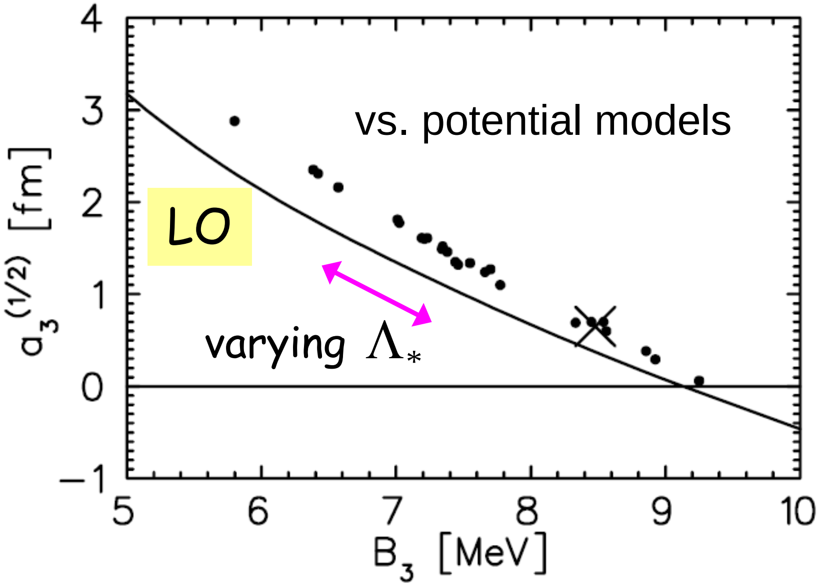


König, Grießhammer, Hammer + v.K. '15

Phillips line

cf. standard PC

Bedaque, Hammer + v.K. '99

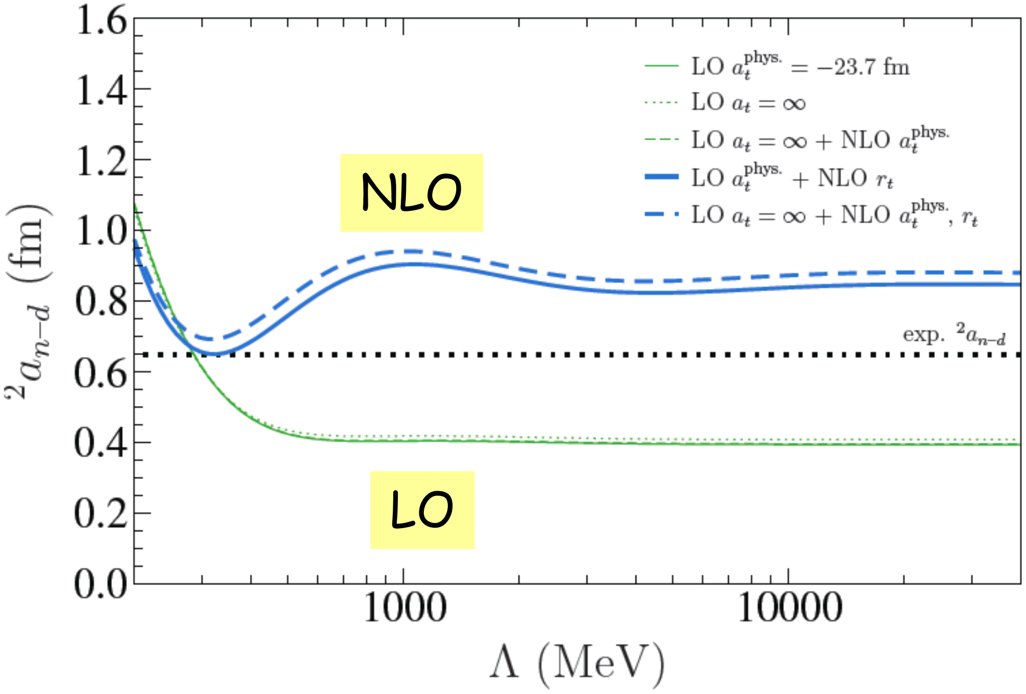


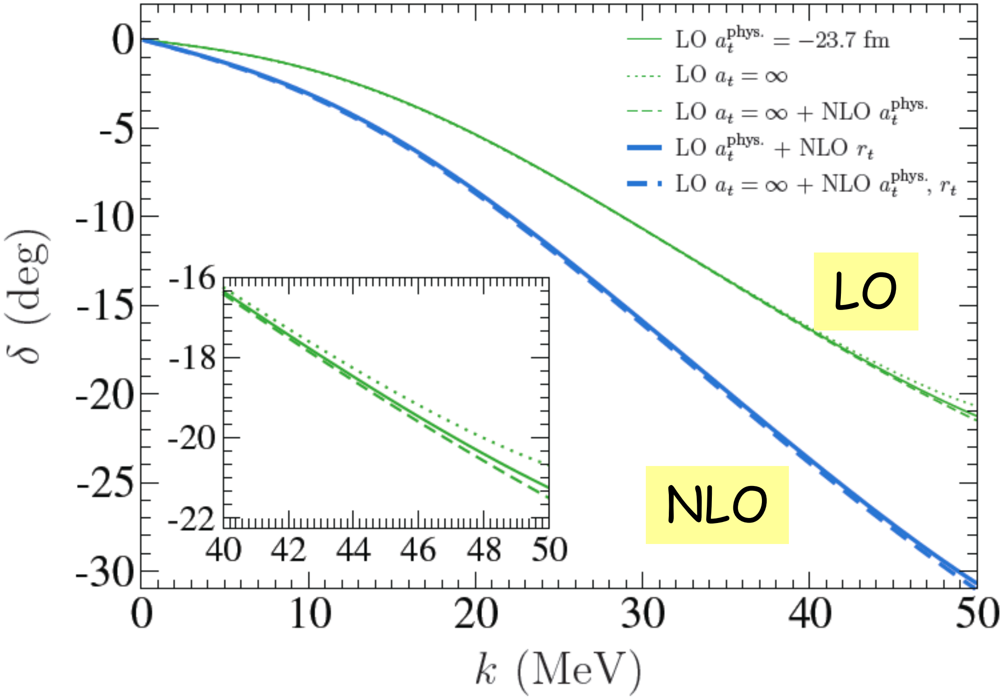
$S_{1/2}$

König, Grießhammer, Hammer + v.K. '15

$B_t = -8.48$ fm (exp) ⇨ D_0

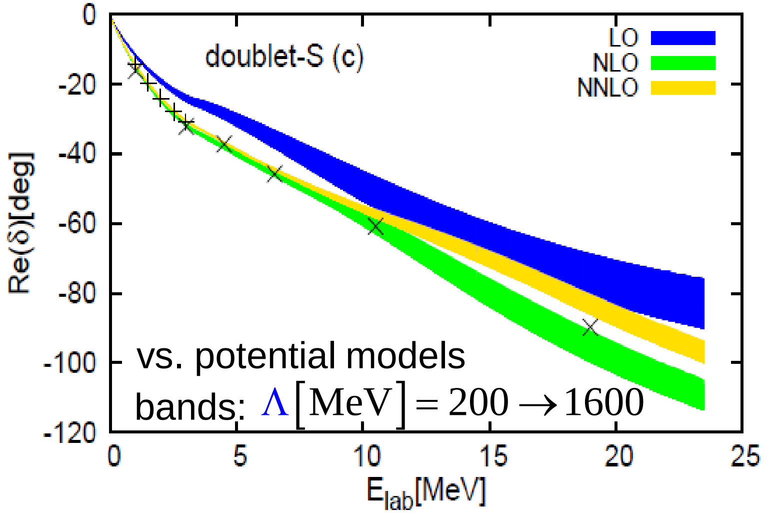
$S_{1/2}$





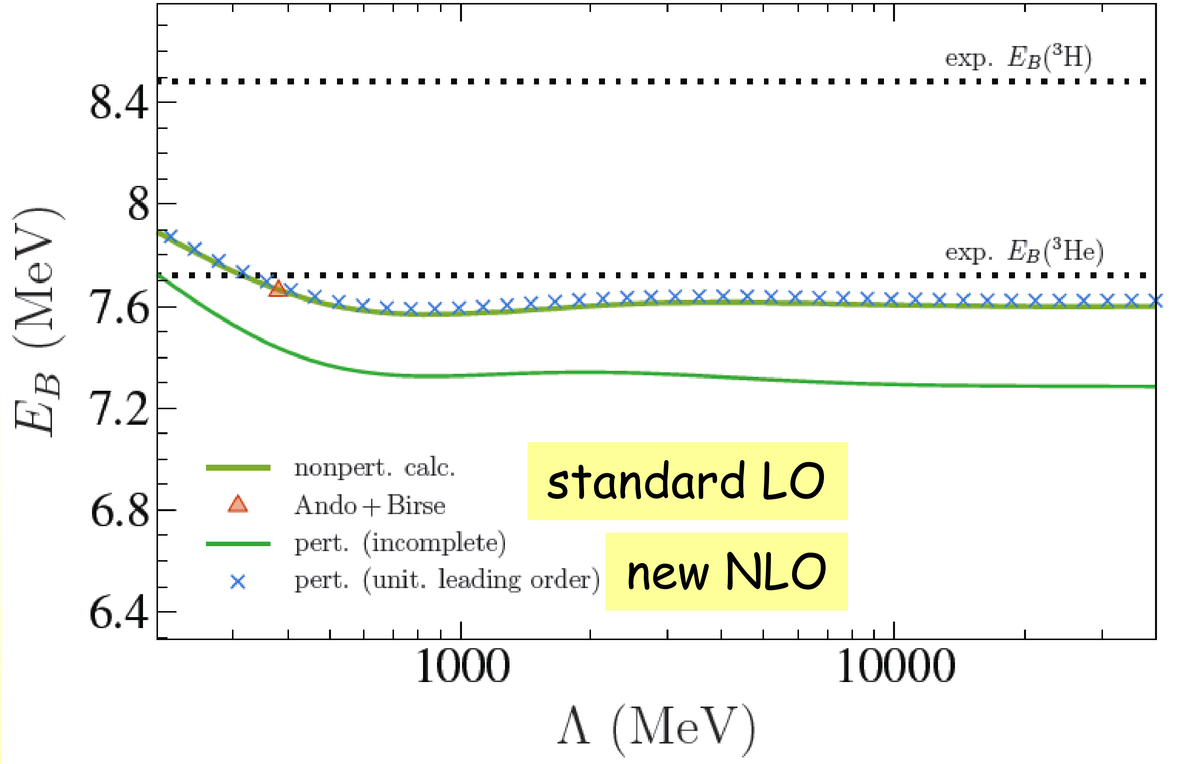
cf. standard PC

Vanasse '13



König, Grießhammer, Hammer + v.K. '15

combined $\alpha m_N/M_{lo}, M_{lo}/M_{hi}$ expansions



standard LO

new NLO

$$E_B(^3\mathrm{He})^{\mathrm{LO+NLO}} = E_B(^3\mathrm{H}) + \Delta E^{\mathrm{NLO}}$$
$$= 8.48\ \mathrm{MeV} - (0.86 \pm 0.12)\ \mathrm{MeV} = (7.62 \pm 0.12)\ \mathrm{MeV}$$

Conclusions

- Pionless EFT provides a model-independent description of few-body universal features

- Convergence and standard power counting
  - well understood up to N^{2}LO for $A \leq 3$,
  - needs to be further studied for $A \geq 4$

Bazak's talk

- New power counting more predictive for bound states in expansion around unitarity in two-nucleon spin singlet
  - works well for $A = 3$,
  - needs to be tested for $A \geq 4$