Short Range EFT for Few-Body Systems

Betzalel Bazak

Physique Theorique

Institut de Physique Nucleaire d'Orsay

Future directions for nuclear structure and reaction theories:
Ab initio approaches for 2020

March 15 2016

GANIL, Caen

Outline

Universality

- Consider particles interacting through 2-body potential with range R.
- Classically, the particles 'feel' each other only within the potential range.
- But, in the case of resonant interaction, the wave function can have much larger extent.
- At low energies, the 2-body physics is completely govern by the scattering length, a.

$$\lim_{k \to 0} k \cot \delta(k) = -\frac{1}{a} + \frac{1}{2} r_{\text{eff}} k^2$$

From Sakurai's book

- When $|a| \gg R$ the potential details has no influence: *Universality*.

Universality

- Consider particles interacting through 2-body potential with range R.
- Classically, the particles 'feel' each other only within the potential range.
- But, in the case of resonant interaction, the wave function can have much larger extent.
- At low energies, the 2-body physics is completely govern by the scattering length, a.

$$\lim_{k\to 0} k\cot\delta(k) = -\frac{1}{a} + \frac{1}{2} r_{\text{eff}} k^2$$

From Sakurai's book

- When $|a| \gg R$ the potential details has no influence: *Universality*.

Universality

- Consider particles interacting through 2-body potential with range R.
- Classically, the particles 'feel' each other only within the potential range.
- But, in the case of resonant interaction, the wave function can have much larger extent.
- At low energies, the 2-body physics is completely govern by the scattering length, a.

$$\lim_{k \to 0} k \cot \delta(k) = -\frac{1}{a} + \frac{1}{2} r_{\text{eff}} k^2$$

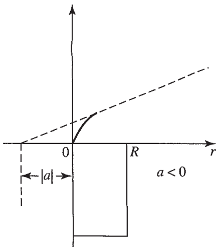
From Sakurai's book

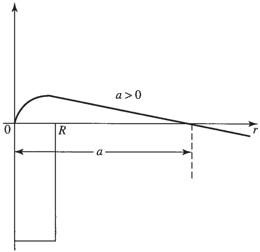
- When $|a| \gg R$ the potential details has no influence: *Universality*.

Universality

- Consider particles interacting through 2-body potential with range R.
- Classically, the particles ‘feel’ each other only within the potential range.
- But, in the case of resonant interaction, the wave function can have much larger extent.
- At low energies, the 2-body physics is completely govern by the scattering length, a.

$$\lim_{k \to 0} k \cot \delta(k) = -\frac{1}{a} + \frac{1}{2} r_{\text{eff}} k^2$$





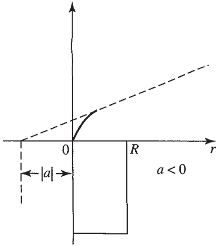
From Sakurai’s book

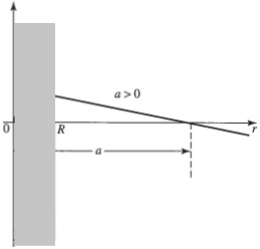
- When $|a| \gg R$ the potential details has no influence: *Universality*.

Universality

- Consider particles interacting through 2-body potential with range R.
- Classically, the particles 'feel' each other only within the potential range.
- But, in the case of resonant interaction, the wave function can have much larger extent.
- At low energies, the 2-body physics is completely govern by the scattering length, a.

$$\lim_{k\to 0} k\cot\delta(k) = -\frac{1}{a} + \frac{1}{2} r_{\text{eff}} k^2$$





From Sakurai's book

- When $|a| \gg R$ the potential details has no influence: *Universality*.

Universality

- Generally, $a \approx r_{\text{eff}} \approx R$.
  Universal systems are fine-tuned to get $a \gg r_{\text{eff}}, R$.
- Corrections to universal theory are of order of r_{eff}/a and R/a.
- For $a > 0$, we have universal dimer with energy $E = -\hbar^2/ma^2$.
- ^{4}He Atoms: $a \approx 170.9a_0$, (a_0 = the Bohr radius), is much larger than its van der Waals radius, $r_{vdW} \approx 2.6a_0$.
- Nucleus: $a_s \approx -23.4$ fm, $a_t \approx 5.42$ fm, $R = \hbar/m_\pi c \approx 1.4$ fm.
  Deuteron binding energy, 2.22 MeV, is close to $\hbar/ma_t{}^2 \approx 1.4$ MeV.
- Ultracold atoms near a Feshbach resonance,

$$a(B) = a_{bg}\left(1 + \frac{\Delta}{B - B_0}\right)$$

Universality

- Generally, $a \approx r_{\text{eff}} \approx R$.
  Universal systems are fine-tuned to get $a \gg r_{\text{eff}}, R$.
- Corrections to universal theory are of order of r_{eff}/a and R/a.
- For $a > 0$, we have universal dimer with energy $E = -\hbar^2/ma^2$.
- ^{4}He Atoms: $a \approx 170.9a_0$, (a_0 = the Bohr radius), is much larger than its van der Waals radius, $r_{vdW} \approx 2.6a_0$.
- Nucleus: $a_s \approx -23.4$ fm, $a_t \approx 5.42$ fm, $R = \hbar/m_\pi c \approx 1.4$ fm.
  Deuteron binding energy, 2.22 MeV, is close to $\hbar/ma_t{}^2 \approx 1.4$ MeV.
- Ultracold atoms near a Feshbach resonance,

$$a(B) = a_{bg}\left(1 + \frac{\Delta}{B - B_0}\right)$$

Universality

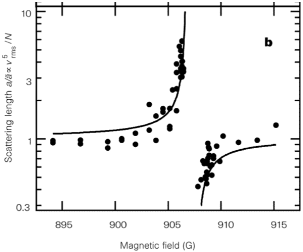
- Generally, $a \approx r_{\text{eff}} \approx R$.
  Universal systems are fine-tuned to get $a \gg r_{\text{eff}}, R$.
- Corrections to universal theory are of order of r_{eff}/a and R/a.
- For $a > 0$, we have universal dimer with energy $E = -\hbar^2/ma^2$.
- ^{4}He Atoms: $a \approx 170.9 a_0$, (a_0 = the Bohr radius), is much larger than its van der Waals radius, $r_{vdW} \approx 2.6 a_0$.
- Nucleus: $a_s \approx -23.4$ fm, $a_t \approx 5.42$ fm, $R = \hbar/m_\pi c \approx 1.4$ fm.
  Deuteron binding energy, 2.22 MeV, is close to $\hbar/m{a_t}^2 \approx 1.4$ MeV.
- Ultracold atoms near a Feshbach resonance,

$$a(B) = a_{bg}\left(1 + \frac{\Delta}{B - B_0}\right)$$

Universality

- Generally, $a \approx r_{\rm eff} \approx R$.
  Universal systems are fine-tuned to get $a \gg r_{\rm eff}, R$.
- Corrections to universal theory are of order of $r_{\rm eff}/a$ and R/a.
- For $a > 0$, we have universal dimer with energy $E = -\hbar^2/ma^2$.
- ^{4}He Atoms: $a \approx 170.9a_0$, (a_0 = the Bohr radius), is much larger than its van der Waals radius, $r_{vdW} \approx 2.6a_0$.
- Nucleus: $a_s \approx -23.4$ fm, $a_t \approx 5.42$ fm, $R = \hbar/m_\pi c \approx 1.4$ fm.
  Deuteron binding energy, 2.22 MeV, is close to $\hbar/m{a_t}^2 \approx 1.4$ MeV.
- Ultracold atoms near a Feshbach resonance,

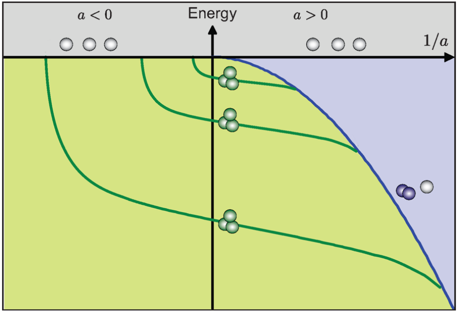
$$a(B) = a_{bg}\left(1 + \frac{\Delta}{B - B_0}\right)$$



S. Inouye *et al.*, Nature 392, 151 (1998)

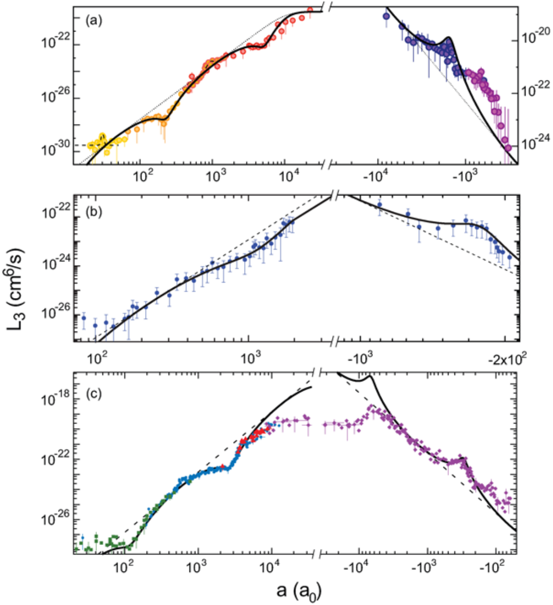
Efimov Physics

- *The unitary limit*: $E_2 = 0$, $a \longrightarrow \infty$.
- In 1970 V. Efimov found out that if $E_2 = 0$ the 3-body system will have an **infinite** number of bound states.
- The 3-body spectrum is $E_n = E_0 e^{-2\pi n/s_0}$ with $s_0 = 1.00624$.



F. Ferlaino and R. Grimm, Physics **3**, 9 (2010)

Efimov Physics in Ultracold Atoms



F. Ferlaino and R. Grimm, Physics **3**, 9 (2010)

- ^{39}K
  M. Zaccanti *et al.*,
  Nature Phys. **5**, 586 (2009).
- ^{7}Li
  N. Gross, Z. Shotan, S. Kokkelmans, and L. Khaykovich,
  Phys. Rev. Lett. **103**, 163202 (2009).
- ^{7}Li
  S.E. Pollack, D. Dries, and R.G. Hulet,
  Science **326**, 1683 (2009)

Effective Field Theory (EFT)

- Typically in physics we have an "underlying" theory, valid at a mass scale M_{hi}, but we want to study processes at momenta $Q \approx M_{lo} \ll M_{hi}$.
- For example, nuclear structure involves energies that are much smaller than the typical QCD mass scale, $M_{QCD} \approx 1$ GeV.
- Effective Field Theory (EFT) is a framework to construct the interactions systematically. The high-energy degrees of freedom are integrated out, while the effective Lagrangian have the same symmetries as the underlying theory.
- The details of the underlying dynamics are contained in the interaction strengths.

Effective Field Theory (EFT)

- Typically in physics we have an "underlying" theory, valid at a mass scale M_{hi}, but we want to study processes at momenta $Q \approx M_{lo} \ll M_{hi}$.
- For example, nuclear structure involves energies that are much smaller than the typical QCD mass scale, $M_{QCD} \approx 1$ GeV.
- Effective Field Theory (EFT) is a framework to construct the interactions systematically. The high-energy degrees of freedom are integrated out, while the effective Lagrangian have the same symmetries as the underlying theory.
- The details of the underlying dynamics are contained in the interaction strengths.

Effective Field Theory (EFT)

- Typically in physics we have an "underlying" theory, valid at a mass scale M_{hi}, but we want to study processes at momenta $Q \approx M_{lo} \ll M_{hi}$.
- For example, nuclear structure involves energies that are much smaller than the typical QCD mass scale, $M_{QCD} \approx 1$ GeV.
- Effective Field Theory (EFT) is a framework to construct the interactions systematically. The high-energy degrees of freedom are integrated out, while the effective Lagrangian have the same symmetries as the underlying theory.
- The details of the underlying dynamics are contained in the interaction strengths.

Effective Field Theory (EFT)

- Typically in physics we have an "underlying" theory, valid at a mass scale M_{hi}, but we want to study processes at momenta $Q \approx M_{lo} \ll M_{hi}$.
- For example, nuclear structure involves energies that are much smaller than the typical QCD mass scale, $M_{QCD} \approx 1$ GeV.
- Effective Field Theory (EFT) is a framework to construct the interactions systematically. The high-energy degrees of freedom are integrated out, while the effective Lagrangian have the same symmetries as the underlying theory.
- The details of the underlying dynamics are contained in the interaction strengths.

Pion-less EFT

- The degrees of freedom in pionless EFT are the nucleons.
- We have to include all terms conserving our theory symmetries, order by order.
- For nucleons, the Leading Order (LO) is,

$$V_{LO} = a_1 + a_2\sigma_i \cdot \sigma_j + a_3\tau_i \cdot \tau_j + a_4(\sigma_i \cdot \sigma_j)(\tau_i \cdot \tau_j)$$

  where due to symmetry, only 2 are independent, corresponding to the two scattering lengths.
- The Next to Leading Order (NLO) is,

$$\begin{aligned} V_{NLO} &= b_1(k^2+q^2) + b_2(k^2+q^2)\sigma_i \cdot \sigma_j + b_3(k^2+q^2)\tau_i \cdot \tau_j \\ &+ b_4(k^2+q^2)(\sigma_i \cdot \sigma_j)(\tau_i \cdot \tau_j) \end{aligned}$$

  here also only 2 parameters are independent, corresponding to the two effective ranges.
- p-wave enters at N^3LO!

Pion-less EFT

- The degrees of freedom in pionless EFT are the nucleons.
- We have to include all terms conserving our theory symmetries, order by order.
- For nucleons, the Leading Order (LO) is,

$$V_{LO} = a_1 + a_2 \sigma_i \cdot \sigma_j + a_3 \tau_i \cdot \tau_j + a_4 (\sigma_i \cdot \sigma_j)(\tau_i \cdot \tau_j)$$

  where due to symmetry, only 2 are independent, corresponding to the two scattering lengths.
- The Next to Leading Order (NLO) is,

$$\begin{aligned} V_{NLO} &= b_1(k^2+q^2) + b_2(k^2+q^2)\sigma_i \cdot \sigma_j + b_3(k^2+q^2)\tau_i \cdot \tau_j \\ &+ b_4(k^2+q^2)(\sigma_i \cdot \sigma_j)(\tau_i \cdot \tau_j) \end{aligned}$$

  here also only 2 parameters are independent, corresponding to the two effective ranges.
- p-wave enters at N^3LO!

Pion-less EFT

- The degrees of freedom in pionless EFT are the nucleons.
- We have to include all terms conserving our theory symmetries, order by order.
- For nucleons, the Leading Order (LO) is,

$$V_{LO} = a_1 + a_2 \sigma_i \cdot \sigma_j + a_3 \tau_i \cdot \tau_j + a_4 (\sigma_i \cdot \sigma_j)(\tau_i \cdot \tau_j)$$

  where due to symmetry, only 2 are independent, corresponding to the two scattering lengths.
- The Next to Leading Order (NLO) is,

$$\begin{aligned} V_{NLO} &= b_1(k^2+q^2) + b_2(k^2+q^2)\sigma_i \cdot \sigma_j + b_3(k^2+q^2)\tau_i \cdot \tau_j \\ &+ b_4(k^2+q^2)(\sigma_i \cdot \sigma_j)(\tau_i \cdot \tau_j) \end{aligned}$$

  here also only 2 parameters are independent, corresponding to the two effective ranges.
- p-wave enters at N^3LO!

Pion-less EFT

- The degrees of freedom in pionless EFT are the nucleons.
- We have to include all terms conserving our theory symmetries, order by order.
- For nucleons, the Leading Order (LO) is,

$$V_{LO} = a_1 + a_2\sigma_i \cdot \sigma_j + a_3\tau_i \cdot \tau_j + a_4(\sigma_i \cdot \sigma_j)(\tau_i \cdot \tau_j)$$

  where due to symmetry, only 2 are independent, corresponding to the two scattering lengths.
- The Next to Leading Order (NLO) is,

$$\begin{aligned} V_{NLO} &= b_1(k^2+q^2) + b_2(k^2+q^2)\sigma_i \cdot \sigma_j + b_3(k^2+q^2)\tau_i \cdot \tau_j \\ &+ b_4(k^2+q^2)(\sigma_i \cdot \sigma_j)(\tau_i \cdot \tau_j) \end{aligned}$$

  here also only 2 parameters are independent, corresponding to the two effective ranges.
- p-wave enters at N^3LO!

Pion-less EFT

- The degrees of freedom in pionless EFT are the nucleons.
- We have to include all terms conserving our theory symmetries, order by order.
- For nucleons, the Leading Order (LO) is,

$$V_{LO} = a_1 + a_2 \sigma_i \cdot \sigma_j + a_3 \tau_i \cdot \tau_j + a_4 (\sigma_i \cdot \sigma_j)(\tau_i \cdot \tau_j)$$

  where due to symmetry, only 2 are independent, corresponding to the two scattering lengths.
- The Next to Leading Order (NLO) is,

$$\begin{aligned} V_{NLO} &= b_1(k^2+q^2) + b_2(k^2+q^2)\sigma_i \cdot \sigma_j + b_3(k^2+q^2)\tau_i \cdot \tau_j \\ &+ b_4(k^2+q^2)(\sigma_i \cdot \sigma_j)(\tau_i \cdot \tau_j) \end{aligned}$$

  here also only 2 parameters are independent, corresponding to the two effective ranges.
- p-wave enters at N^3LO!

Short-Range EFT for Bosonic system

- For spin-less bosons, most of the terms are dropped, and we have at LO,

$$V_{LO} = a_1.$$

- At NLO,

$$V_{NLO} = b_1(k^2 + q^2).$$

- Terms proportional to $\boldsymbol{k} \cdot \boldsymbol{q}$ are omitted due to time reversal symmetry.
- The LO term is iterated at NLO.

Short-Range EFT for Bosonic system

- For spin-less bosons, most of the terms are dropped, and we have at LO,

$$V_{LO} = a_1.$$

- At NLO,

$$V_{NLO} = b_1(k^2 + q^2).$$

- Terms proportional to $\boldsymbol{k} \cdot \boldsymbol{q}$ are omitted due to time reversal symmetry.
- The LO term is iterated at NLO.

Short-Range EFT for Bosonic system

- For spin-less bosons, most of the terms are dropped, and we have at LO,

$$V_{LO} = a_1.$$

- At NLO,

$$V_{NLO} = b_1(k^2 + q^2).$$

- Terms proportional to $\boldsymbol{k} \cdot \boldsymbol{q}$ are omitted due to time reversal symmetry.
- The LO term is iterated at NLO.

Short-Range EFT for Bosonic system

- For spin-less bosons, most of the terms are dropped, and we have at LO,

$$V_{LO} = a_1.$$

- At NLO,

$$V_{NLO} = b_1(k^2 + q^2).$$

- Terms proportional to $\boldsymbol{k} \cdot \boldsymbol{q}$ are omitted due to time reversal symmetry.
- The LO term is iterated at NLO.

Stochastic Variational Method

- To solve the A-body Schrodinger equation, we use correlated Gaussian basis,

$$\psi(\eta_1, \eta_2 ... \eta_{A-1}) = \sum_i c_i \mathcal{A} \exp(-\eta^T A_i \eta)$$

η = Jacobi coordinates, A_i = matrix of $(A-1) \times (A-1)$ numbers.

- The matrix elements can be calculated analytically in most cases.
- Since $a \ll \Lambda^{-1}$, we need large spread of basis functions.
- Symmetrization gives factor of $A!$ which limits the number of particles.
- Works for any angular momentum, for bosons and fermions. Spin and isospin can be introduced.

Y. Suzuki and K. Varga, ''Stochastic Variational Approach to Quantum-Mechanical Few-Body Problems'', Springer 1998.

Stochastic Variational Method

- To solve the A-body Schrodinger equation, we use correlated Gaussian basis,

$$\psi(\eta_1, \eta_2 ... \eta_{A-1}) = \sum_i c_i \mathcal{A} \exp(-\eta^T A_i \eta)$$

$\eta =$ Jacobi coordinates, $A_i =$ matrix of $(A-1) \times (A-1)$ numbers.

- The matrix elements can be calculated analytically in most cases.
- Since $a \ll \Lambda^{-1}$, we need large spread of basis functions.
- Symmetrization gives factor of $A!$ which limits the number of particles.
- Works for any angular momentum, for bosons and fermions. Spin and isospin can be introduced.

Y. Suzuki and K. Varga, ''Stochastic Variational Approach to Quantum-Mechanical Few-Body Problems'', Springer 1998.

Stochastic Variational Method

- To solve the A-body Schrodinger equation, we use correlated Gaussian basis,

$$\psi(\eta_1, \eta_2 ... \eta_{A-1}) = \sum_i c_i \mathcal{A} \exp(-\eta^T A_i \eta)$$

  $\eta =$ Jacobi coordinates, $A_i =$ matrix of $(A-1) \times (A-1)$ numbers.
- The matrix elements can be calculated analytically in most cases.
- Since $a \ll \Lambda^{-1}$, we need large spread of basis functions.
- Symmetrization gives factor of $A!$ which limits the number of particles.
- Works for any angular momentum, for bosons and fermions. Spin and isospin can be introduced.

Y. Suzuki and K. Varga, "Stochastic Variational Approach to Quantum-Mechanical Few-Body Problems", Springer 1998.

Stochastic Variational Method

- To solve the A-body Schrodinger equation, we use correlated Gaussian basis,

$$\psi(\eta_1, \eta_2 ... \eta_{A-1}) = \sum_i c_i \mathcal{A} \exp(-\eta^T A_i \eta)$$

  $\eta =$ Jacobi coordinates, $A_i =$ matrix of $(A-1) \times (A-1)$ numbers.
- The matrix elements can be calculated analytically in most cases.
- Since $a \ll \Lambda^{-1}$, we need large spread of basis functions.
- Symmetrization gives factor of $A!$ which limits the number of particles.
- Works for any angular momentum, for bosons and fermions. Spin and isospin can be introduced.

Y. Suzuki and K. Varga, ''Stochastic Variational Approach to Quantum-Mechanical Few-Body Problems'', Springer 1998.

Stochastic Variational Method

- To solve the A-body Schrodinger equation, we use correlated Gaussian basis,

$$\psi(\eta_1, \eta_2 ... \eta_{A-1}) = \sum_i c_i \mathcal{A} \exp(-\eta^T A_i \eta)$$

  η = Jacobi coordinates, A_i = matrix of $(A-1) \times (A-1)$ numbers.
- The matrix elements can be calculated analytically in most cases.
- Since $a \ll \Lambda^{-1}$, we need large spread of basis functions.
- Symmetrization gives factor of $A!$ which limits the number of particles.
- Works for any angular momentum, for bosons and fermions. Spin and isospin can be introduced.

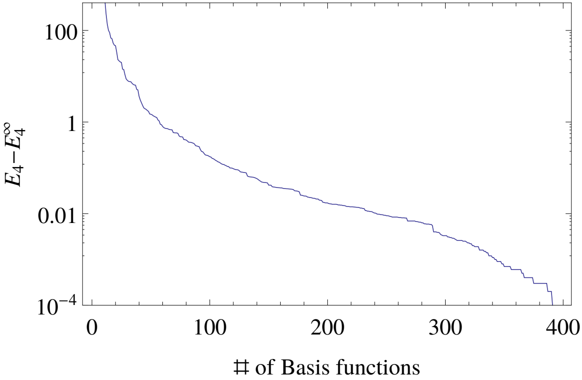
Y. Suzuki and K. Varga, "Stochastic Variational Approach to Quantum-Mechanical Few-Body Problems", Springer 1998.

Stochastic Variational Method

- To find the best A_i, we use the Stochastic Variational Method (SVM):
- We add basis function one by one, or try to replace an exist basis function by a new one.
- We choose randomly the matrix A_i element by element, trying to minimize the energy.
- According to the variational principle, an upper bound for the ground (excited) state is achieved.

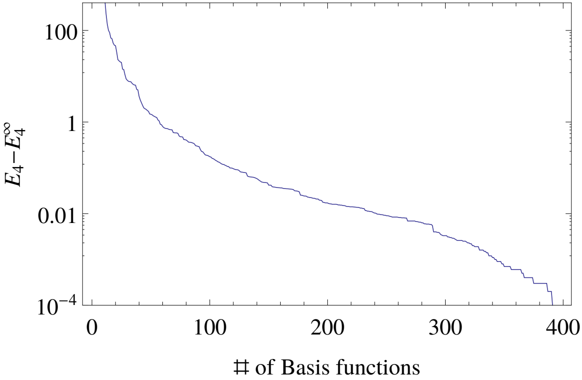
Stochastic Variational Method

- To find the best A_i, we use the Stochastic Variational Method (SVM):
- We add basis function one by one, or try to replace an exist basis function by a new one.
- We choose randomly the matrix A_i element by element, trying to minimize the energy.
- According to the variational principle, an upper bound for the ground (excited) state is achieved.



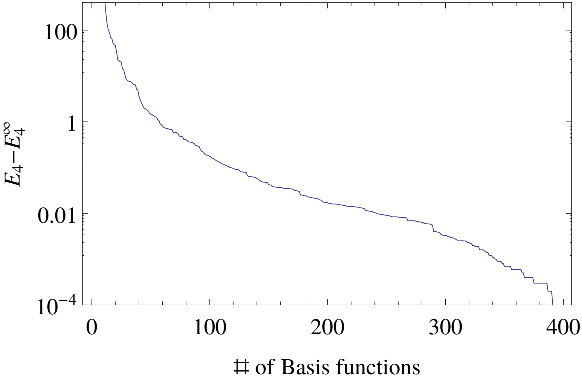
Stochastic Variational Method

- To find the best A_i, we use the Stochastic Variational Method (SVM):
- We add basis function one by one, or try to replace an exist basis function by a new one.
- We choose randomly the matrix A_i element by element, trying to minimize the energy.
- According to the variational principle, an upper bound for the ground (excited) state is achieved.



Stochastic Variational Method

- To find the best A_i, we use the Stochastic Variational Method (SVM):
- We add basis function one by one, or try to replace an exist basis function by a new one.
- We choose randomly the matrix A_i element by element, trying to minimize the energy.
- According to the variational principle, an upper bound for the ground (excited) state is achieved.



Regularization I: non local potential

- At LO, we have only contact interaction,

$$V(r_{ij}) = g\delta(r_{ij}).$$

- This interaction needs regularization and renormalization.
- The bound state of two identical bosons (here $\hbar = m = 1$),

$$-\nabla^2\psi(r) + g\delta(r)\psi(r) = -B_2\psi(r)$$

and in momentum space,

$$p^2\phi(p) + g\int \frac{d^3p'}{(2\pi)^3}\phi(p') = -B_2\phi(p)$$

- Therefore,

$$\frac{1}{g} = \int \frac{d^3p'}{(2\pi)^3}\frac{1}{p'^2 + B_2}$$

which diverges!

Regularization I: non local potential

- At LO, we have only contact interaction,

$$V(r_{ij}) = g\delta(r_{ij}).$$

- This interaction needs regularization and renormalization.
- The bound state of two identical bosons (here $\hbar = m = 1$),

$$-\nabla^2\psi(r) + g\delta(r)\psi(r) = -B_2\psi(r)$$

and in momentum space,

$$p^2\phi(p) + g\int \frac{d^3p'}{(2\pi)^3}\phi(p') = -B_2\phi(p)$$

- Therefore,

$$\frac{1}{g} = \int \frac{d^3p'}{(2\pi)^3}\frac{1}{p'^2 + B_2}$$

which diverges!

Regularization I: non local potential

- At LO, we have only contact interaction,

$$V(r_{ij}) = g\delta(r_{ij}).$$

- This interaction needs regularization and renormalization.
- The bound state of two identical bosons (here $\hbar = m = 1$),

$$-\nabla^2\psi(r) + g\delta(r)\psi(r) = -B_2\psi(r)$$

and in momentum space,

$$p^2\phi(p) + g\int \frac{d^3p'}{(2\pi)^3}\phi(p') = -B_2\phi(p)$$

- Therefore,

$$\frac{1}{g} = \int \frac{d^3p'}{(2\pi)^3}\frac{1}{p'^2 + B_2}$$

which diverges!

Regularization I: non local potential

- At LO, we have only contact interaction,

$$V(r_{ij}) = g\delta(r_{ij}).$$

- This interaction needs regularization and renormalization.
- The bound state of two identical bosons (here $\hbar = m = 1$),

$$-\nabla^2\psi(r) + g\delta(r)\psi(r) = -B_2\psi(r)$$

and in momentum space,

$$p^2\phi(p) + g\int \frac{d^3p'}{(2\pi)^3}\phi(p') = -B_2\phi(p)$$

- Therefore,

$$\frac{1}{g} = \int \frac{d^3p'}{(2\pi)^3}\frac{1}{p'^2 + B_2}$$

which diverges!

Regularization I: non local potential

- To regularize, we can smear the interaction over a range of $1/\Lambda$,

$$\delta_\Lambda(r) \xrightarrow{\Lambda\to\infty} \delta(r).$$

- Doing so for the incoming and outcoming momenta we have,

$$\frac{1}{g} = \int \frac{d^3p'}{(2\pi)^3} \frac{exp(-2p'^2/\Lambda^2)}{p'^2 + B_2}$$

- Which can be expand by powers of Q/Λ, $(Q = \sqrt{B_2})$

$$g = \frac{8\sqrt{2}\pi^{3/2}}{\Lambda}\left(1 + \sqrt{\pi}\frac{Q}{\Lambda} + ...\right).$$

- With the price of non-local potential,

$$\langle r|V|r'\rangle = g\delta_\Lambda(r)\delta_\Lambda(r')$$

R.F. Mohr *et al.*, Ann. Phys. **321**, 225 (2006).

Regularization I: non local potential

- To regularize, we can smear the interaction over a range of $1/\Lambda$,

$$\delta_\Lambda(r) \xrightarrow{\Lambda\to\infty} \delta(r).$$

- Doing so for the incoming and outcoming momenta we have,

$$\frac{1}{g} = \int \frac{d^3p'}{(2\pi)^3} \frac{exp(-2p'^2/\Lambda^2)}{p'^2 + B_2}$$

- Which can be expand by powers of Q/Λ, $(Q = \sqrt{B_2})$

$$g = \frac{8\sqrt{2}\pi^{3/2}}{\Lambda}\left(1 + \sqrt{\pi}\frac{Q}{\Lambda} + ...\right).$$

- With the price of non-local potential,

$$\langle r|V|r'\rangle = g\delta_\Lambda(r)\delta_\Lambda(r')$$

R.F. Mohr *et al.*, Ann. Phys. **321**, 225 (2006).

Regularization I: non local potential

- To regularize, we can smear the interaction over a range of $1/\Lambda$,

$$\delta_\Lambda(r) \xrightarrow{\Lambda\to\infty} \delta(r).$$

- Doing so for the incoming and outcoming momenta we have,

$$\frac{1}{g} = \int \frac{d^3p'}{(2\pi)^3} \frac{exp(-2p'^2/\Lambda^2)}{p'^2 + B_2}$$

- Which can be expand by powers of Q/Λ, ($Q = \sqrt{B_2}$)

$$g = \frac{8\sqrt{2}\pi^{3/2}}{\Lambda}\left(1 + \sqrt{\pi}\frac{Q}{\Lambda} + ...\right).$$

- With the price of non-local potential,

$$\langle r|V|r'\rangle = g\delta_\Lambda(r)\delta_\Lambda(r')$$

R.F. Mohr *et al.*, Ann. Phys. **321**, 225 (2006).

Regularization I: non local potential

- To regularize, we can smear the interaction over a range of $1/\Lambda$,

$$\delta_\Lambda(r) \xrightarrow{\Lambda\to\infty} \delta(r).$$

- Doing so for the incoming and outcoming momenta we have,

$$\frac{1}{g} = \int \frac{d^3p'}{(2\pi)^3} \frac{exp(-2p'^2/\Lambda^2)}{p'^2 + B_2}$$

- Which can be expand by powers of Q/Λ, ($Q = \sqrt{B_2}$)

$$g = \frac{8\sqrt{2}\pi^{3/2}}{\Lambda}\left(1 + \sqrt{\pi}\frac{Q}{\Lambda} + ...\right).$$

- With the price of non-local potential,

$$\langle r|V|r'\rangle = g\delta_\Lambda(r)\delta_\Lambda(r')$$

R.F. Mohr *et al.*, Ann. Phys. **321**, 225 (2006).

Regularization II: local potential

- We can cut the momentum transfer $q = p - p'$, to get a local potential,

$$\langle r|V|r'\rangle = g\delta_\Lambda(r)\delta(r - r')$$

  but now the two-body equation is to be solved numerically.
- The LEC is renormalized by fixing one observable, like the dimer binding energy or the scattering length, to its physical value.
- Using dimension less LEC,

$$V(r) = \frac{4\pi\hbar^2}{m\Lambda}C^{(0)}(\Lambda)\delta_\Lambda(\mathbf{r}), \;\; C^{(0)}(\Lambda) = 2.38\left(1 + \frac{2.25}{a\Lambda} - \frac{4.68}{(a\Lambda)^2} + ...\right)$$

Regularization II: local potential

- We can cut the momentum transfer $q = p - p'$, to get a local potential,

$$\langle r|V|r'\rangle = g\delta_\Lambda(r)\delta(r - r')$$

  but now the two-body equation is to be solved numerically.
- The LEC is renormalized by fixing one observable, like the dimer binding energy or the scattering length, to its physical value.
- Using dimension less LEC,

$$V(r) = \frac{4\pi\hbar^2}{m\Lambda} C^{(0)}(\Lambda)\delta_\Lambda(\mathbf{r}), \;\; C^{(0)}(\Lambda) = 2.38\left(1 + \frac{2.25}{a\Lambda} - \frac{4.68}{(a\Lambda)^2} + ...\right)$$

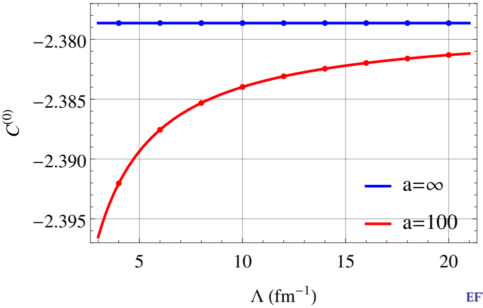
Regularization II: local potential

- We can cut the momentum transfer $q = p - p'$, to get a local potential,

$$\langle r|V|r'\rangle = g\delta_\Lambda(r)\delta(r-r')$$

  but now the two-body equation is to be solved numerically.
- The LEC is renormalized by fixing one observable, like the dimer binding energy or the scattering length, to its physical value.
- Using dimension less LEC,

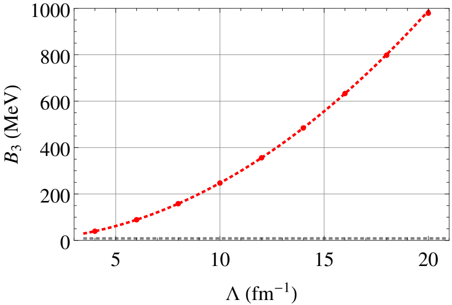
$$V(r) = \frac{4\pi\hbar^2}{m\Lambda}C^{(0)}(\Lambda)\delta_\Lambda(\boldsymbol{r}), \;\; C^{(0)}(\Lambda) = 2.38\left(1 + \frac{2.25}{a\Lambda} - \frac{4.68}{(a\Lambda)^2} + ...\right)$$



Three-boson system

Trying to calculate the trimer binding energy we get the Thomas collapse:

$$B_3 \approx 0.0596 \frac{\hbar \Lambda^2}{m}$$



- To stabilize the system, a 3-body counter term must be introduced at LO,

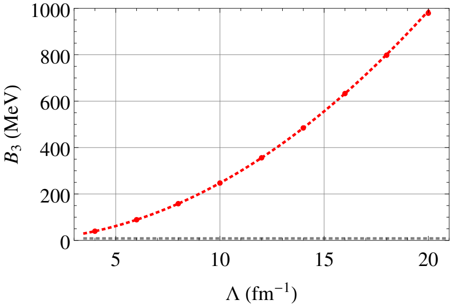
$$V_{LO}^{3N} = \frac{4\pi\hbar^2}{m\Lambda^2\Lambda_*^2} D^{(0)} \sum_{i<j<k} \sum_{cyc} \delta_\Lambda(\boldsymbol{r}_{ij})\delta_\Lambda(\boldsymbol{r}_{jk}),$$

  Λ_* is a new momentum scale, $D^{(0)} = f(\Lambda/\Lambda_*)$
- $D^{(0)}$ is fixed by an A-body ($A > 2$) observable.
  Here we use the "triton" binding energy, $B_3 \approx 8.482$ MeV.

Three-boson system

Trying to calculate the trimer binding energy we get the Thomas collapse:

$$B_3 \approx 0.0596 \frac{\hbar \Lambda^2}{m}$$



- To stabilize the system, a 3-body counter term must be introduced at LO,

$$V_{LO}^{3N} = \frac{4\pi\hbar^2}{m\Lambda^2\Lambda_*^2} D^{(0)} \sum_{i<j<k} \sum_{cyc} \delta_\Lambda(\boldsymbol{r}_{ij})\delta_\Lambda(\boldsymbol{r}_{jk}),$$

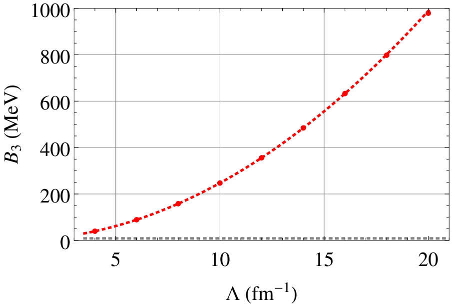
  Λ_* is a new momentum scale, $D^{(0)} = f(\Lambda/\Lambda_*)$

- $D^{(0)}$ is fixed by an A-body ($A > 2$) observable.
  Here we use the "triton" binding energy, $B_3 \approx 8.482$ MeV.

Three-boson system

Trying to calculate the trimer binding energy we get the Thomas collapse:

$$B_3 \approx 0.0596 \frac{\hbar \Lambda^2}{m}$$

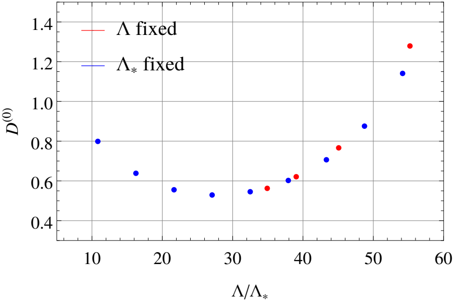


- To stabilize the system, a 3-body counter term must be introduced at LO,

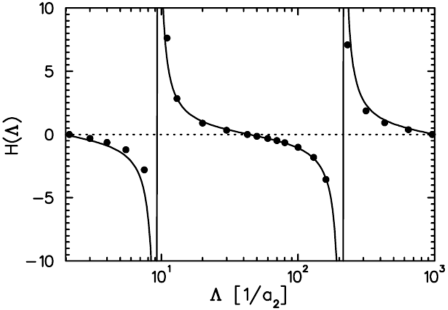
$$V_{LO}^{3N} = \frac{4\pi\hbar^2}{m\Lambda^2\Lambda_*^2} D^{(0)} \sum_{i<j<k} \sum_{cyc} \delta_\Lambda(\boldsymbol{r}_{ij})\delta_\Lambda(\boldsymbol{r}_{jk}),$$

  Λ_* is a new momentum scale, $D^{(0)} = f(\Lambda/\Lambda_*)$
- $D^{(0)}$ is fixed by an A-body ($A > 2$) observable.
  Here we use the "triton" binding energy, $B_3 \approx 8.482$ MeV.

Three-boson system



$\Lambda_3 = \Lambda_2$, local, smooth cutoff

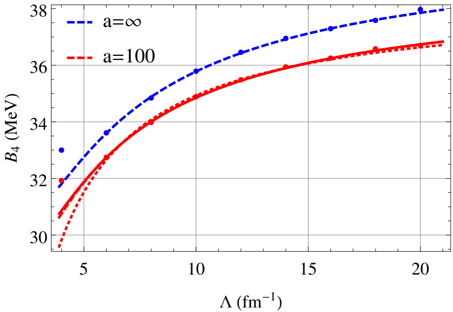


$\Lambda_3 \ll \Lambda_2$, non-local, sharp cutoff
P. F. Bedaque, H.W. Hammer, and U. van Kolck Phys. Rev. Lett. **82** 463 (1999).

$\Lambda_3 = \Lambda_2$, non-local, smooth cutoff R.F. Mohr *et al.*, Ann. Phys. **321**, 225 (2006).

Four-boson system

Are more terms needed to stabilize heavier systems?



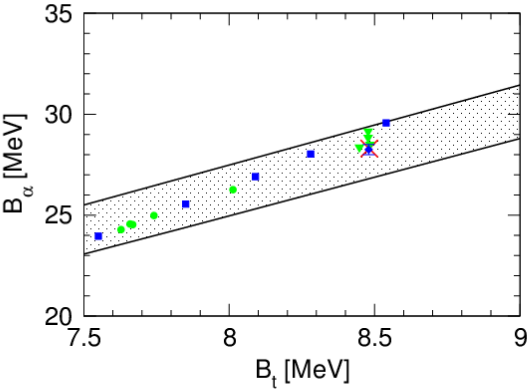
$$B_4(\Lambda) = B_\infty \left[1 + \alpha \frac{Q_4}{\Lambda} + \beta \left(\frac{Q_4}{\Lambda}\right)^2 + \gamma \left(\frac{Q_4}{\Lambda}\right)^3 + ...\right], Q_4 = \sqrt{\frac{2mB_4}{4\hbar^2}} \approx 0.68\text{fm}^{-1}$$

B_∞	α	β	γ
38.34	-1.30	–	–
38.89	-1.71	2.86	–
38.83	-1.65	1.98	3.88

Tjon line

Another evidence is the Tjon line, the correlation between the binding energies of the triton and the α-particle.

J.A. Tjon, Phys. Lett. B **56**, 217 (1975).

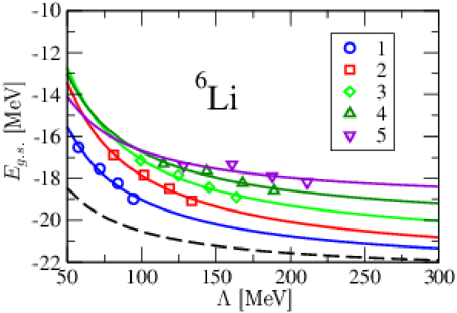


L. Platter, H.-W. Hammer, U.-G. Meissner, Phys. Lett. B **607**, 254 (2005).

5- and 6- boson system

Are more terms needed to stabilize heavier systems?

For nucleons,



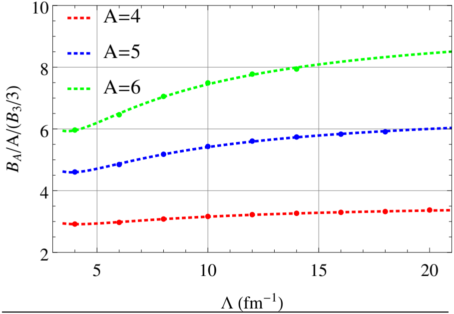
I. Stetcu, B.R. Barrett, and U. van Kolck, Phys. Lett. B **653** , 358 (2007).

A	x	x^2	x^3	Ref. [1]
4	9.63	10.22	9.94	9.88
5	17.51	19.08	18.40	18.05
6	24.37	27.46	24.98	26.28

[1] J. von Stecher, J. Phys. B: At. Mol. Opt. Phys. **43**, 101002 (2010)

5- and 6- boson system

Are more terms needed to stabilize heavier systems?

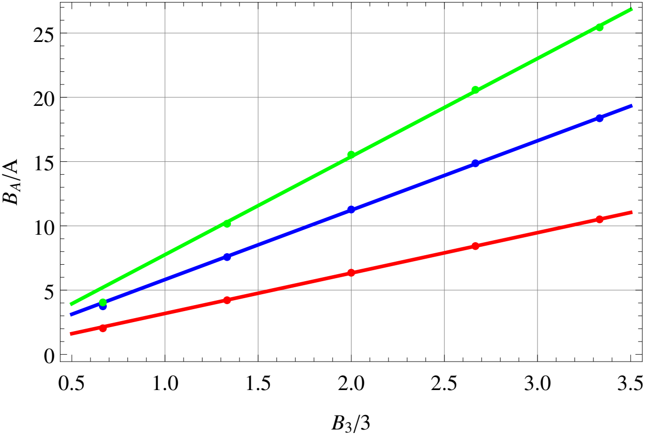


A	x	x^2	x^3	Ref. [1]
4	9.63	10.22	9.94	9.88
5	17.51	19.08	18.40	18.05
6	24.37	27.46	24.98	26.28

[1] J. von Stecher, J. Phys. B: At. Mol. Opt. Phys. **43**, 101002 (2010).

Generalized Tjon-lines

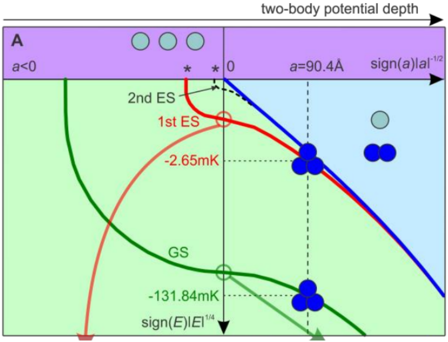
Correlation between B_3 to B_4, B_5, and B_6:



...Therefore, no 4, 5 or 6-body terms are needed at LO.

^{4}He Atoms

- The system of ^{4}He atoms is known to be a natural candidate for universal physics, since $a \approx 170.9a_0 \gg r_{vdW} \approx 2.6a_0$.
- The ^{4}He dimer is bound by 1.62 mK ($\hbar^2/ma^2 \approx 1.48$ mK).
- $E_3 \approx 131.84$ mK, $E_3^* \approx 2.6502$ mK, giving a ratio of $E_3^*/E_3 \approx 49.7$.
- Recently this excited state was observed experimentally.

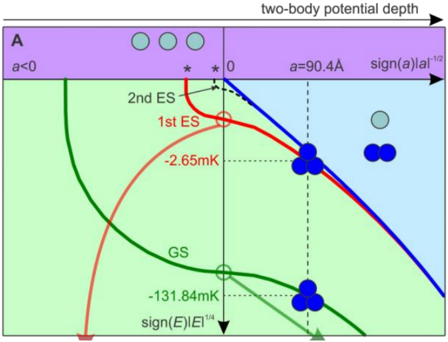


Theory: E. Hiyama and M. Kamimura, Phys Rev A. **85**, 062505 (2012);

Experiment: M. Kunitski *et al.*, Science **348** 551 (2015).

^{4}He Atoms

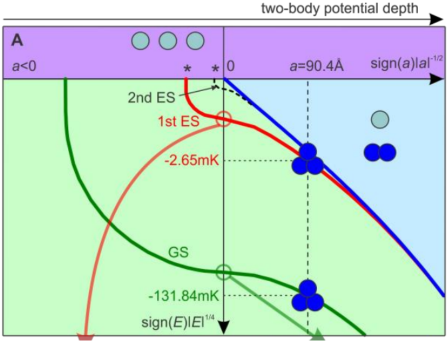
- The system of ^{4}He atoms is known to be a natural candidate for universal physics, since $a \approx 170.9a_0 \gg r_{vdW} \approx 2.6a_0$.
- The ^{4}He dimer is bound by 1.62 mK ($\hbar^2/ma^2 \approx 1.48$ mK).
- $E_3 \approx 131.84$ mK, $E_3^* \approx 2.6502$ mK, giving a ratio of $E_3^*/E_3 \approx 49.7$.
- Recently this excited state was observed experimentally.



Theory: E. Hiyama and M. Kamimura, Phys Rev A. **85**, 062505 (2012);
Experiment: M. Kunitski *et al.*, Science **348** 551 (2015).

^{4}He Atoms

- The system of ^{4}He atoms is known to be a natural candidate for universal physics, since $a \approx 170.9 a_0 \gg r_{vdW} \approx 2.6 a_0$.
- The ^{4}He dimer is bound by 1.62 mK ($\hbar^2/ma^2 \approx 1.48$ mK).
- $E_3 \approx 131.84$ mK, $E_3^* \approx 2.6502$ mK, giving a ratio of $E_3^*/E_3 \approx 49.7$.
- Recently this excited state was observed experimentally.

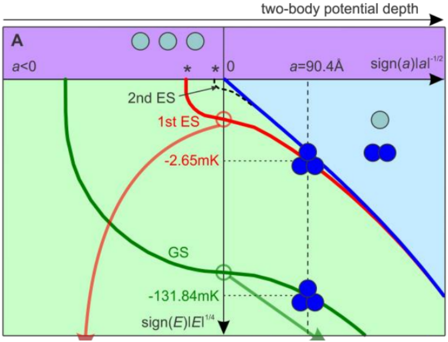


Theory: E. Hiyama and M. Kamimura, Phys Rev A. **85**, 062505 (2012);

Experiment: M. Kunitski *et al.*, Science **348** 551 (2015).

^{4}He Atoms

- The system of ^{4}He atoms is known to be a natural candidate for universal physics, since $a \approx 170.9 a_0 \gg r_{vdW} \approx 2.6 a_0$.
- The ^{4}He dimer is bound by 1.62 mK ($\hbar^2/ma^2 \approx 1.48$ mK).
- $E_3 \approx 131.84$ mK, $E_3^* \approx 2.6502$ mK, giving a ratio of $E_3^*/E_3 \approx 49.7$.
- Recently this excited state was observed experimentally.



Theory: E. Hiyama and M. Kamimura, Phys Rev A. **85**, 062505 (2012);

Experiment: M. Kunitski *et al.*, Science **348** 551 (2015).

Short-Range EFT for ^{4}He Atoms

- We apply short-range EFT for the ^{4}He atomic system.
- The two-body LEC is fitted to the dimer binding energy.
- The three-body LEC is fitted to the binding energy of the trimer excited state.
- Then the ground state energies of the trimer and the tetramer states are calculated.

See also: L. Platter, H.W. Hammer, and Ulf-G. Meissner, Phys. Rev. A **70**, 052101 (2004).

Short-Range EFT for ^{4}He Atoms

- We apply short-range EFT for the ^{4}He atomic system.
- The two-body LEC is fitted to the dimer binding energy.
- The three-body LEC is fitted to the binding energy of the trimer excited state.
- Then the ground state energies of the trimer and the tetramer states are calculated.

See also: L. Platter, H.W. Hammer, and Ulf-G. Meissner, Phys. Rev. A **70**, 052101 (2004).

Short-Range EFT for ^{4}He Atoms

- We apply short-range EFT for the ^{4}He atomic system.
- The two-body LEC is fitted to the dimer binding energy.
- The three-body LEC is fitted to the binding energy of the trimer excited state.
- Then the ground state energies of the trimer and the tetramer states are calculated.

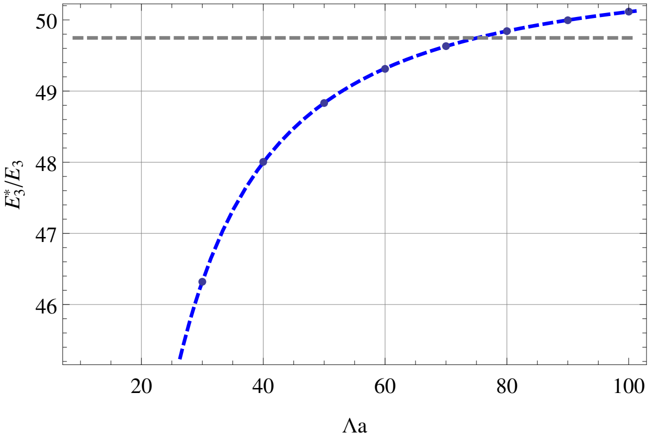
See also: L. Platter, H.W. Hammer, and Ulf-G. Meissner, Phys. Rev. A **70**, 052101 (2004).

Short-Range EFT for ^{4}He Atoms

- We apply short-range EFT for the ^{4}He atomic system.
- The two-body LEC is fitted to the dimer binding energy.
- The three-body LEC is fitted to the binding energy of the trimer excited state.
- Then the ground state energies of the trimer and the tetramer states are calculated.

See also: L. Platter, H.W. Hammer, and Ulf-G. Meissner, Phys. Rev. A **70**, 052101 (2004).

^{4}He Trimer



Real Nuclei

- At LO, the 2-body potential reads,

$$V_{LO} = a_1 + a_2\sigma_i \cdot \sigma_j + a_3\tau_i \cdot \tau_j + a_4\sigma_i \cdot \sigma_j\tau_i \cdot \tau_j$$

- using the fermionic symmetry,

$$V_{LO} = C_S\hat{P}_S + C_T\hat{P}_T$$

  where $\hat{P}_\alpha$ is projection operator on channel α
- The 2-body LECs are fitted to the deuteron binding energy and the singlet $^1S_0$ np scattering length.
- The 3-body LEC is fitted to the triton binding energy
- No Coulumb interaction

Real Nuclei

- At LO, the 2-body potential reads,

$$V_{LO} = a_1 + a_2\sigma_i \cdot \sigma_j + a_3\tau_i \cdot \tau_j + a_4\sigma_i \cdot \sigma_j\tau_i \cdot \tau_j$$

- using the fermionic symmetry,

$$V_{LO} = C_S\hat{P}_S + C_T\hat{P}_T$$

  where $\hat{P}_\alpha$ is projection operator on channel α
- The 2-body LECs are fitted to the deuteron binding energy and the singlet $^1S_0$ np scattering length.
- The 3-body LEC is fitted to the triton binding energy
- No Coulumb interaction

Real Nuclei

- At LO, the 2-body potential reads,

$$V_{LO} = a_1 + a_2 \sigma_i \cdot \sigma_j + a_3 \tau_i \cdot \tau_j + a_4 \sigma_i \cdot \sigma_j \tau_i \cdot \tau_j$$

- using the fermionic symmetry,

$$V_{LO} = C_S \hat{P}_S + C_T \hat{P}_T$$

  where $\hat{P}_\alpha$ is projection operator on channel α
- The 2-body LECs are fitted to the deuteron binding energy and the singlet ${}^1S_0$ np scattering length.
- The 3-body LEC is fitted to the triton binding energy
- No Coulumb interaction

Real Nuclei

- At LO, the 2-body potential reads,

$$V_{LO} = a_1 + a_2 \sigma_i \cdot \sigma_j + a_3 \tau_i \cdot \tau_j + a_4 \sigma_i \cdot \sigma_j \tau_i \cdot \tau_j$$

- using the fermionic symmetry,

$$V_{LO} = C_S \hat{P}_S + C_T \hat{P}_T$$

  where $\hat{P}_\alpha$ is projection operator on channel α
- The 2-body LECs are fitted to the deuteron binding energy and the singlet ${}^1S_0$ np scattering length.
- The 3-body LEC is fitted to the triton binding energy
- No Coulumb interaction

Real Nuclei

- At LO, the 2-body potential reads,

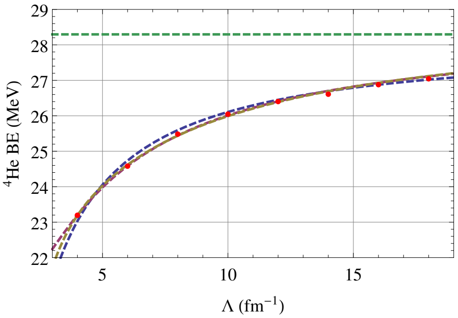
$$V_{LO} = a_1 + a_2 \sigma_i \cdot \sigma_j + a_3 \tau_i \cdot \tau_j + a_4 \sigma_i \cdot \sigma_j \tau_i \cdot \tau_j$$

- using the fermionic symmetry,

$$V_{LO} = C_S \hat{P}_S + C_T \hat{P}_T$$

  where $\hat{P}_\alpha$ is projection operator on channel α
- The 2-body LECs are fitted to the deuteron binding energy and the singlet ${}^1S_0$ np scattering length.
- The 3-body LEC is fitted to the triton binding energy
- No Coulumb interaction

α - ^{4}He nuclei



$$B_4(\Lambda) = B_\infty \left[1 + \alpha \frac{Q_4}{\Lambda} + \beta \left(\frac{Q_4}{\Lambda} \right)^2 + \gamma \left(\frac{Q_4}{\Lambda} \right)^3 + ... \right]$$

B_∞	α	β	γ
28.16	-1.25	–	–
28.66	-1.77	3.17	–
28.88	-2.12	8.07	-19.21

Conclusion

- A pionless EFT was constructed for various systems: bosonic nucleons, ^{4}He atoms, and real nucleus, fitting nicely the known results.
- The convergence of bosonic EFT for $A = 4, 5$ and 6 was studied.
- Generalized Tjon-lines were introduced, showing that at LO no 4,5 or 6-body term is needed.
- Work in progress for $A = 6$ nuclei.

Conclusion

- A pionless EFT was constructed for various systems: bosonic nucleons, ^{4}He atoms, and real nucleus, fitting nicely the known results.
- The convergence of bosonic EFT for $A = 4, 5$ and 6 was studied.
- Generalized Tjon-lines were introduced, showing that at LO no 4,5 or 6-body term is needed.
- Work in progress for $A = 6$ nuclei.

Conclusion

- A pionless EFT was constructed for various systems: bosonic nucleons, ^{4}He atoms, and real nucleus, fitting nicely the known results.
- The convergence of bosonic EFT for $A = 4, 5$ and 6 was studied.
- Generalized Tjon-lines were introduced, showing that at LO no 4,5 or 6-body term is needed.
- Work in progress for $A = 6$ nuclei.

Conclusion

- A pionless EFT was constructed for various systems: bosonic nucleons, ^{4}He atoms, and real nucleus, fitting nicely the known results.
- The convergence of bosonic EFT for $A = 4, 5$ and 6 was studied.
- Generalized Tjon-lines were introduced, showing that at LO no 4,5 or 6-body term is needed.
- Work in progress for $A = 6$ nuclei.