

# Composition and clusters in the neutron-star crust

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J. M. Pearson (Université de Montréal)

Supernova Remnant 1987A in the Large Magellanic Cloud.  HUBBLE SITE.org

Fustipen Topical Meeting  
GANIL, Caen (France), 17 – 19 May 2016



# Outline

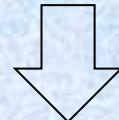
- ❖ Introduction: NS & EoS
- ❖ Effective nuclear models
  - Brussels-Montreal BSk model
- ❖ NS composition & clusters
  - outer crust
  - inner crust
- ❖ Conclusions



# NS: general properties

Contrarily to a normal star, in a NS:

- ✓ matter is highly **degenerate!**  
( **T = 0** approximation )
- ✓ **very high density!**  
composition uncertain



different states of matter :  
inhomogeneous, homogeneous,  
exotic particles ?

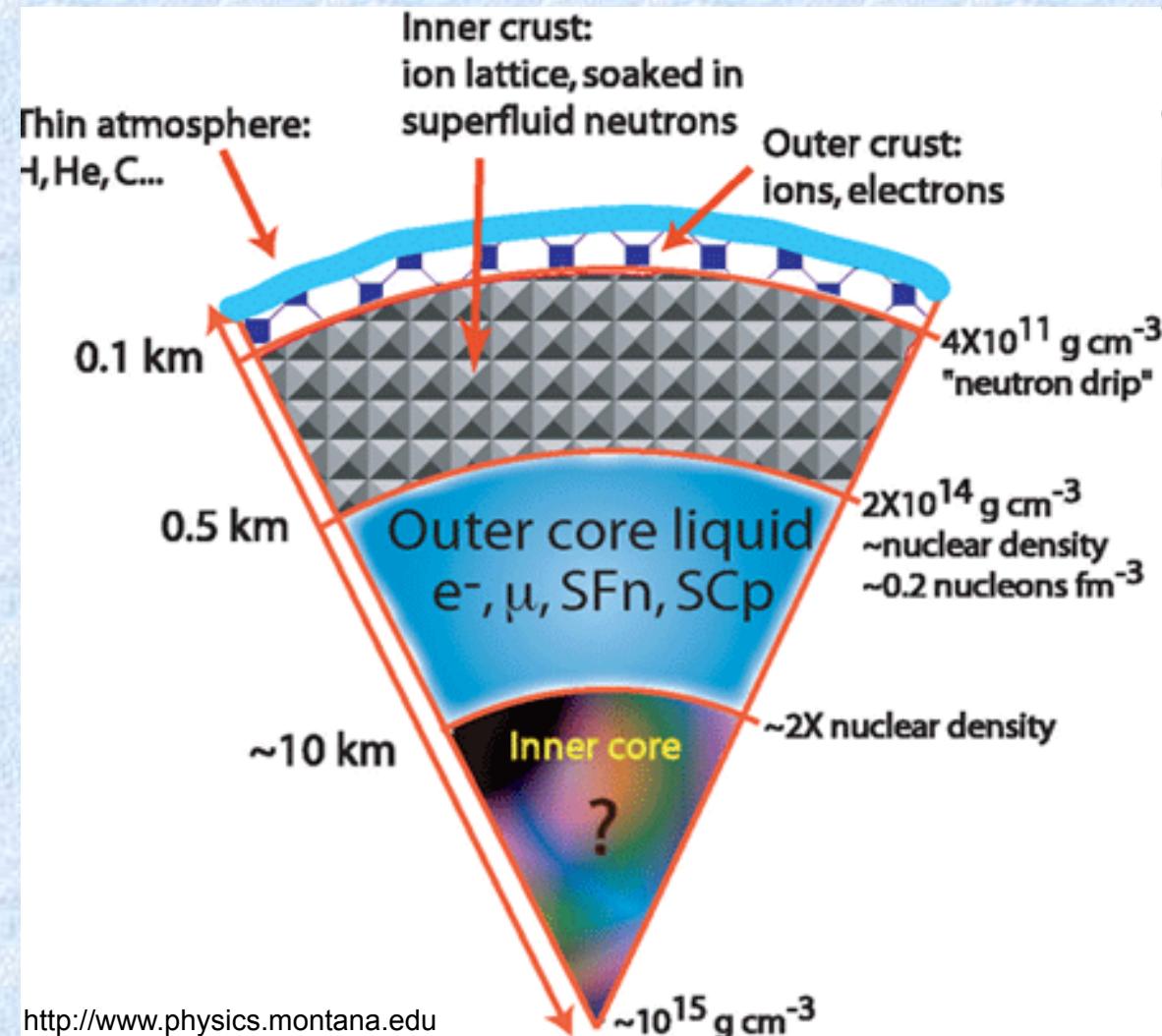
$$M \approx 1 - 2 M_{\odot}$$

$$R \approx 10 \text{ km}$$

$$\bar{\rho} \approx 10^{14} - 10^{15} \text{ g cm}^{-3}$$

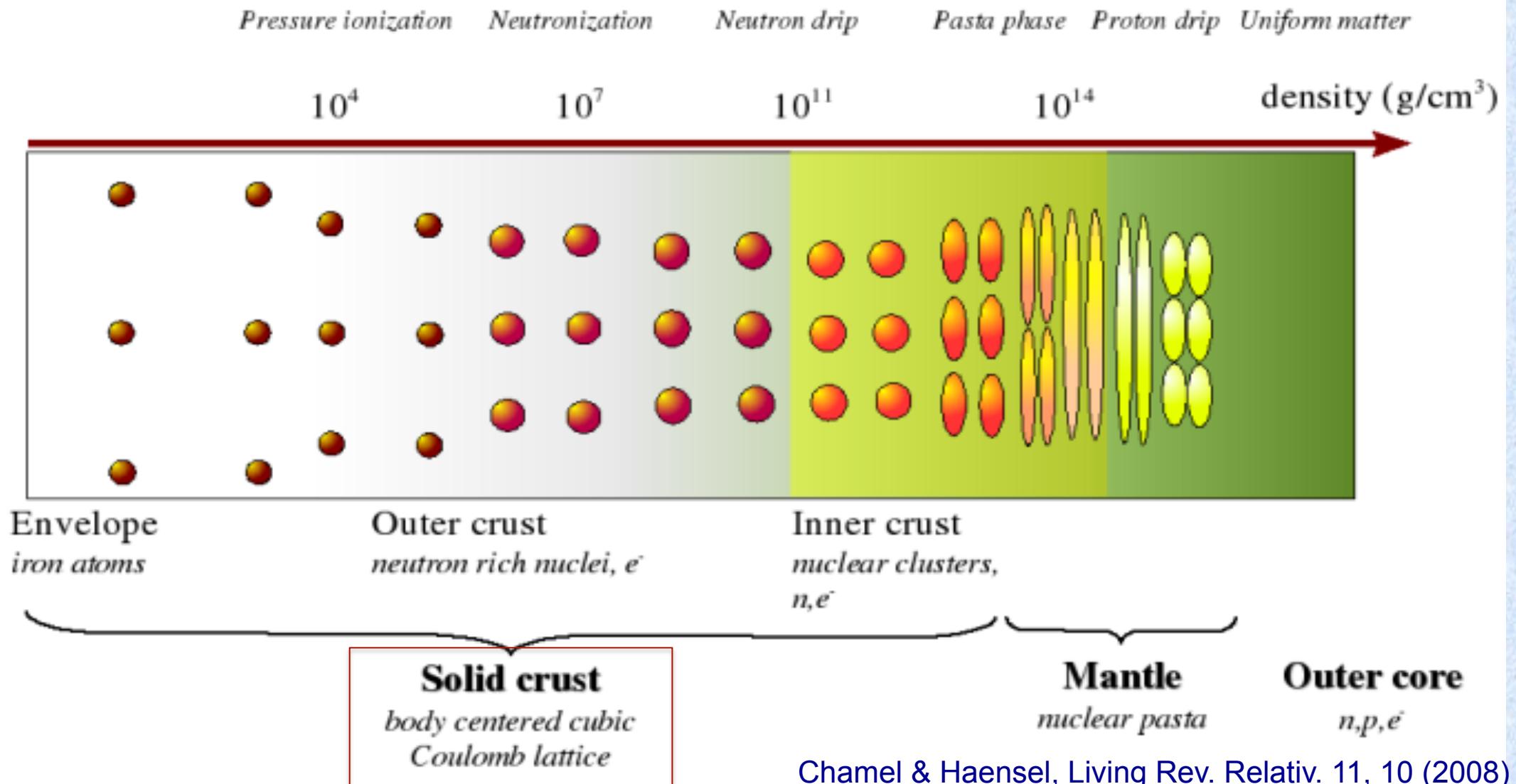
$$\frac{2GM}{Rc^2} \approx 0.2 - 0.4$$

$$B_{\text{surf}} \text{ up to } 10^{14} - 10^{15} \text{ G} \longrightarrow B_{\text{int}} ???$$

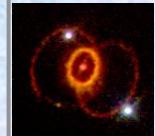




# NS crust structure

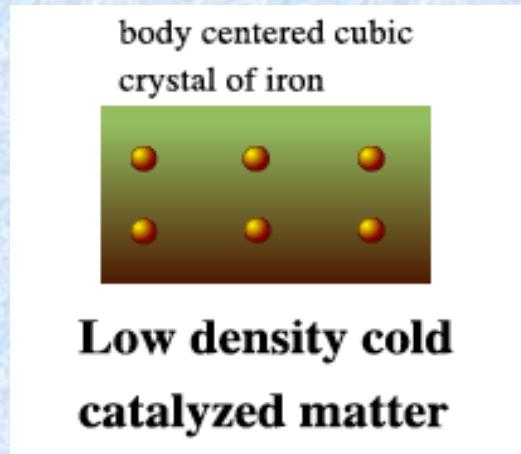


Neutron star crust :  $\approx 1\%$  mass,  $\approx 10\%$  radius

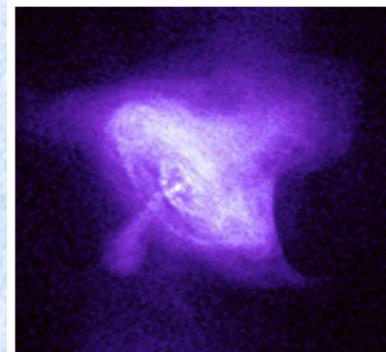


# Different NS crusts

## ➤ Catalysed matter

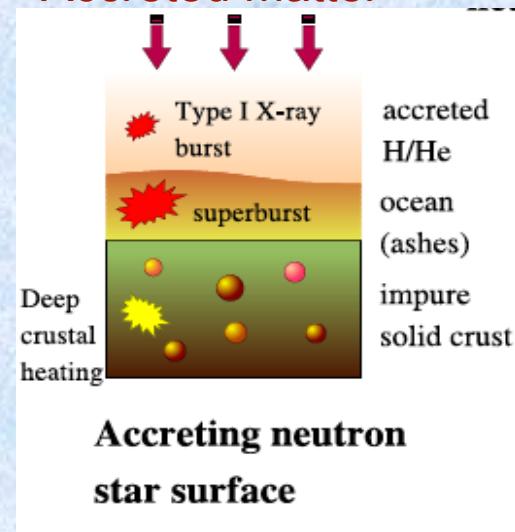


- NS born at high  $T \approx 10^{11}$  K → “hot” scenario
- **full thermodynamical equilibrium at  $T=0$**

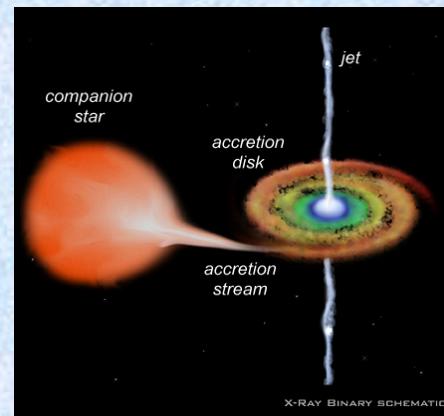


Example of isolated NS  
Credits: NASA/CXC/SAO (Crab, X ray)

## ➤ Accreted matter



- $T < 10^9$  K → “cold” scenario
- **matter off-equilibrium (local min of E)**



Artist view of accreting NS

for a review: Chamel & Haensel, Living Rev. Relativ. 11, 10 (2008) and Refs. therein



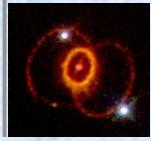
# Different theoretical approaches

## Phenomenological approaches

- Based on effective nuclear interactions
- Parameter adjusted to reproduced nuclear properties
- Liquid drop models (e.g. BBP 1971, Douchin&Haensel 2001, LS, Oertel *et al.* 2012 T>0)
- Semi-classical models: (Extended) Thomas-Fermi (e.g. Onsi *et al.* 2008, Shen 1998)
- Self-consistent mean-field models : Hartree-Fock / Hartree-Fock Bogoliubov (e.g. Negele&Vautherin 1973, Baldo *et al.* 2007, Grill *et al.* 2011)  
RMF (with and w/o hyperons) (e.g. Weissenborn *et al.* 2012, Bednarek *et al.* 2012)
- Nambu-Jona-Lasinio (quark) (e.g. Zdunik&Haensel 2013, Blaschke *et al.* 2010, Bonanno&Sedrakian 2012)
- (Modified) Bag Model of quark (e.g. Weissenborn *et al.* 2011)

## Microscopic approaches

- Based on quantum many-body theories from realistic nuclear interactions (→ *ab-initio* methods)
- But : not (yet) affordable for the crust
- Variational methods (e.g. APR 1998)
- (Dirac) Brueckner Hartree-Fock (e.g. Sammaruca 2010, Li&Schulze 2008, Fuchs 2008, Vidaña *et al.* 2011, Burgio *et al.* 2011)
- Perturbative QCD (e.g. Kurkela *et al.* 2010)
- Monte Carlo (e.g. Carlson *et al.* 2003, Gandolfi *et al.* 2012)



# Properties of nuclear matter

In applying nuclear models in astrophysics → two kinds of quantities :

1. **Thermodynamic variables** → physical conditions in the star (e.g.  $P, T, B, \dots$ )
2. **Nuclear parameters** → properties of nuclear matter around saturation at  $T=0$

- Energy around saturation (in a liquid drop model):

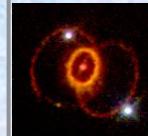
$$E(n, \eta) = E(n_0, \eta = 0) + \frac{1}{2} K_\infty \left( \frac{n - n_0}{3n_0} \right)^2 + E_{\text{sym}}$$

- In SN & NS → n-rich matter → symmetry energy important:

$$S_1(n) = \frac{1}{2} \left. \frac{\partial^2(\mathcal{E}/n)}{\partial \eta^2} \right|_{\eta=0} \approx J + \frac{1}{3} L \left( \frac{n - n_0}{n_0} \right) + \frac{1}{18} K_{\text{sym}} \left( \frac{n - n_0}{n_0} \right)^2$$

≠

$$S_2(n) = \frac{\mathcal{E}(n, \eta = 1) - \mathcal{E}(n, \eta = 0)}{n} \quad \eta = \frac{n_n - n_p}{n}$$



# Properties of nuclear matter

In applying nuclear models in astrophysics we need to know:

1. Thermodynamic variables  $\rightarrow$  energy, entropy, pressure, ...

2. Nuclear parameters  $\rightarrow$  properties of the interaction

- Energy around saturation (*i.e.*  $n = n_0$ )

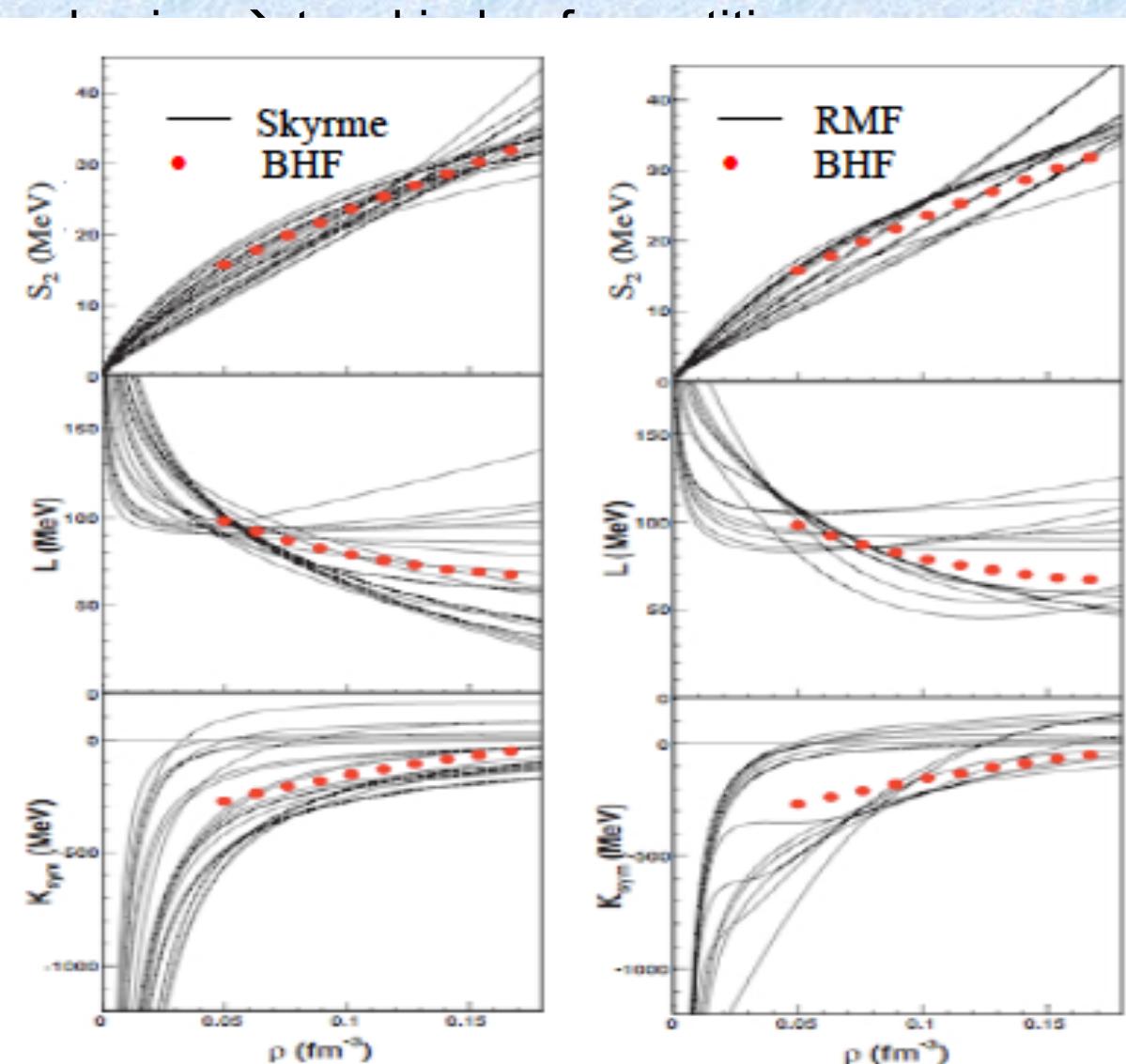
$$E(n, \eta) = E(n_0, \eta = 1) + \dots$$

- In SN & NS  $\rightarrow$  n-rich matter

$$S_1(n) = \frac{1}{2} \left. \frac{\partial^2 (\mathcal{E}/n)}{\partial \eta^2} \right|_{\eta=0} \approx$$

$\neq$

$$S_2(n) = \frac{\mathcal{E}(n, \eta = 1) - \mathcal{E}(n, \eta = 0)}{n}$$



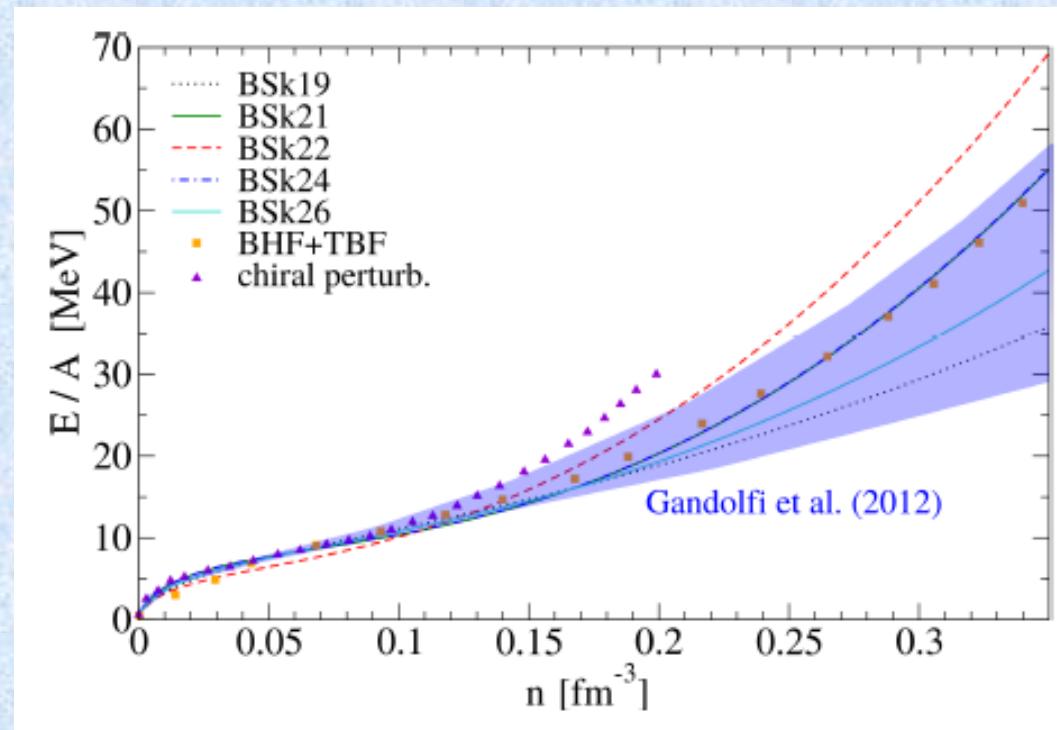
I. Vidana, talk at WG2 meeting NewCompstar(2014)



# How can we constrain the EoS ?

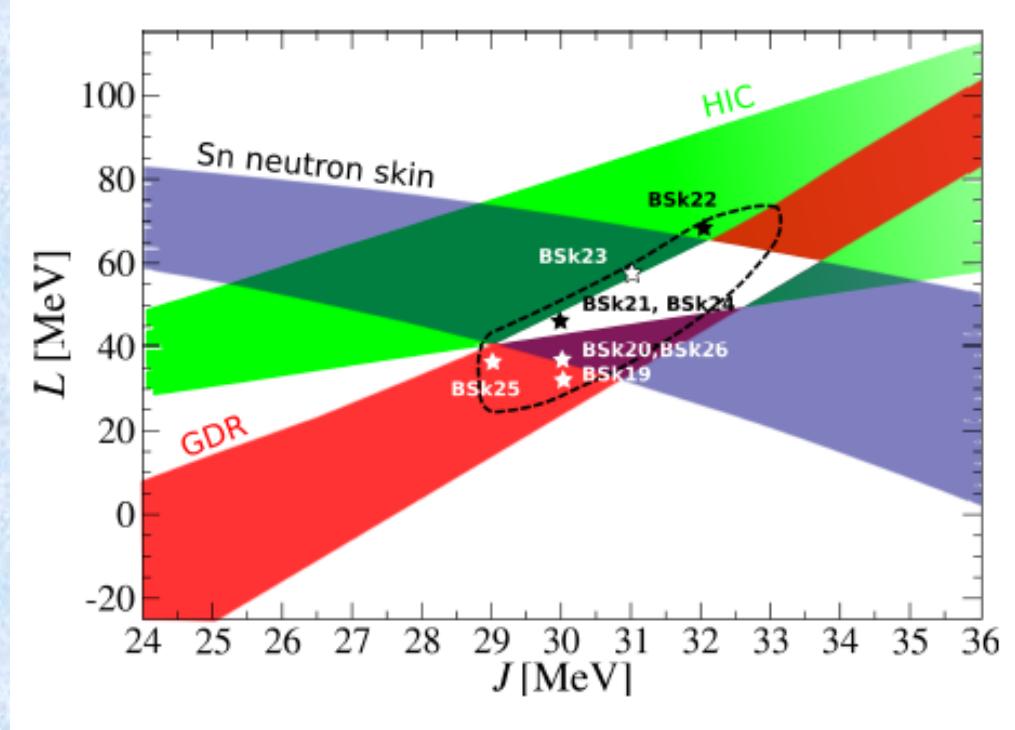
## Nuclear physics experiments

- Measure of **nuclear masses**, radii
- Resonances  $\rightarrow K, E_{sym} \dots$
- Heavy ion collision experiments
- ...
- + *ab-initio* calculations (theory)



## Astrophysical observations

- Measure of NS masses, radii (!)
- Rotation
- Cooling
- moments of inertia,  $M_b$  vs  $M_{grav}$
- ...

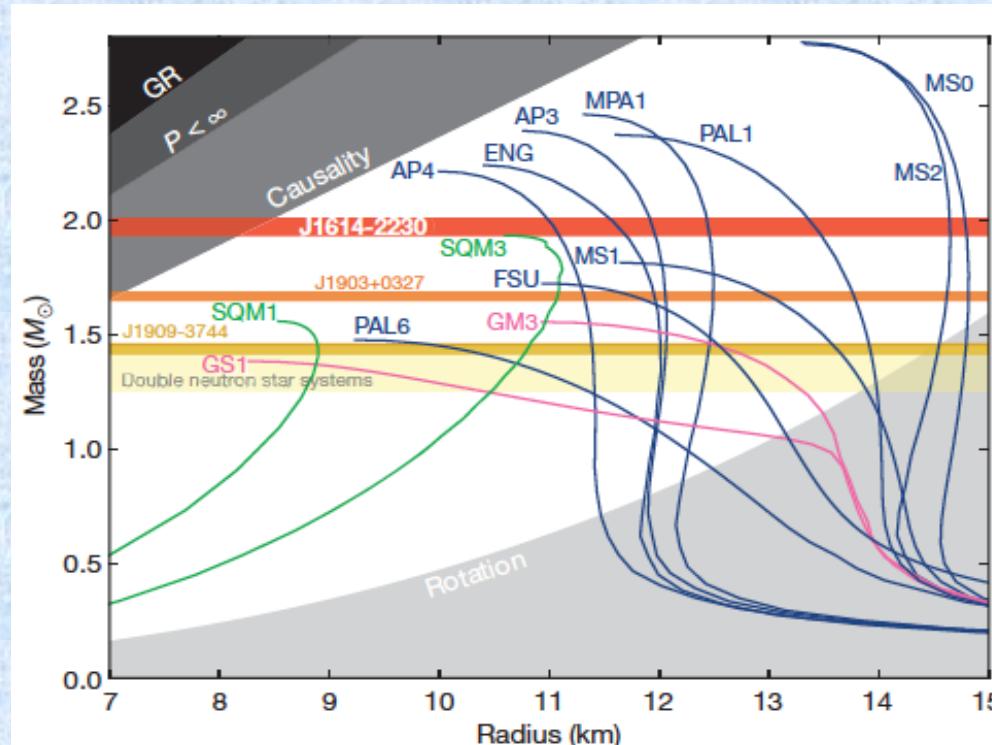




# How can we constrain the EoS ?

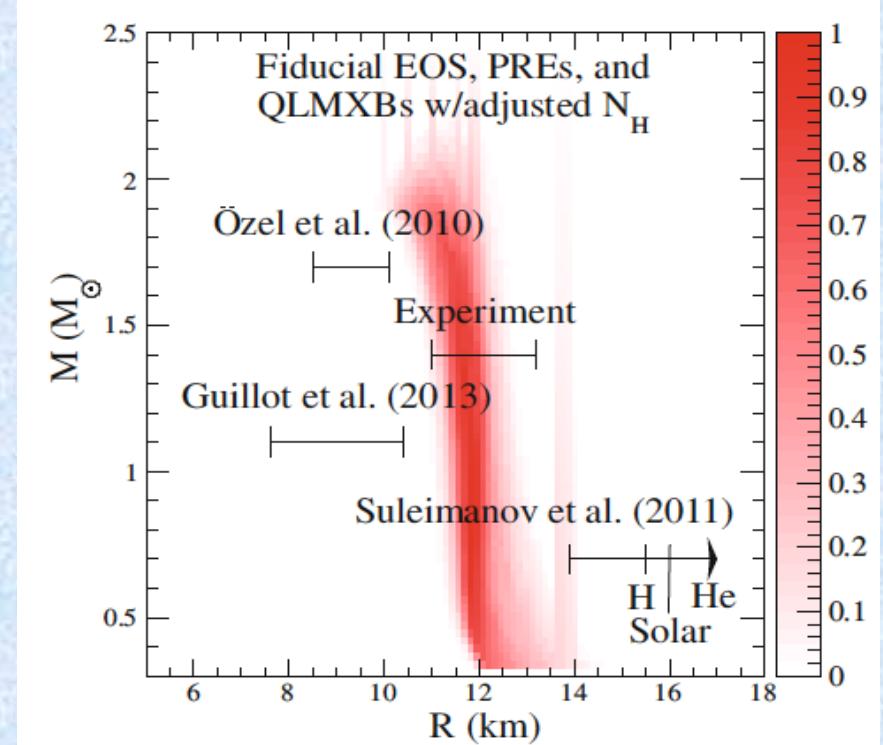
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- Measure of **nuclear masses**, radii
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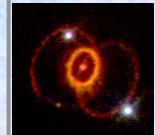


## Astrophysical observations

- **Measure of NS masses**, radii (!)
- **Rotation**
- Cooling
- moments of inertia,  $M_b$  vs  $M_{grav}$
- ...



Demorest *et al.*, Nature 467, 1081 (2010) ; Lattimer & Steiner, EPJ A50, 40 (2014);  
see also Heinke *et al.*, MNRAS 444, 443 (2014)



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# Brussels-Montreal (BSk) functionals

Mass models based on HFB method with Skyrme type energy-density functionals (EDFs) and macroscopically deduced pairing force.

Fitted to experimental data + N-body calculations with realistic forces.

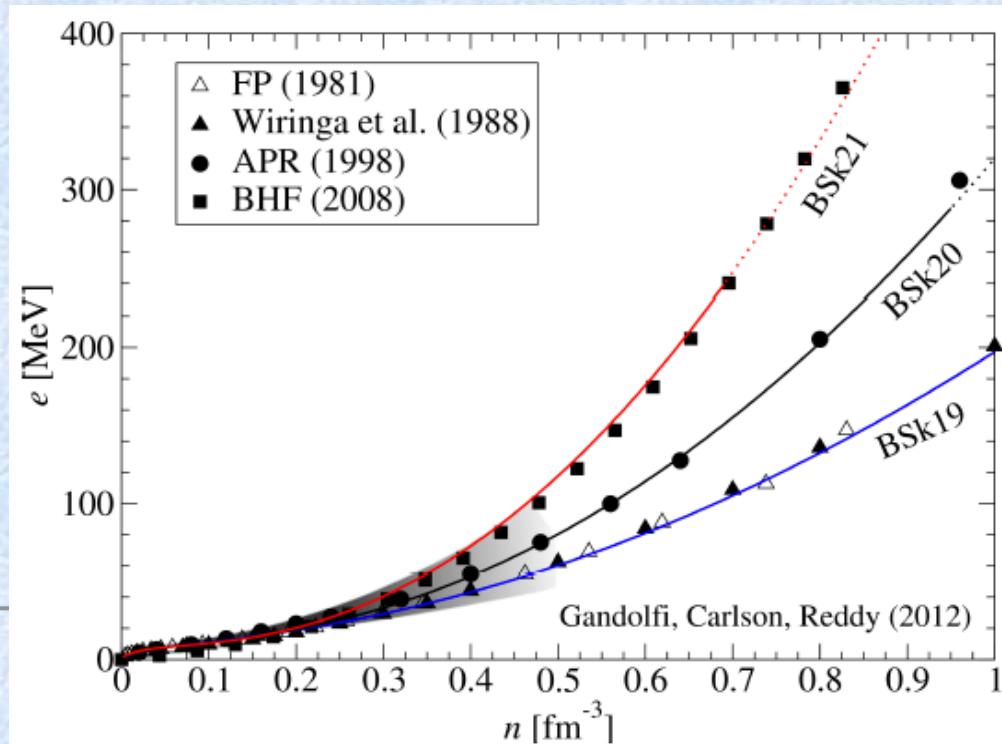
**BSk19**  
**BSk20**  
**BSk21**

- fit **2010 AME data** (2149 masses, rms = 0.581 MeV)
- **different degrees of stiffness** (BSk19 softer → BSk21 stiffer)  
constrained to different microscopic neutron-matter EoSs at T = 0
- all have  $J = 30$  MeV, ,  $K_\infty$  in experimental range ( $\approx 240$  MeV)

Goriely *et al.*, PRC 82, 035804 (2010)

see also: Chamel *et al.*, PRC 80, 065804 (2009)

A. F. Fantina





# Brussels-Montreal (BSk) functionals

Mass models based on HFB mean-field functionals (EDFs) and macroscopic-microscopic Fitted to experimental data + N-body

**BSk19**  
**BSk20**  
**BSk21**

- fit 2010 AME data (2149 masses)
- different degrees of stiffness constrained to different microscopic EoSs
- all have  $J = 30 \text{ MeV}$ ,  $K_\infty$  in experimental range

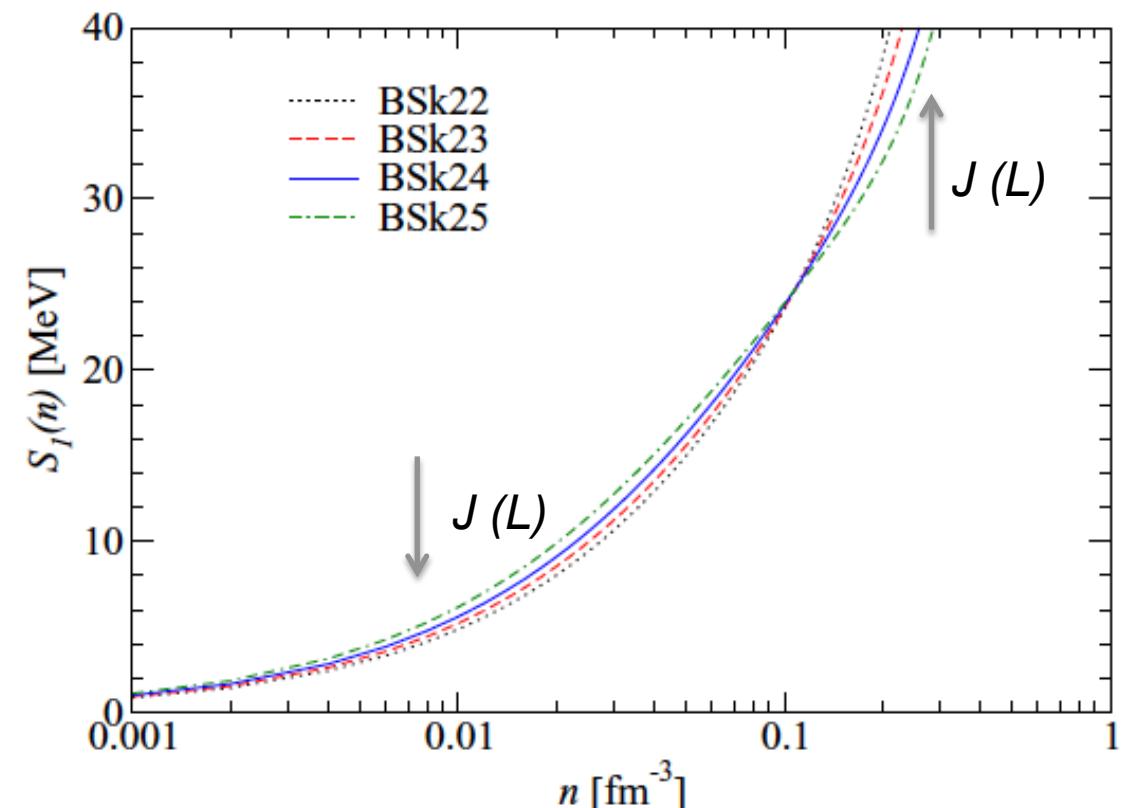
Goriely *et al.*, PRC 82, 035804 (2010)

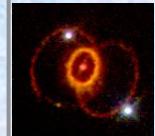
**BSk22**  
**BSk23**  
**BSk24**  
**BSk25**  
**BSk26**

- fit 2012 AME data (2353 masses, rms=0.5-0.6 MeV)
- constrained to microscopic neutron-matter EoSs at T = 0 (rather stiff)
- different  $E_{\text{sym}}$  coefficient ( $J = 32, 31, 30, 29, 30 \text{ MeV}$ ),  $K_\infty$  in experimental range ( $\approx 240 \text{ MeV}$ )

Goriely *et al.*, PRC 88, 024308 (2013)

see also: Chamel *et al.*, PRC 80, 065804 (2009)



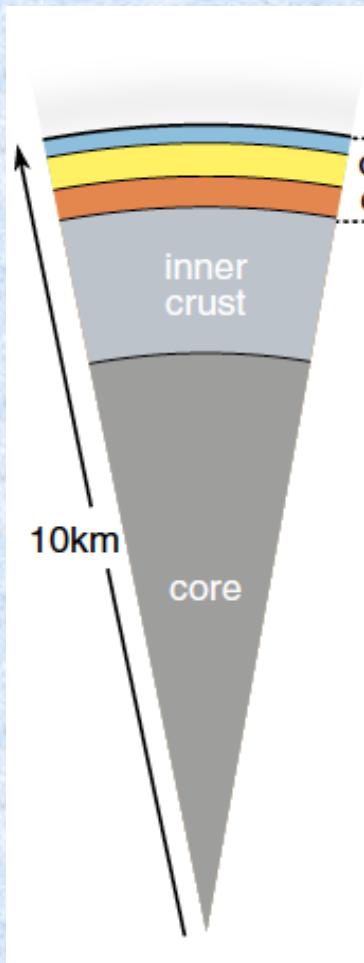


# Brussels-Montreal (BSk) functionals

- ✓ BSk\*\* suitable to describe all the regions of NS



- ✓ BSk\*\* also used to compute properties of infinite homogeneous nuclear matter

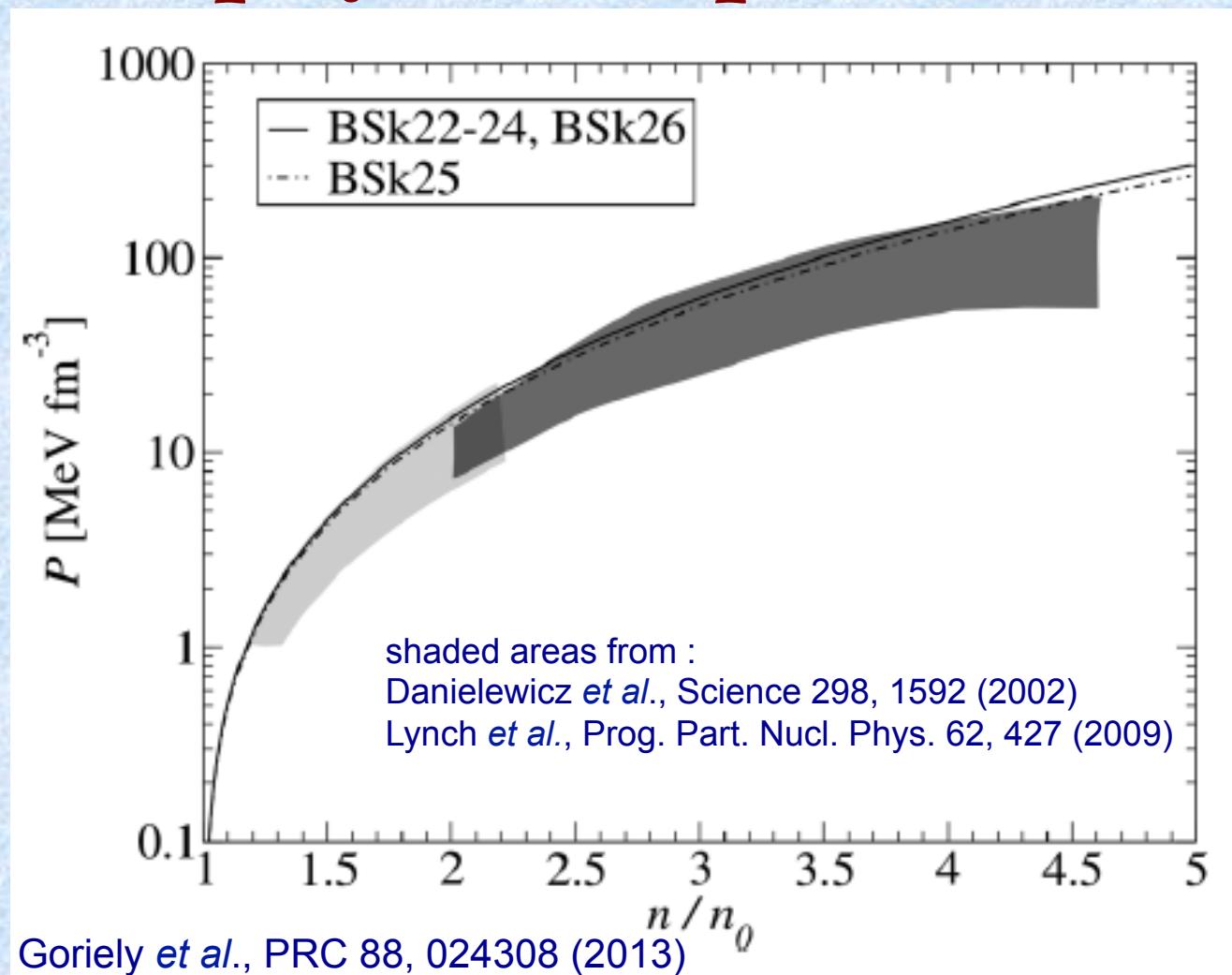


we construct ***unified EoSs*** with these functionals  
(until now, few unified EoSs! e.g. SLy, BCPM)

- *same nuclear model* to treat different NS regions
- avoid ad-hoc matching procedures at boundaries  
(see e.g. Fortin *et al.*, arxiv:1604.01944v1)
- here: case of “ $T = 0$ ”



# Comparison with observables from nuclear physics experiments

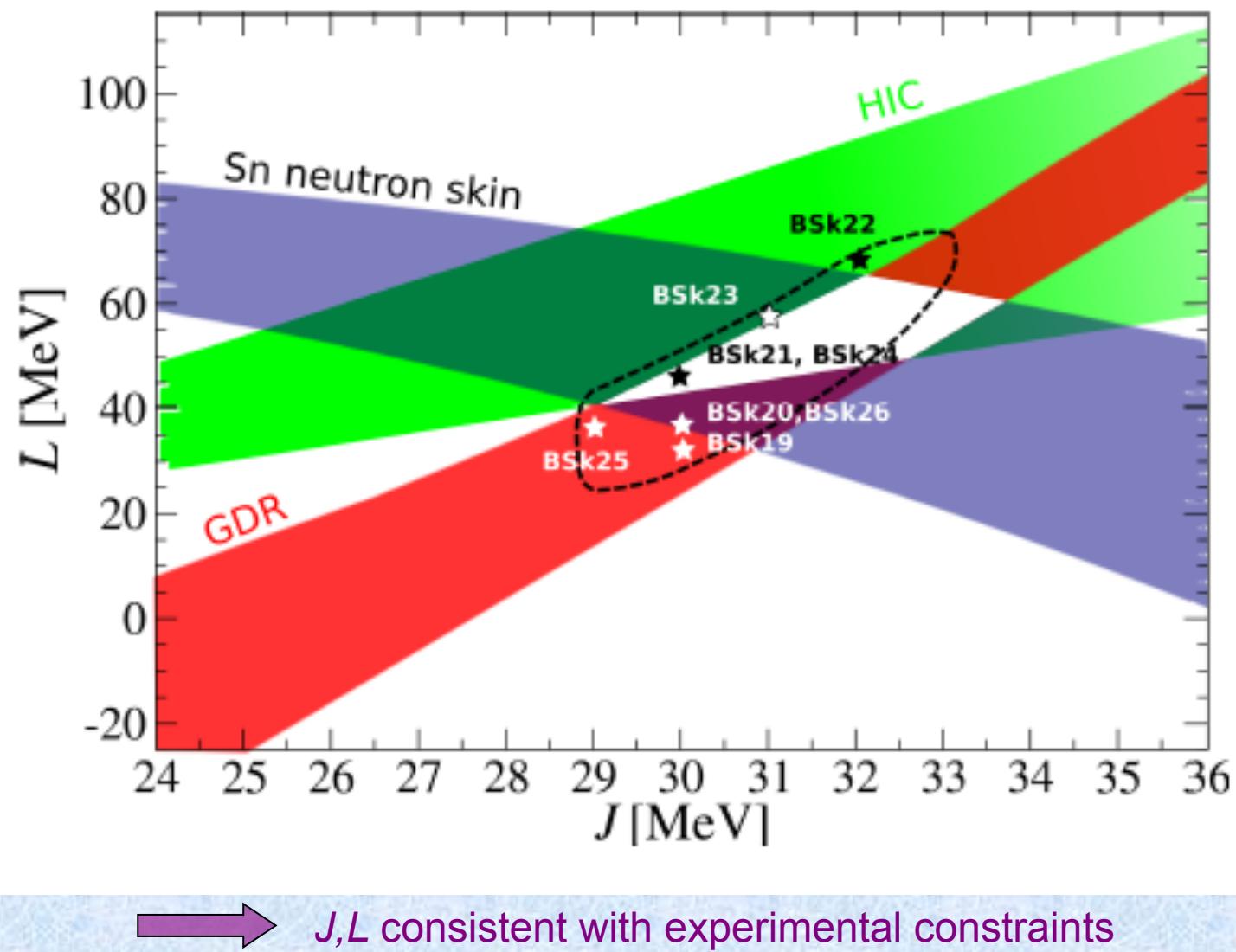


Functionals in good agreement with “experimental” constraints on symmetric matter

N.B.: deduced constraints are not direct experimental data, are model dependent!



# Comparison with observables from nuclear physics experiments

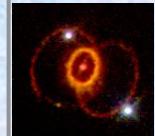


Potekhin, Fantina, Chamel *et al.*, A&A 560, A48 (2013)

Fantina *et al.*, AIP Conf. 1645, 92 (2015)

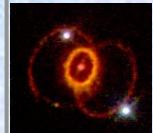
Fantina *et al.*, PRC 93, 015801 (2016)

Tsang et al., PRC 86, 015803 (2012);  
Lattimer and Lim , ApJ 771, 51 (2013);  
Lattimer & Steiner, EPJA 50, 40 (2014)



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# NS EoS: outer crust (nonaccreting)

Ground state of matter below neutron drip (isolated NSs):

- cold catalysed matter hypothesis  $\rightarrow T = 0$ , matter in full thermodynamical equilibrium
- charge neutrality +  $\beta$  equilibrium
- structure made by perfect crystal with ONE nuclear species at lattice sites (e.g. bcc) ( $A, Z$ ), + electrons (no free neutrons!).

→ **BPS model** (Baym, Pethick, Sutherland, ApJ 170, 299 (1971))

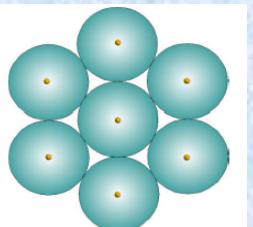
→ minimise the Gibbs energy per nucleon:  $g = \frac{\mathcal{E} + P}{n}$  at constant  $P$

where:  $n$  is the baryon number density,

$P$  is the pressure:  $P = P_e + P_L$  (electrons + lattice)

$\mathcal{E}$  is the energy density:

$$\mathcal{E} = n_N M(Z, A)c^2 + \mathcal{E}_e + \mathcal{E}_L$$

  
density of nuclei      mass of nucleus (exp or theory)      e<sup>-</sup> energy      lattice energy (electrostatic contribution – Wigner-Seitz)

$n_N = n/A$

For ref. other calculations of the EoS have been made, e.g.:

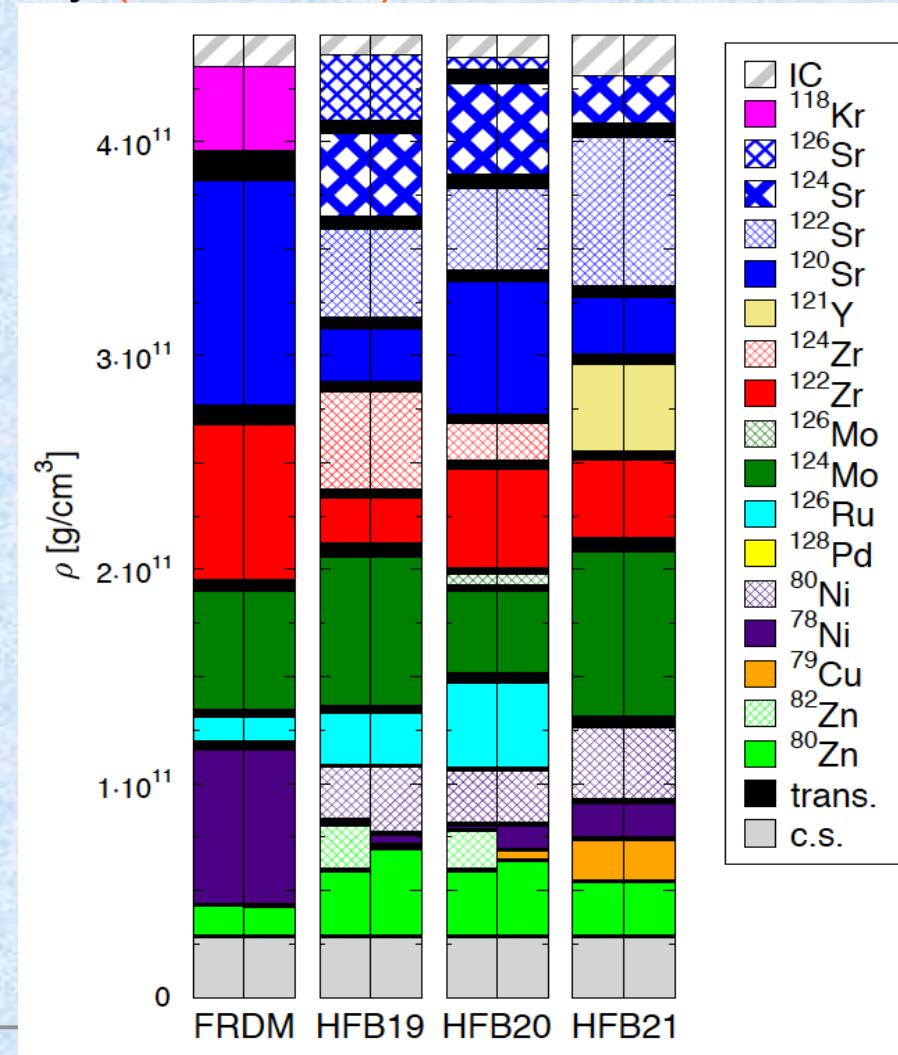
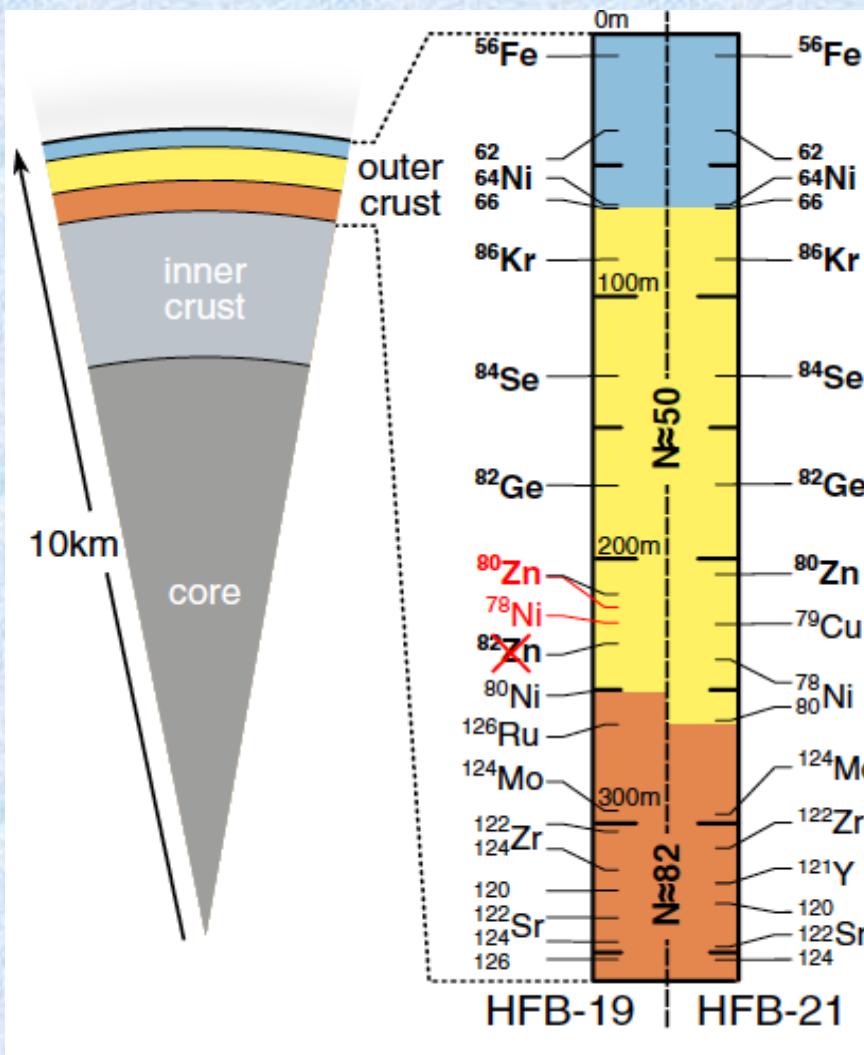
A. F. Fantina    Harrison & Wheeler (1958), Salpeter, ApJ 134, 669 (1961)

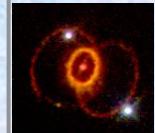


# NS EoS: outer crust

$$\mathcal{E} = n_N M(Z, A) c^2 + \mathcal{E}_e + \mathcal{E}_L$$

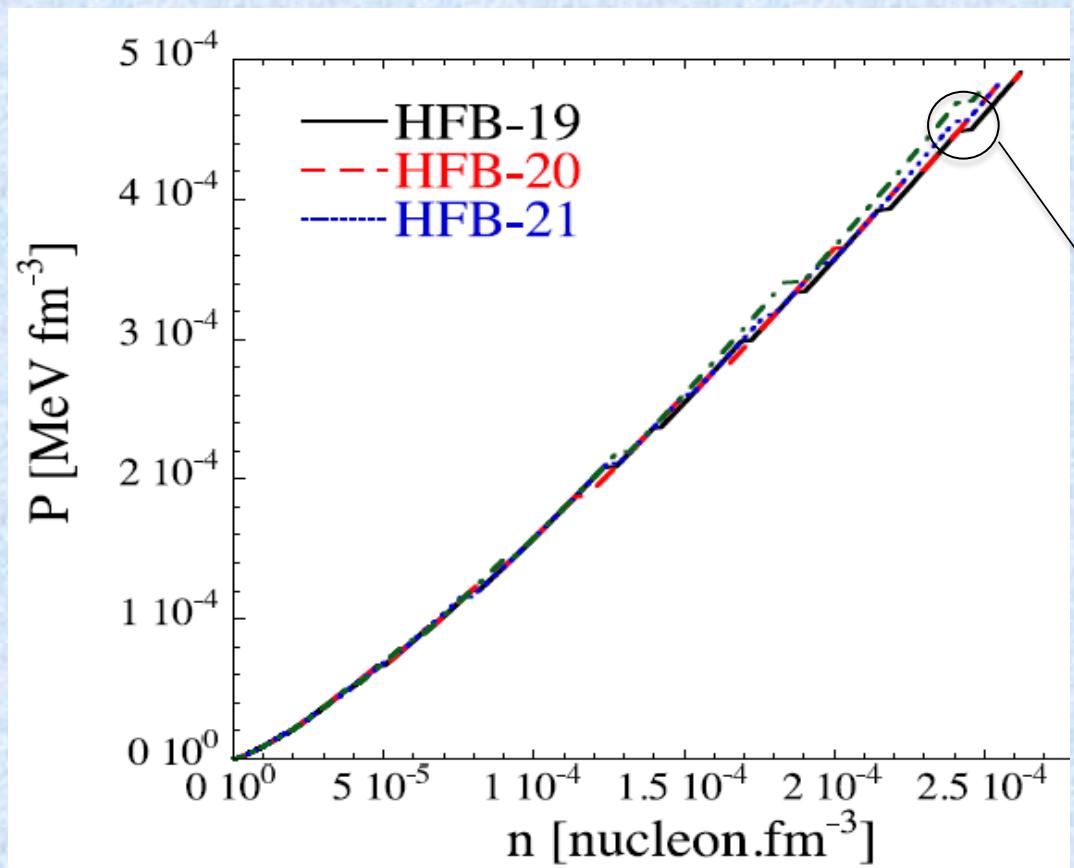
→ outer crust structure determined by (measured) masses of n-rich nuclei





# NS EoS: outer crust

Mass models: HFB (no approximations!)



J. M. Pearson *et al.*, PRC 83, 065810 (2011)

HFB-19	HFB-20	HFB-21
$^{56}\text{Fe}$	$^{56}\text{Fe}$	$^{56}\text{Fe}$
$^{62}\text{Ni}$	$^{62}\text{Ni}$	$^{62}\text{Ni}$
$^{64}\text{Ni}$	$^{64}\text{Ni}$	$^{64}\text{Ni}$
$^{66}\text{Ni}$	$^{66}\text{Ni}$	$^{66}\text{Ni}$
$^{86}\text{Kr}$	$^{86}\text{Kr}$	$^{86}\text{Kr}$
$^{84}\text{Se}$	$^{84}\text{Se}$	$^{84}\text{Se}$
$^{82}\text{Ge}$	$^{82}\text{Ge}$	$^{82}\text{Ge}$
$^{80}\text{Zn}$	$^{80}\text{Zn}$	$^{80}\text{Zn}$
$^{82}\text{Zn}$	$^{82}\text{Zn}$	-
-	-	$^{79}\text{Cu}$
	$^{78}\text{Ni}$	$^{78}\text{Ni}$
$^{80}\text{Ni}$	$^{80}\text{Ni}$	$^{80}\text{Ni}$
$^{126}\text{Ru}$	$^{126}\text{Ru}$	-
$^{124}\text{Mo}$	$^{124}\text{Mo}$	$^{124}\text{Mo}$
-	$^{122}\text{Mo}$	-
$^{122}\text{Zr}$	$^{122}\text{Zr}$	$^{122}\text{Zr}$
$^{124}\text{Zr}$	$^{124}\text{Zr}$	-
-	-	$^{121}\text{Y}$
$^{120}\text{Sr}$	$^{120}\text{Sr}$	$^{120}\text{Sr}$
$^{122}\text{Sr}$	$^{122}\text{Sr}$	$^{122}\text{Sr}$
$^{124}\text{Sr}$	$^{124}\text{Sr}$	$^{124}\text{Sr}$
$^{126}\text{Sr}$	$^{126}\text{Sr}$	-

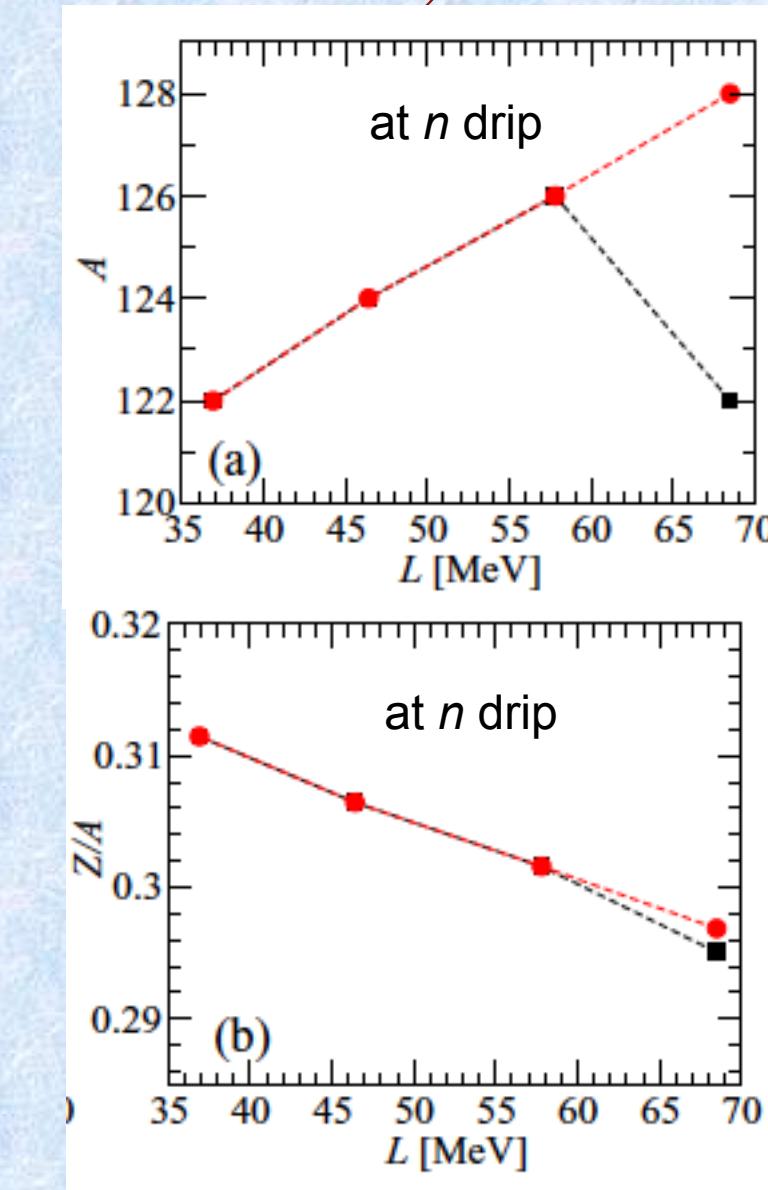
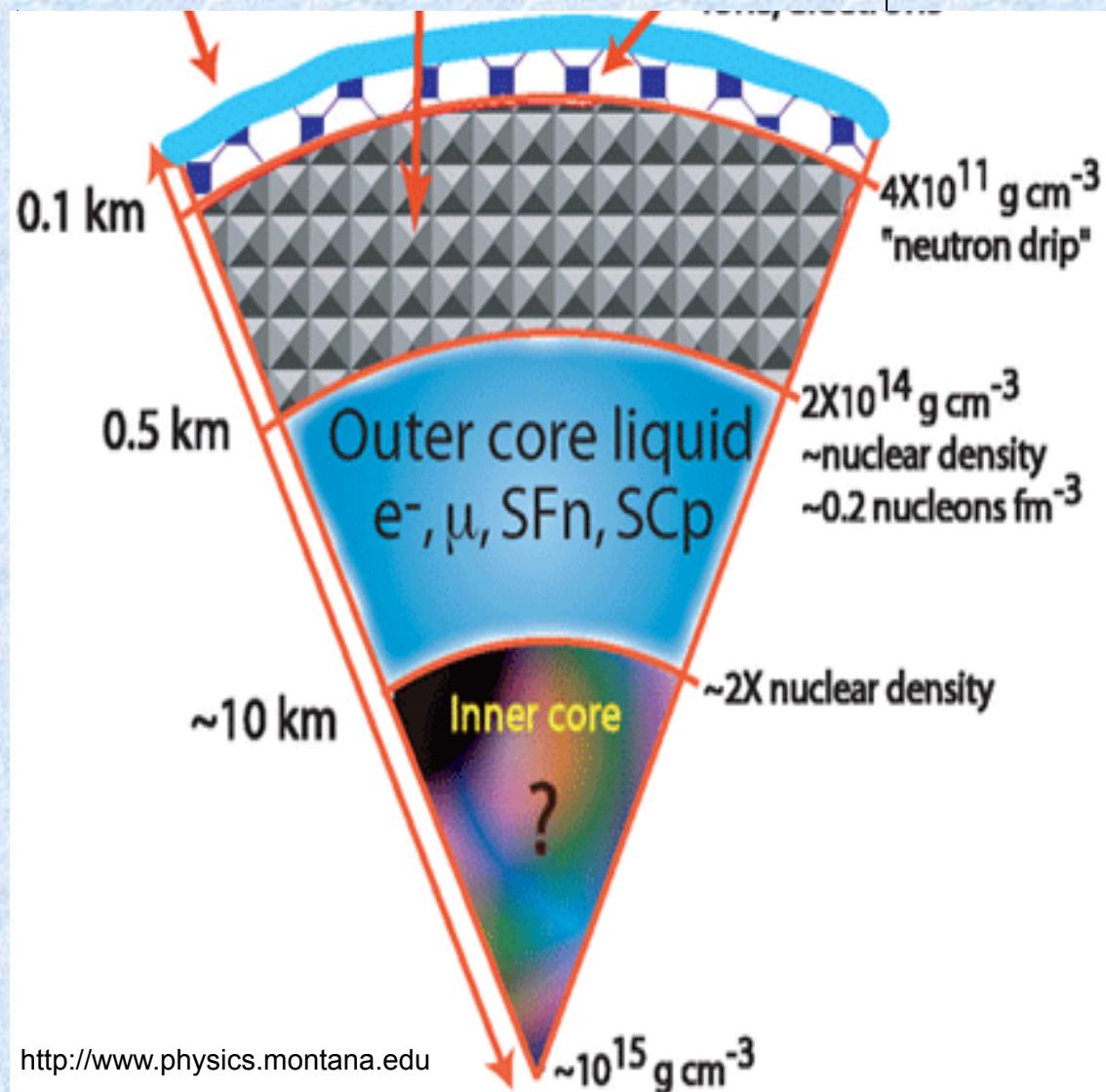
N.B.: Pressure continuous function of star radius

→ density jump if transition from one stable nuclide to the other

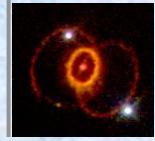


# NS EoS: outer crust and $E_{sym}$

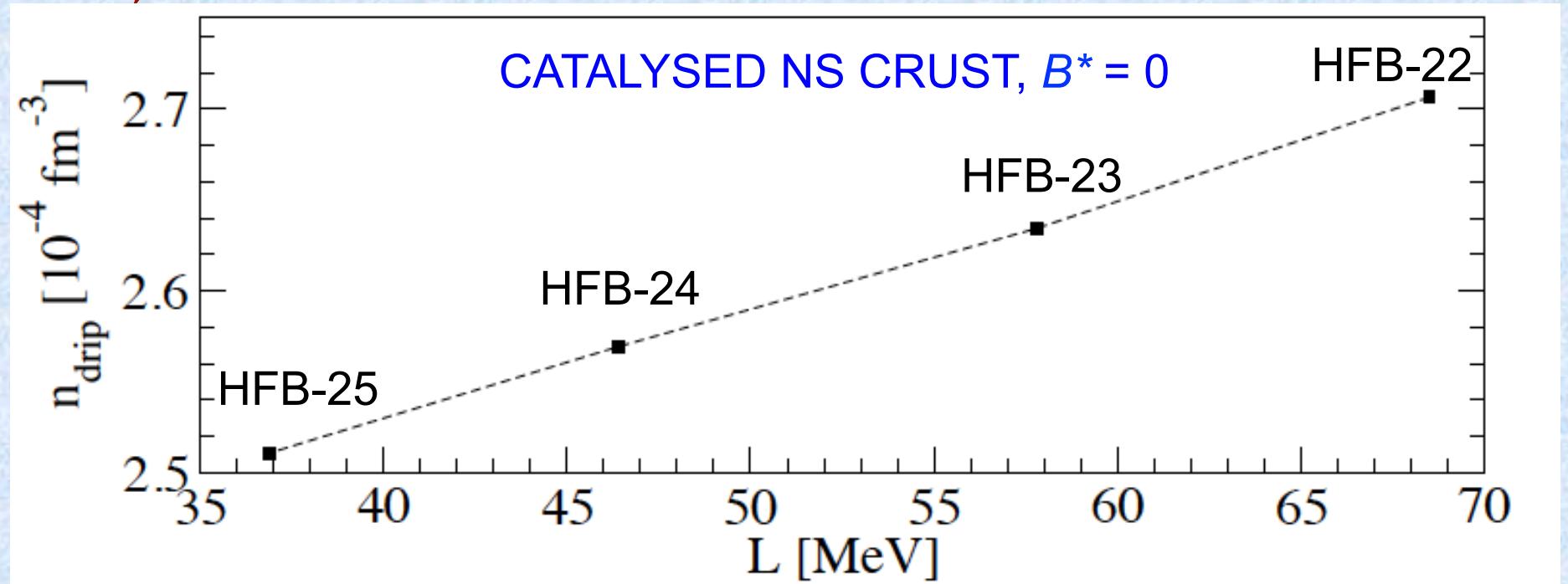
HFB-22   HFB-23   HFB-24   HFB-25



$$\frac{Z}{A} \approx \frac{1}{2} \sqrt{1 + \frac{a_{\text{eff}}}{J_{\text{eff}}}}$$



# $E_{sym}$ and inner-outer crust boundary

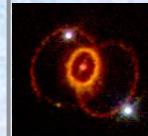


$$n_{\text{drip}}(A, Z) \approx \frac{A}{Z} \frac{\mu_e^{\text{drip}}(A, Z)^3}{3\pi^2(\hbar c)^3} \left[ 1 + \frac{4C\alpha}{(81\pi^2)^{1/3}} Z^{2/3} \right]^{-3}$$

$$\mu_e^{\text{drip}}(A, Z) \equiv \frac{-M'(A, Z)c^2 + Am_n c^2}{Z} + m_e c^2.$$

Fantina *et al.*, PRC 93, 015801 (2016)

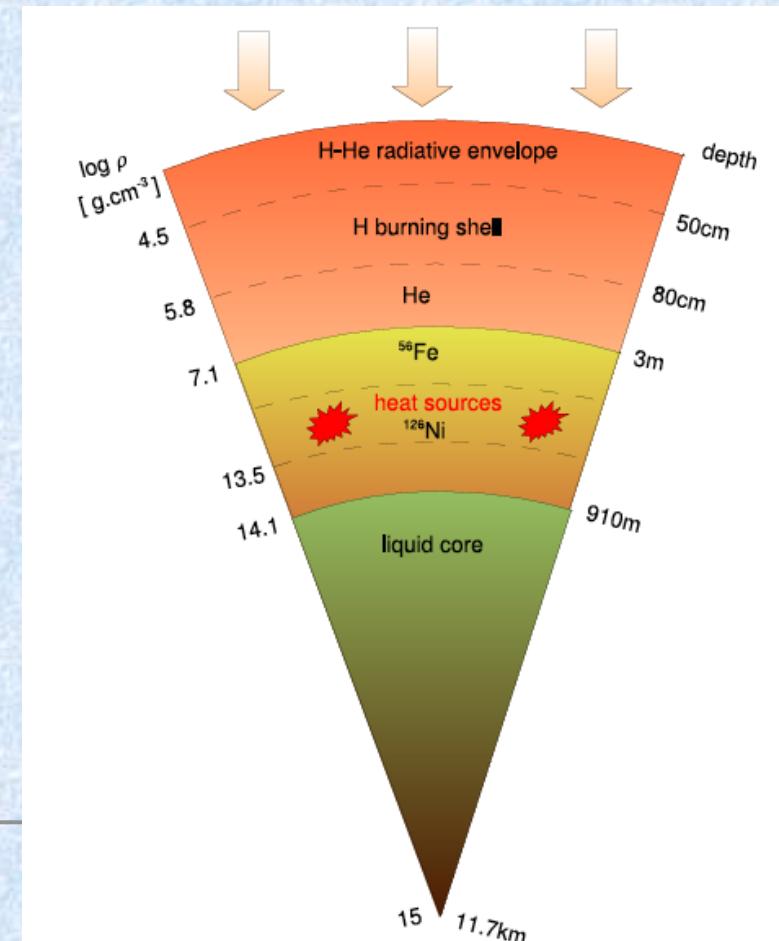
Chamel, Fantina, Zdunik, Haensel, PRC 91, 055803 (2015)

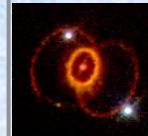


# NS EoS: outer crust for accreting NS

For **catalysed** NS crust → initial composition around Fe →  $A \approx 50-60$   
(determined by full minimisation of  $g$ )

For **accreting** NS crust → initial composition depend on ashes  
produced by *rp*-process during x-ray bursts →  $A$  from  $\approx 60$  to  $\approx 106$   
and in steady state H and He burning during superbursts →  $A \approx 60$

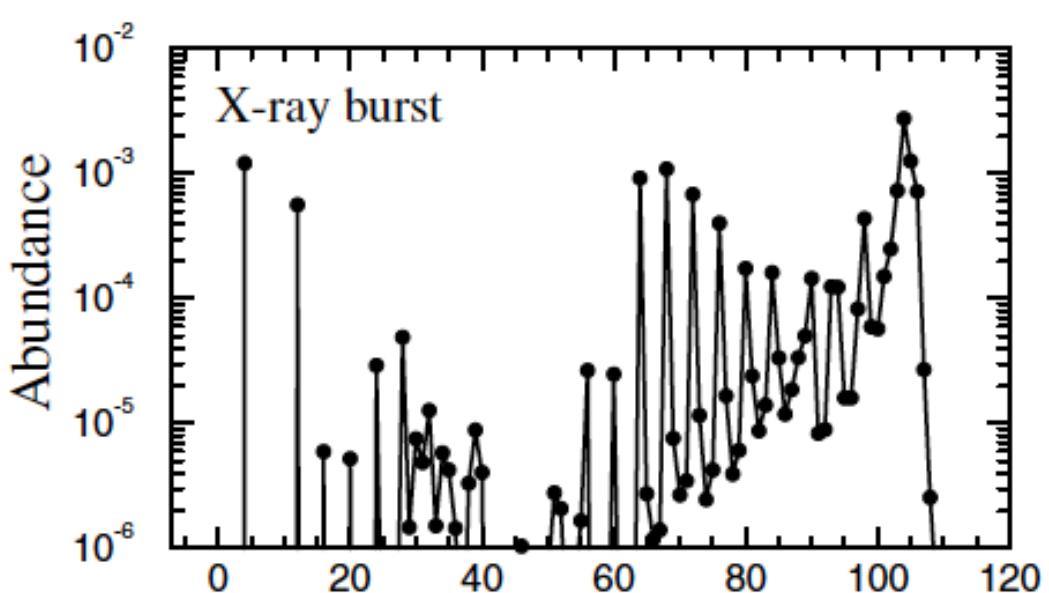




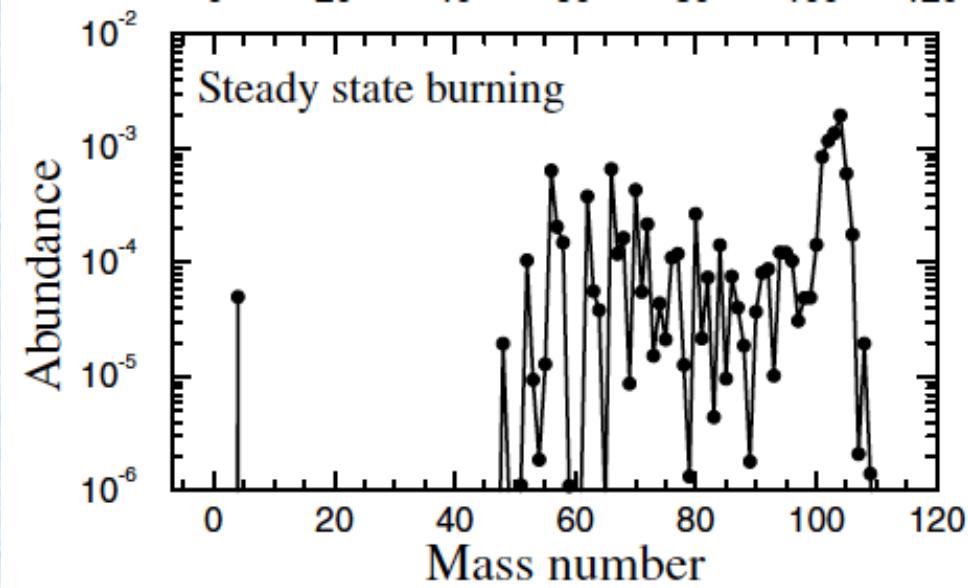
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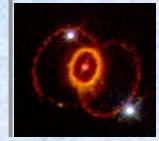
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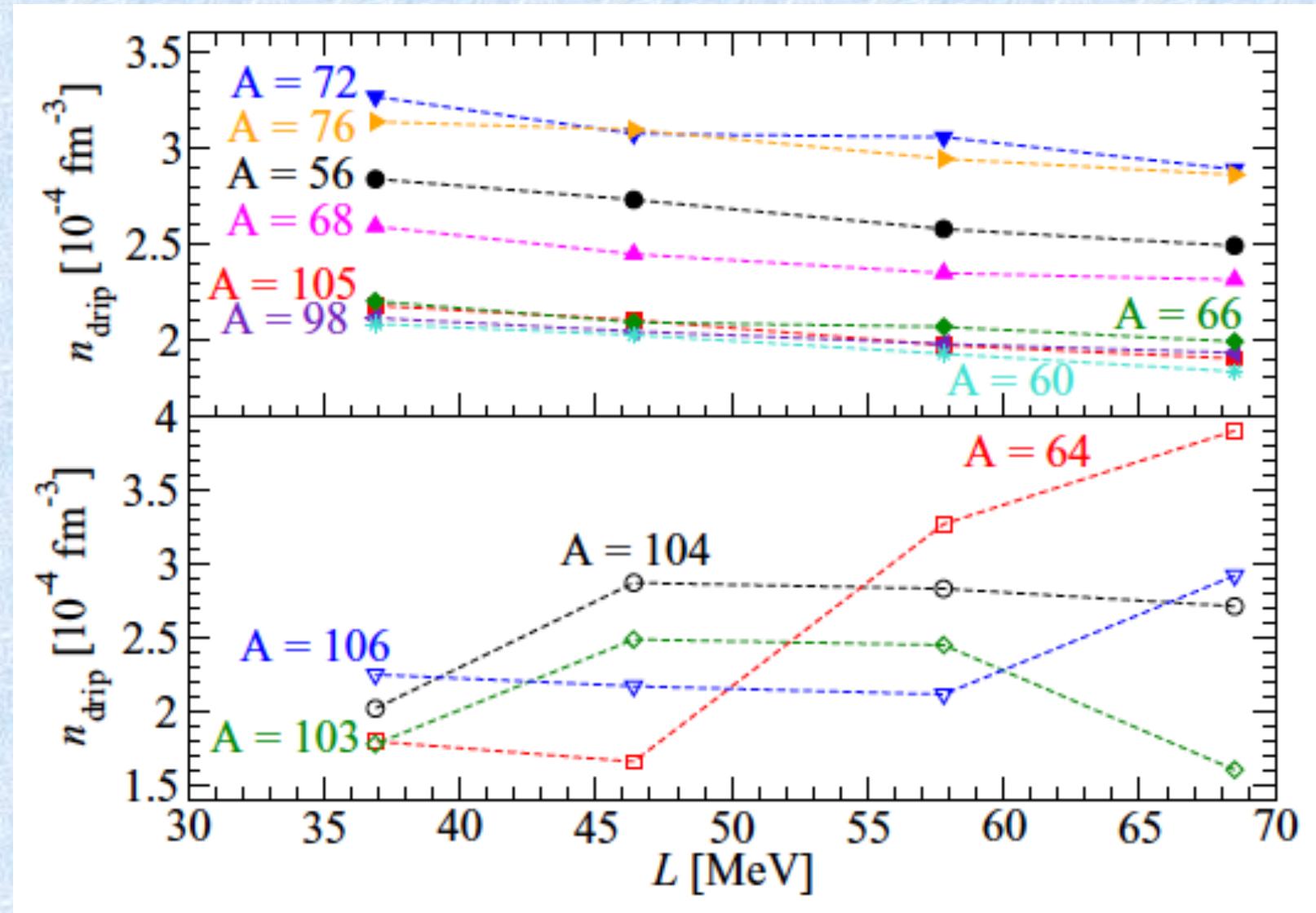
Schatz et al., PRL 86, 3471 (2001)





# $E_{sym}$ and inner-outer crust boundary

ACCRETED NS CRUST



Fantina et al., PRC 93, 015801 (2016)

Chamel, Fantina, Zdunik, Haensel, PRC 91, 055803 (2015)

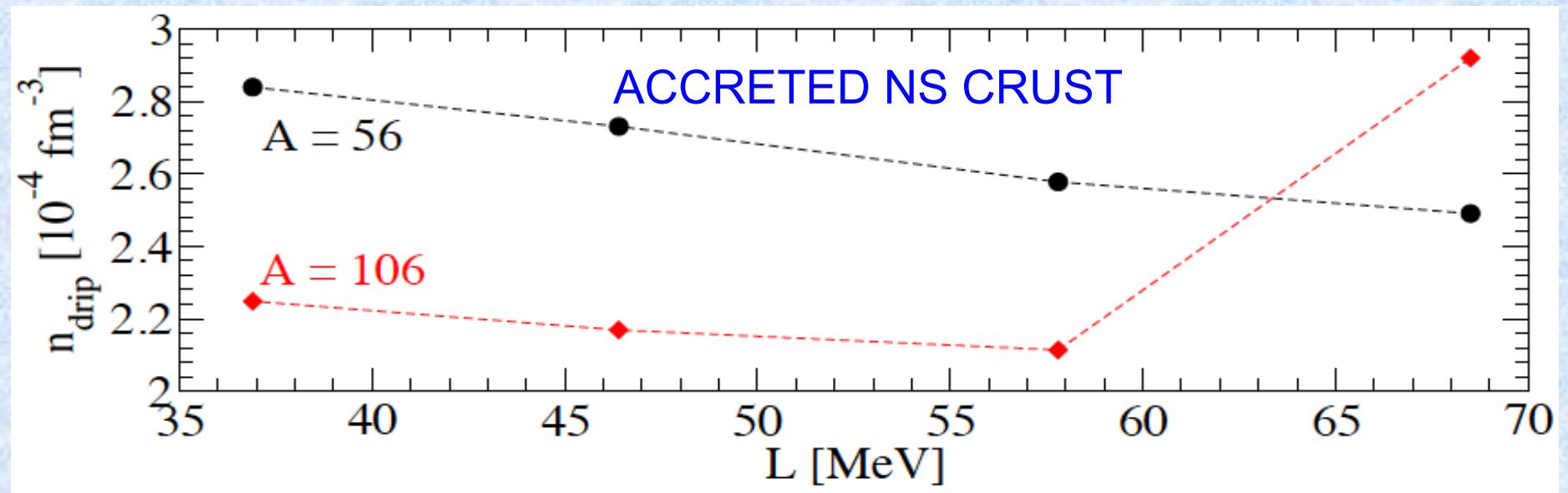


# $E_{sym}$ and inner-outer crust boundary

$$n_{\text{drip-acc}}(A, Z) \approx \frac{A}{Z} \frac{\mu_e^{\text{drip-acc}}(A, Z)^3}{3\pi^2(\hbar c)^3} \left[ 1 + \frac{C\alpha}{(3\pi^2)^{1/3}} \left( Z^{5/3} - (Z-1)^{5/3} + \frac{Z^{2/3}}{3} \right) \right]^{-3}$$

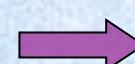
$$\mu_e^{\text{drip-acc}} = S_{\Delta N n}(A, Z-1) + M'(A, Z-1)c^2 - M'(A, Z)c^2 + m_e c^2.$$

$$S_{\Delta N n}(A, Z-1) = M(A - \Delta N, Z-1)c^2 - M(A, Z-1)c^2 + \Delta N m_n c^2 \quad (\ll \Delta M' c^2)$$



Fantina, et al., PRC 93, 015801 (2016)

Chamel, Fantina, Zdunik, Haensel, PRC 91, 055803 (2015)

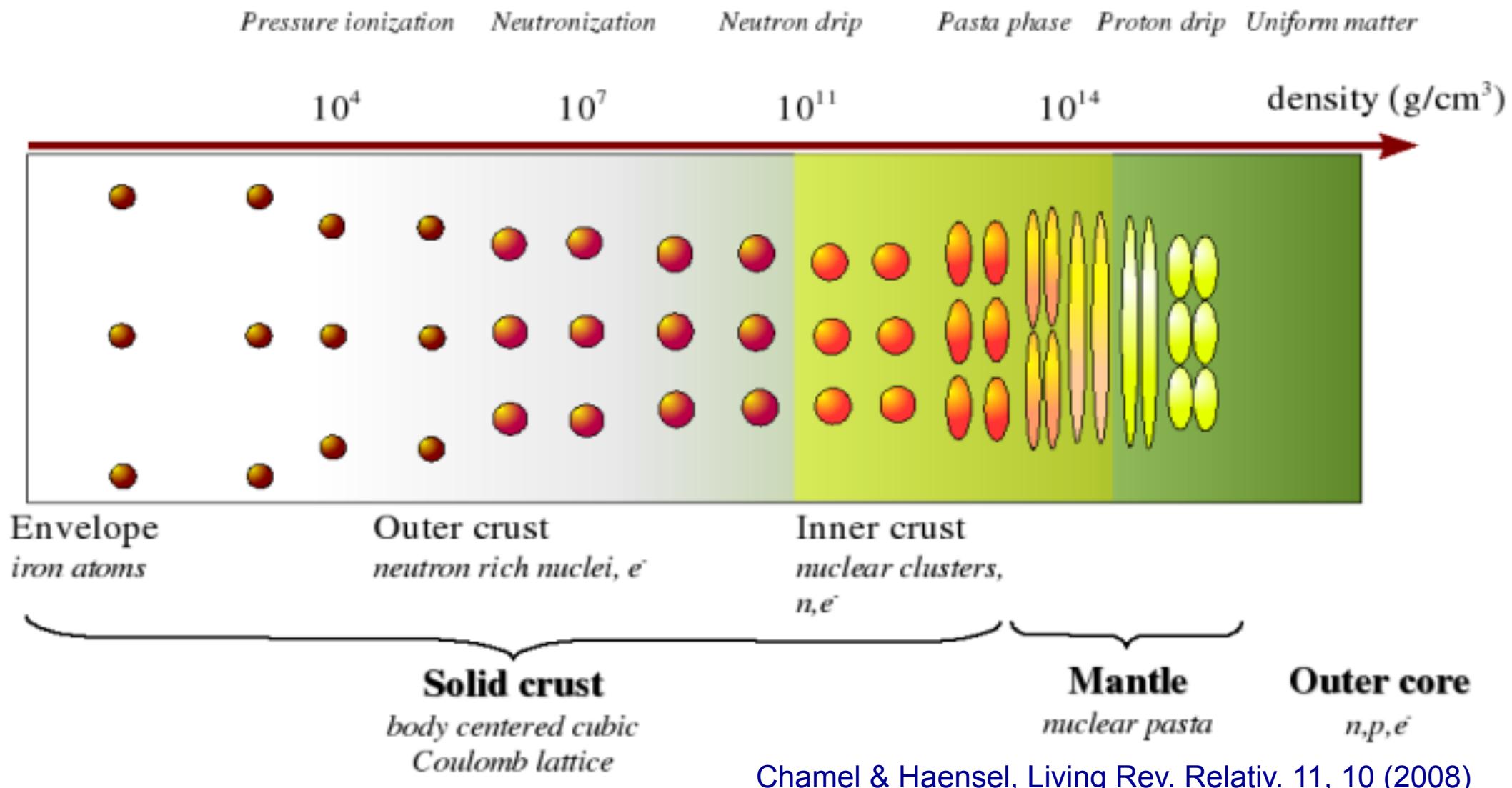


sensitivity also to the details of nuclear structure far from stability!

→ multiple neutron separation energy & isobaric mass difference



# NS crust structure



Neutron star crust :  $\approx 1\%$  mass,  $\approx 10\%$  radius



# NS EoS: inner crust

Beyond n drip ( $\rho \approx 4 \cdot 10^{11} \text{ g cm}^{-3}$ )  $\rightarrow$  n-rich nuclei + e<sup>-</sup> + free n!

i.e. when:  $\mu_n = g(T=0) = \frac{M(A, Z) + Z\mu_e + Z\mu_L}{A} \geq m_nc^2$

→  $E(Z, A)$  has to be extrapolated  $\rightarrow$  which approach to use?



## 1. Compressible liquid-drop model (e.g. Baym, Bethe & Pethick, Nucl. Phys. A 175 225 (1971);

Douchin & Haensel Astron. Astrophys. 380, 151 (2001) )

- nuclei have sharp surface
- nuclear energy given by sum of contribution (bulk, surface, Coulomb)
- separation of nuclear matter “inside” and “outside” nuclei into two homogeneous phases

## 2. (Extended) Thomas-Fermi (e.g. Onsi *et al.*, PRC 77, 065805 (2008) and refs. therein)

- nuclei have smooth surface (smooth density profiles)
- nuclear energy as a functional of the density (and density gradients) of species
- consistent treatment nucleons “inside” and “outside” nuclei

## 3. Hartree-Fock / Hartree-Fock Bogoliubov (e.g. Negele & Vautherin, Nucl. Phys. A 207, 298 (1973); Grill *et al.*, PRC 84, 065801 (2011) )

- quantum calculations: independent particle/quasiparticle  $\rightarrow$  shell / pairing effects
- also 3D calculations! e.g. Magierski & Heenen, PRC 65, 045804 (2002); Newton & Stone, PRC 79, 055801 (2009)



# NS EoS: inner crust: ETFSI

- ❖ **ETF:** Extended (4th order in  $\hbar$ ) Thomas-Fermi (for the T=0 case!)

Onsi *et al.*, PRC 77,065805 (2008); PRC 55, 3139 (1997); PRC 50, 460 (1994) and Refs. therein

→ semi-classical model → fast approximation to HF equations!

- Energy density and energy per nucleon of a unit Wigner-Seitz cell is given by:

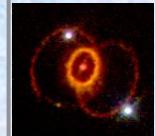
$$\mathcal{E} = \mathcal{E}_{nuc} + \mathcal{E}_e + \mathcal{E}_{Coul} + (\rho_n m_n + \rho_p m_p + n_e m_e) c^2$$
$$e = \frac{1}{A} \int_{\text{cell}} \mathcal{E}(\mathbf{r}) d^3 \mathbf{r} \quad e_{nuc} = \frac{1}{A} \int_{\text{cell}} \mathcal{E}_{\text{Sky}}^{\text{ETF}}(\mathbf{r}) d^3 \mathbf{r}$$

- spherical neutron-proton clusters + n liquid + uniform relativistic  $e^-$  gas in Wigner-Seitz cells, using parameterised density distributions:

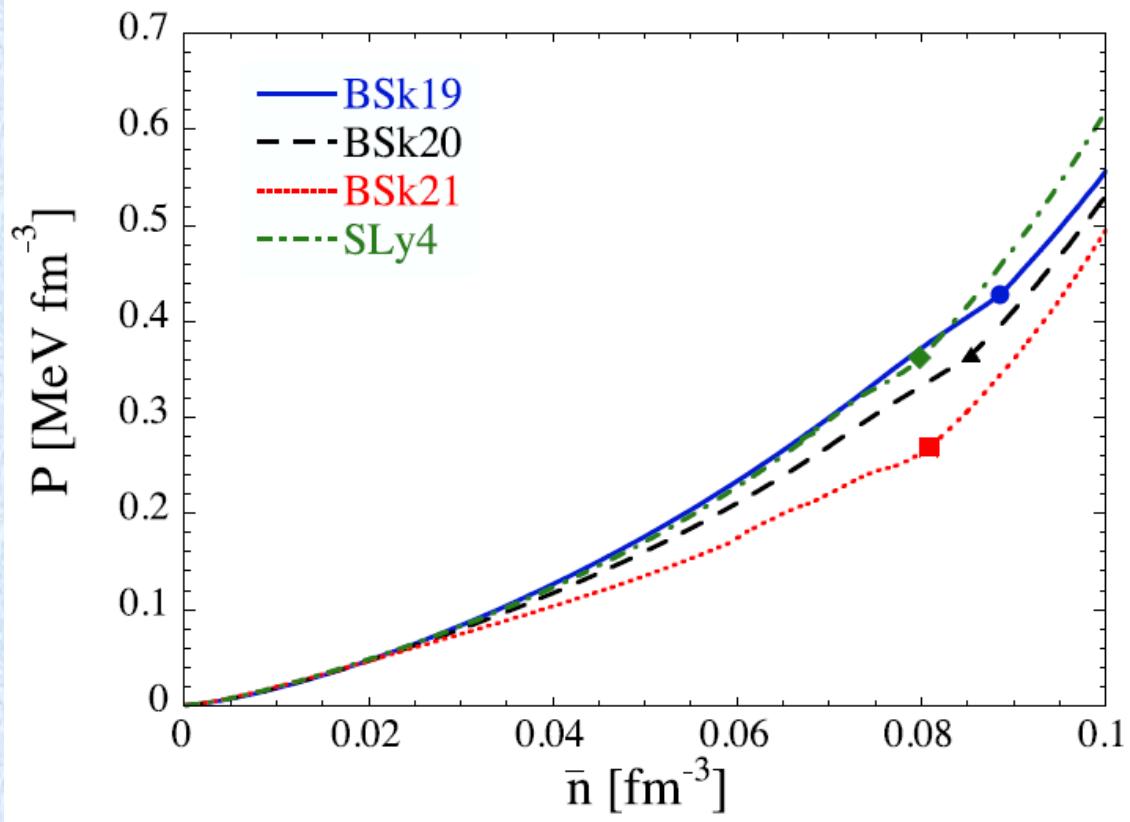
$$n_q(r) = n_{B,q} + n_{\Lambda,q} f_q(r)$$
$$f_q(r) = \frac{1}{1 + \exp \left[ \left( \frac{C_q - R_{\text{cell}}}{r - R_{\text{cell}}} \right)^2 - 1 \right] \exp \left( \frac{r - C_q}{a_q} \right)}$$

First applied by Buchler & Barkat (e.g. Buchler & Barkat, PRL 27, 48 (1971))

Many further applications, e.g. Ogasawara & Sato, Prog. Theor. Phys. 68, 222 (1982); Oyamatsu, NPA 561, 431 (1993); 29  
Centelles *et al.*, Nucl. Phys. A 537, 486 (1992); Cheng *et al.*, PRC 55, 2092 (1997): relativistic ETF



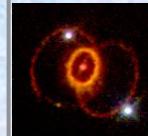
# NS EoS: inner crust / core with BSk models



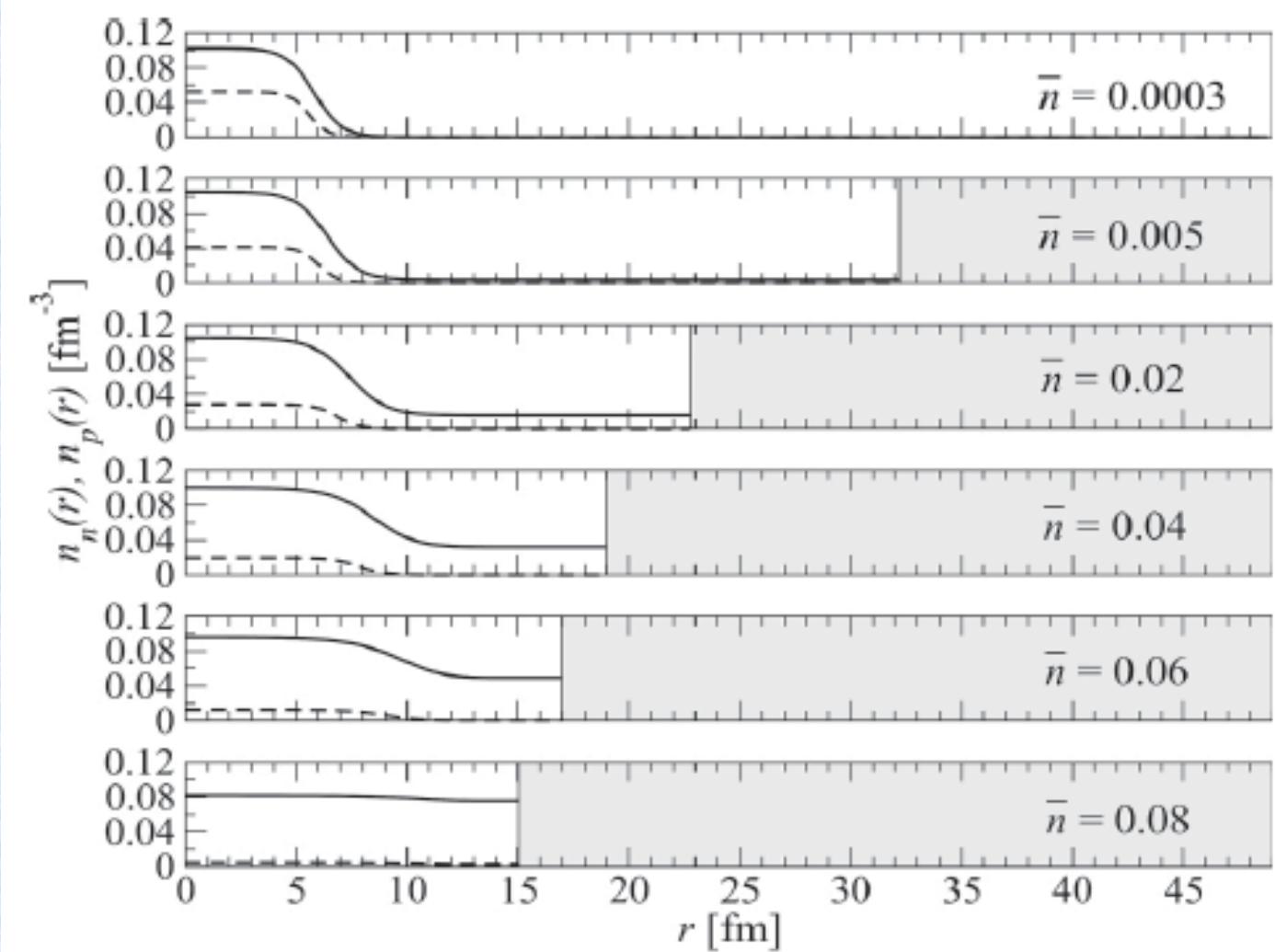
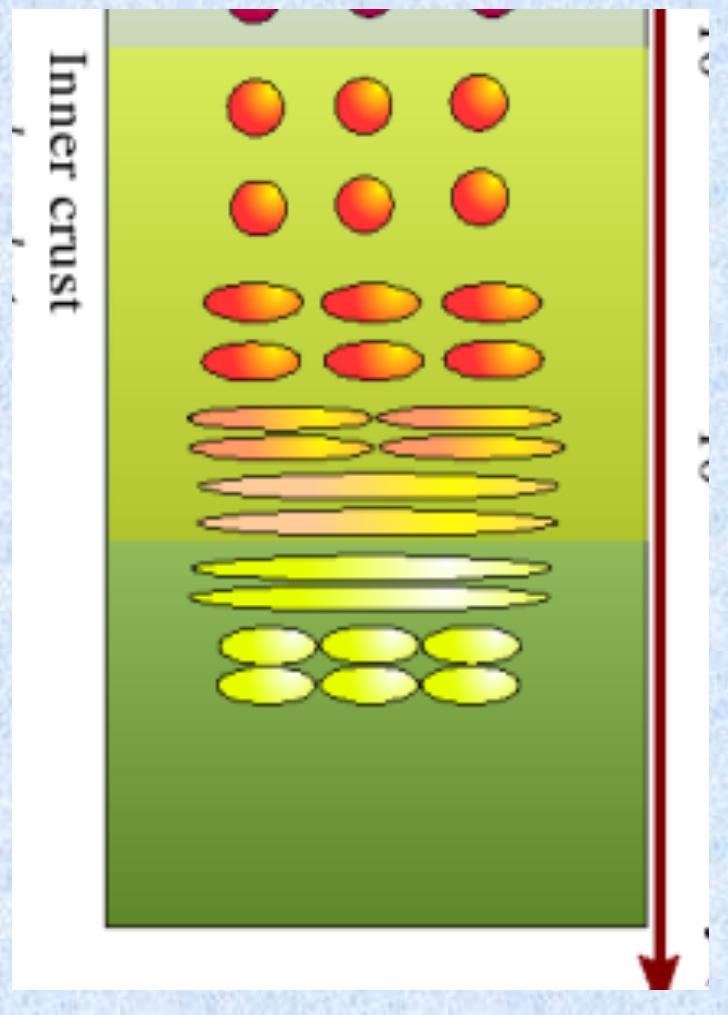
Force	$n_{\text{trans}}$ $\text{fm}^{-3}$	$P_{\text{trans}}$ $\text{MeV fm}^{-3}$
BSk19	0.0885	0.428
BSk20	0.0854	0.365
BSk21	0.0809	0.268
SLy4	0.0798	0.361

Pearson *et al.*, PRC 85, 065803 (2012)

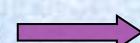
A. F. Fantina      Role of pairing with BSk19-20-21 discussed in Pearson *et al.*, PRC 91, 018801 (2015)  
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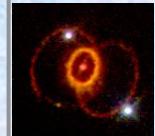


# NS EoS: inner crust / core with BSk models



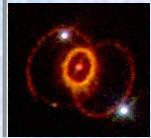
Pearson *et al.*, PRC85, 065803 (2012)





# Outline

- ❖ Introduction: NS & EoS
- ❖ Effective nuclear models
  - Brussels-Montreal BSk model
- ❖ NS composition & clusters
  - outer crust
  - inner crust
- ❖ Conclusions



# Conclusions

- ❖ Composition of NS crusts with unified EoSs for NS matter
  - same EDF nuclear model to describe all regions of NS
    - fitted on *experimental nuclear data* and *nuclear matter properties*
  - smooth transition from inhomogeneous matter (with cluster)  
to homogeneous nuclear matter
- ❖ Masses of nuclei in deeper region of NS crust experimentally unknown
  - need of (microscopic) theoretical mass models
- ❖ Composition in deeper layers of the NS outer crust very sensitive to details of nuclear structure far from valley of stability
  - *masses of very n-rich Sr and Kr isotopes* in *nonaccreting* NS
  - *combination of nuclear masses* (multiple n-separation energy and isobaric 2point mass difference) in *accreting* NS

*Merci!*