

Asymmetry of fission fragments

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Pre-scission points

Pre-scission configuration

Random neck rupture mechanism

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Results - ^{256}Fm

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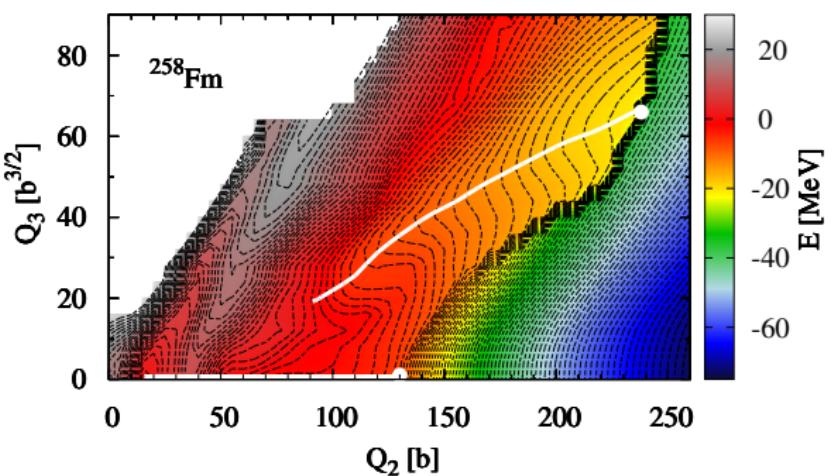
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- ▶ Fission fragment mass yield is a one of the basic, measurable observable
- ▶ The shape of observed fragment mass distribution allows to determine the type of fission (symmetric, asymmetric, bimodal)
- ▶ Accuracy of reproduction of the experimental mass yields as a test of the theoretical models

Potential energy surface



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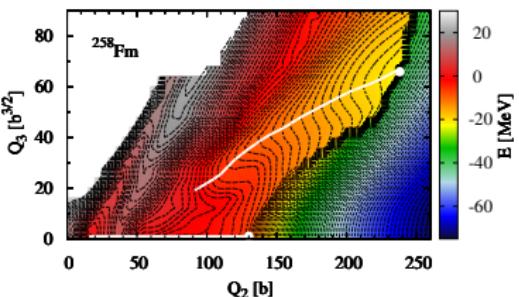
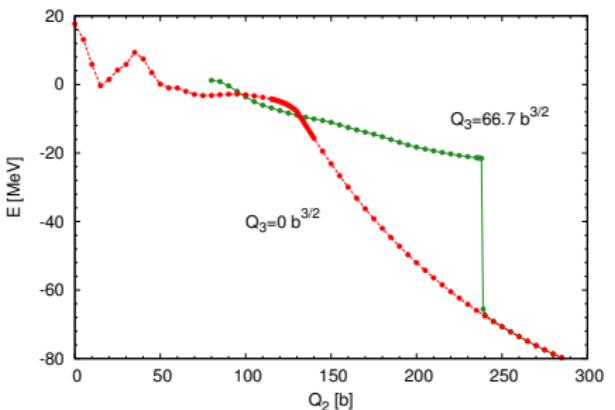
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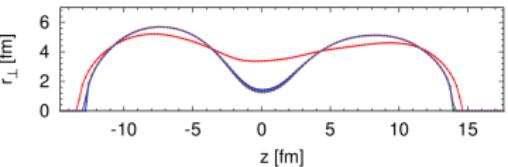
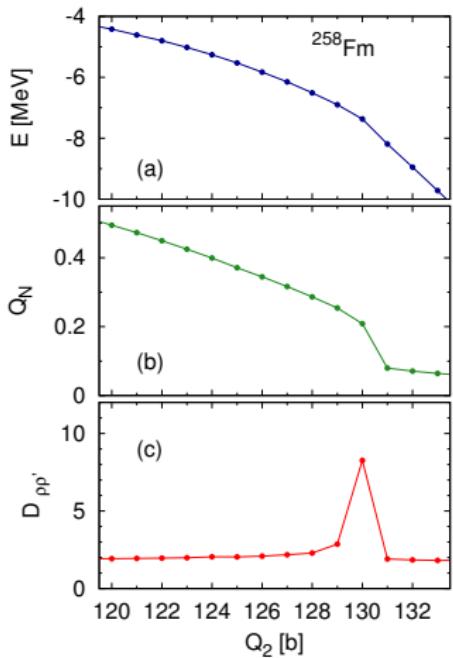
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Searching for pre-scission points



$$D_{\rho\rho'} = \int_0^{\infty} |\rho(\vec{r}) - \rho'(\vec{r})| d\tau$$

N. Dubray et al., Comp. Phys. Comm. **183** (2012) 2035.

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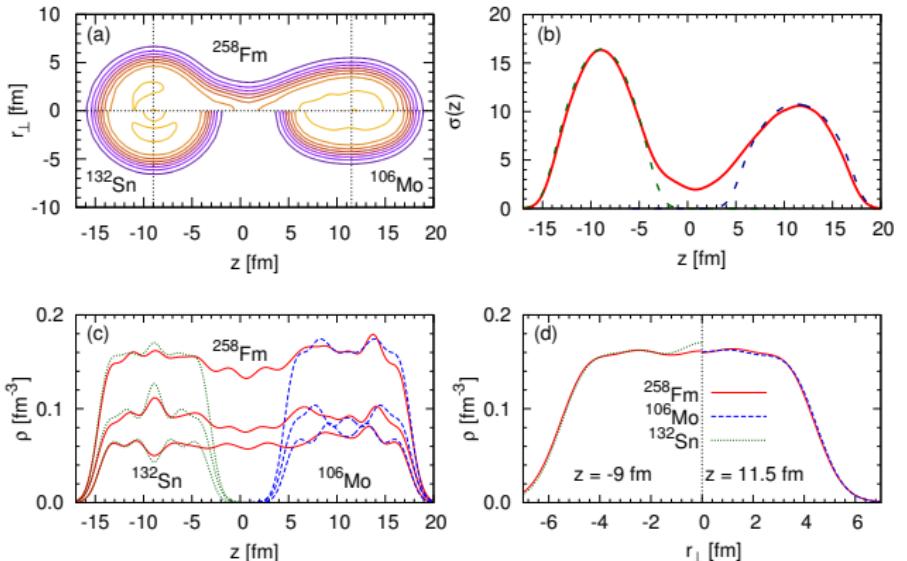
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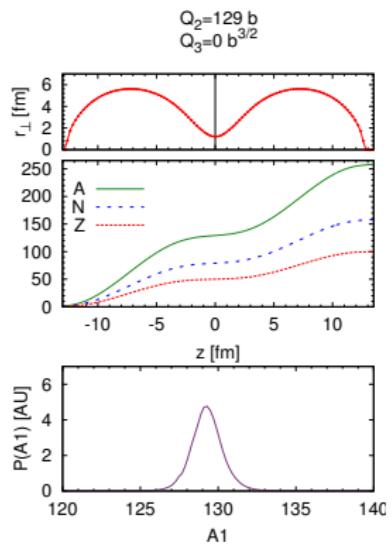


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The neck rupture probability



- ▶ Neck rupture probability:

$$P(z) = \exp[-2\gamma\sigma(z)/T]$$
- ▶ Linear density of a nucleus:

$$\sigma(z) = 2\pi \int_0^\infty r_\perp \rho(z, r_\perp) dr_\perp$$
- ▶ Surface tension coefficient:

$$\gamma = 0.9517[1 - 1.7826(1 - 2Z/A)^2]$$
- ▶ Temperature:

$$T = \sqrt{12E^{\text{sc}}/A}$$
- ▶ Excitation energy:

$$E^{\text{sc}} = E_{\text{g.s.}} - E_{\text{def}}^{\text{sc}}$$

Brosa U., Phys. Rev. C38 1944 (1988).

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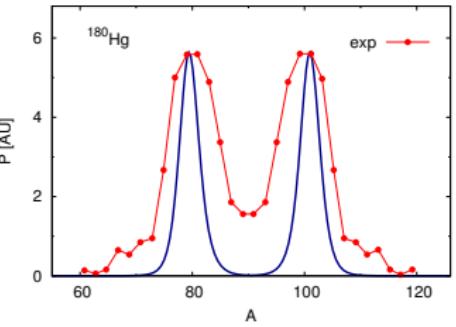
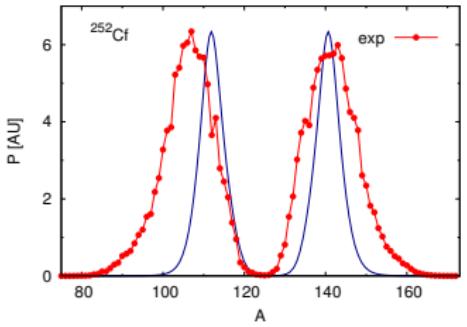
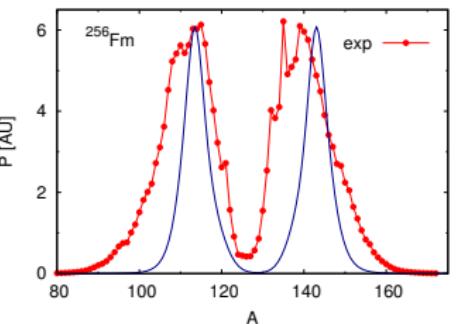
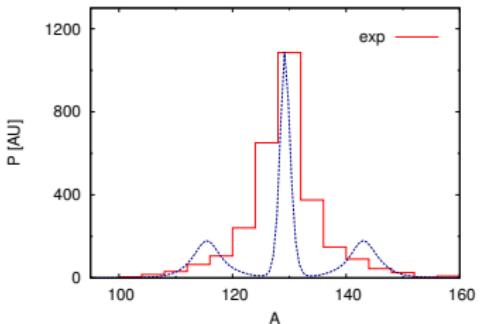
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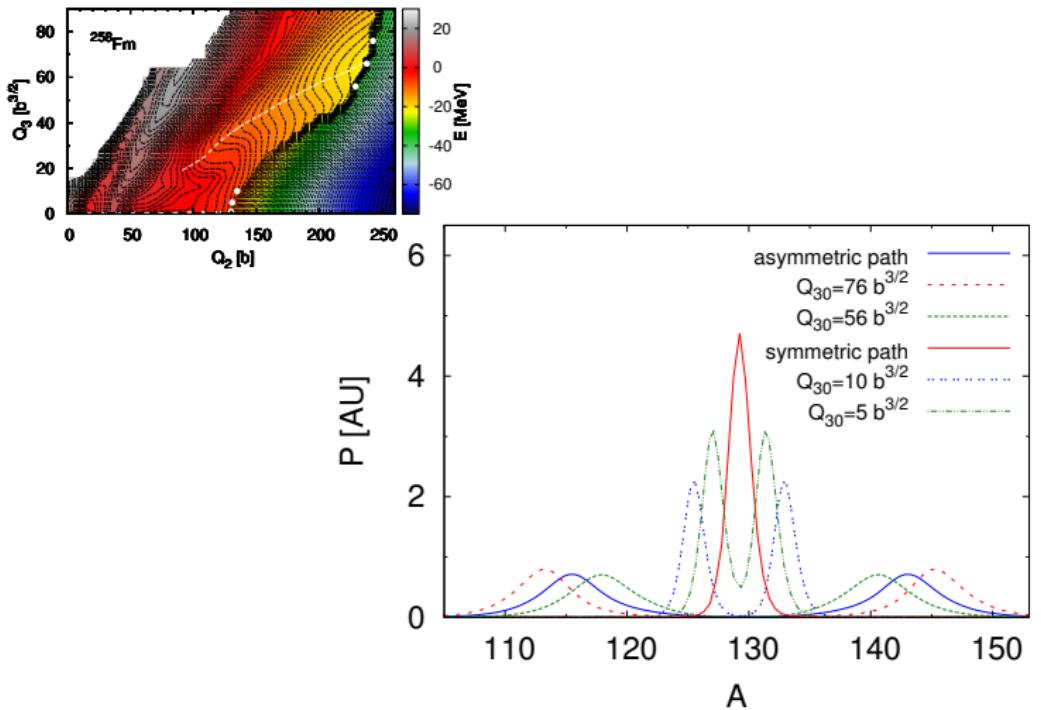
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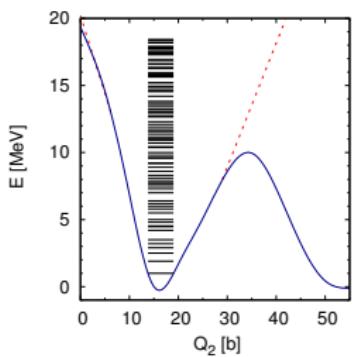


The model

$$\hat{H}_{\text{coll}} = -\frac{\hbar^2}{2} \sum_{i,j=2}^3 \frac{\partial}{\partial Q_i} B_{ij}(Q_2, Q_3) \frac{\partial}{\partial Q_j} + V(Q_2, Q_3),$$

$$B_{ij} = \mathcal{M}_{ij}^{-1}$$

$$\hat{H}'_{\text{coll}} g_n^\pi(Q_2, Q_3, t=0) = E_n^\pi g_n^\pi(Q_2, Q_3, t=0)$$



$$\hat{H}_{\text{coll}} g(Q_2, Q_3, t) = i\hbar \frac{\partial g(Q_2, Q_3, t)}{\partial t}$$

H. Goutte et. al, Phys. Rev. C71 024316 (2005).

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Probability current density

$$\frac{\partial |g|^2}{\partial t} = -\frac{\hbar}{2i} \nabla \cdot (\underbrace{g^* \underline{B} \nabla g - g \underline{B} \nabla g^*}_{\Downarrow})$$

$$\frac{\partial |g|^2}{\partial t} = -\operatorname{div} \vec{J}$$

$$J_{Q_2} = \frac{\hbar}{2i} [g^* B_{22} \frac{\partial g}{\partial Q_2} - g B_{22} \frac{\partial g^*}{\partial Q_2} + g^* B_{23} \frac{\partial g}{\partial Q_2} - g B_{23} \frac{\partial g^*}{\partial Q_2}],$$

$$J_{Q_3} = \frac{\hbar}{2i} [g^* B_{33} \frac{\partial g}{\partial Q_3} - g B_{33} \frac{\partial g^*}{\partial Q_3} + g^* B_{32} \frac{\partial g}{\partial Q_3} - g B_{32} \frac{\partial g^*}{\partial Q_3}],$$

$$Y(Q_2, Q_3) = \int_0^T \vec{J}(Q_2, Q_3, t) \cdot \vec{n} \ dt$$

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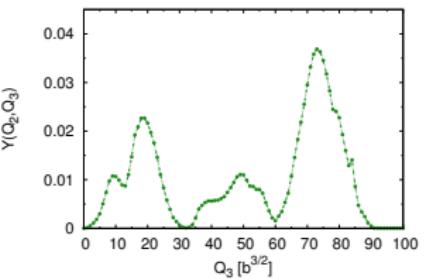
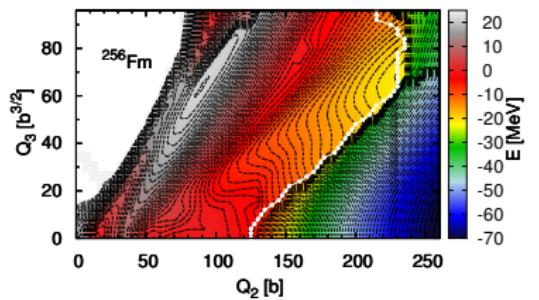
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$$Y(Q_2, Q_3) = \int_0^T \vec{J}(Q_2, Q_3, t) \cdot \vec{n} dt$$

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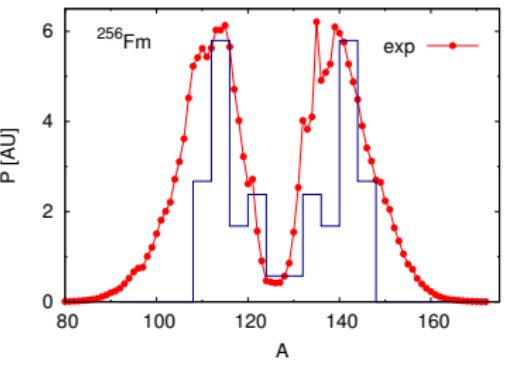
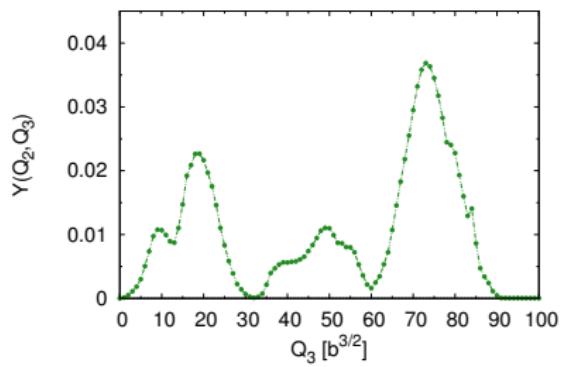
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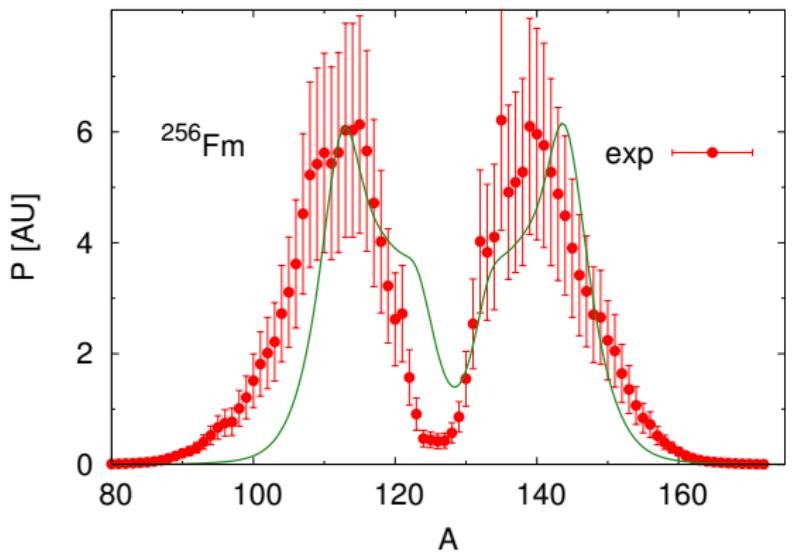
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^{256}Fm - dynamic effects + RNR machanism



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Conclusions I

- ▶ Fragment mass distribution obtained after application of macroscopic method (Brosa) to microscopically calculated pre-scission configuration
- ▶ Time-dependent formalism applied to describe fission process
- ▶ The broadness of the distribution is improved, when the time-dependent formalism is combined with the macroscopic method of Brosa

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Shell corrections and spontaneous fission

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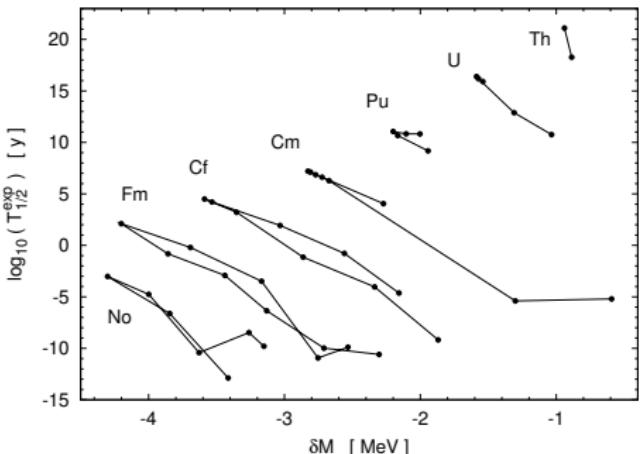
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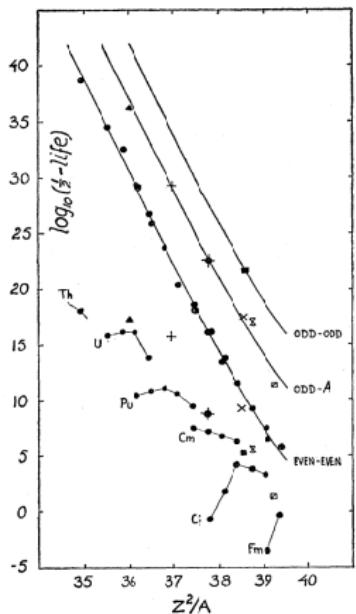
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$$\delta M = M_{\text{exp}} - M_{\text{LSD}}$$

K. Pomorski, J. Dudek, *Phys. Rev.* **C67**, 044316 (2003).

Swiatecki observation



W J. Swiatecki, *Phys. Rev.* **100**, 937 (1955).

$$\log_{10}(T_{1/2}^{sf}) = f(Z^2/A) - k\delta M$$

$$f(\theta) = -7.8\theta + 0.35\theta^2 + 0.073\theta^3 + a_i$$

$$a_{ee} = 18.2$$

$$a_{oe} = 24.8$$

$$a_{oo} = 29.7$$

$$\theta = Z^2/A - 37.5$$

$$k = 5 - \theta$$

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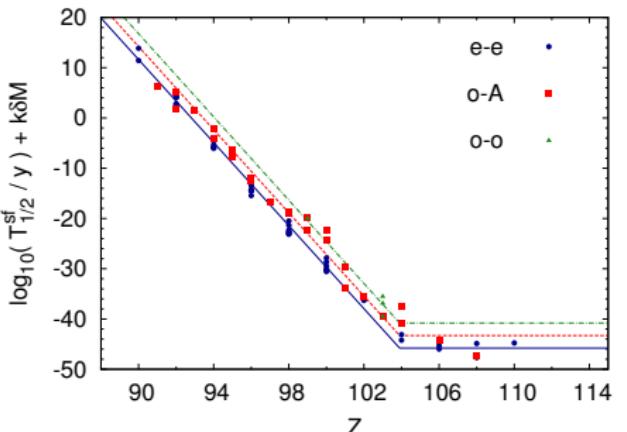
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Swiatecki formula - modern version

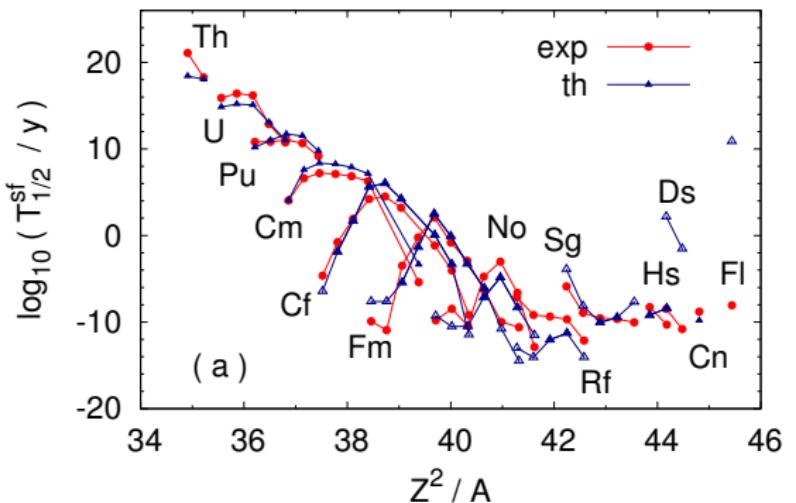


$$\log_{10}(T_{1/2}^{sf}) = f(Z) - k\delta M$$

$$k = 7.7 \text{ MeV}^{-1}$$

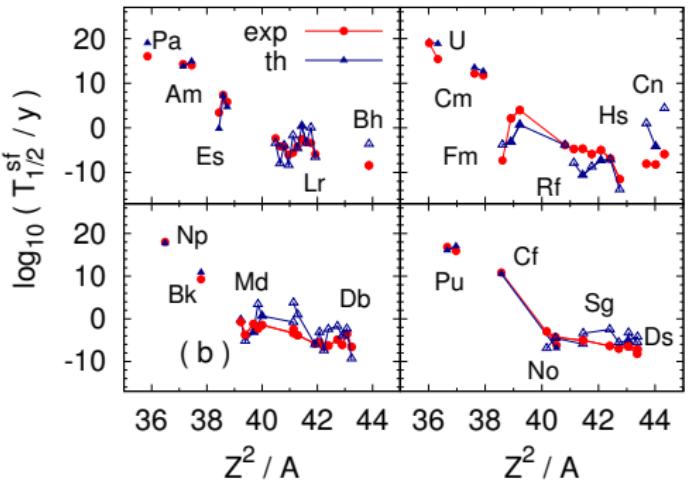
$$f(Z) = \begin{cases} -4Z + 380.2 & Z \leq 103 \\ -42.8 & Z \geq 104 \end{cases}$$

Even-even isotopes



$$\log_{10}(T_{1/2}^{sf}) = -4.1Z + 380.2 - 7.7\delta M$$

Odd isotopes



$$\log_{10}(T_{1/2}^{sf}) = -4.1Z + 380.2 - 7.7\delta M + h$$

$$h_{odd} = 2.5, h_{odd-odd} = 5$$

Conclusions II

- ▶ Semi-empirical formula for the spontaneous fission half-lives, depending on proton number and the ground state microscopic corrections, reproduces data for even-even super-heavy nuclei with reasonable accuracy.
- ▶ Quality of spontaneous fission half-lives evaluation breaks down for nuclei with not measured yet masses.
- ▶ Simple formula might be useful for predictions of spontaneous fission half-lives of unknown isotopes in the region $90 \leq Z \leq 104$.

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