Synthesis of Super-Heavy Elements

— Role of the fission barrier in uncertainty analysis

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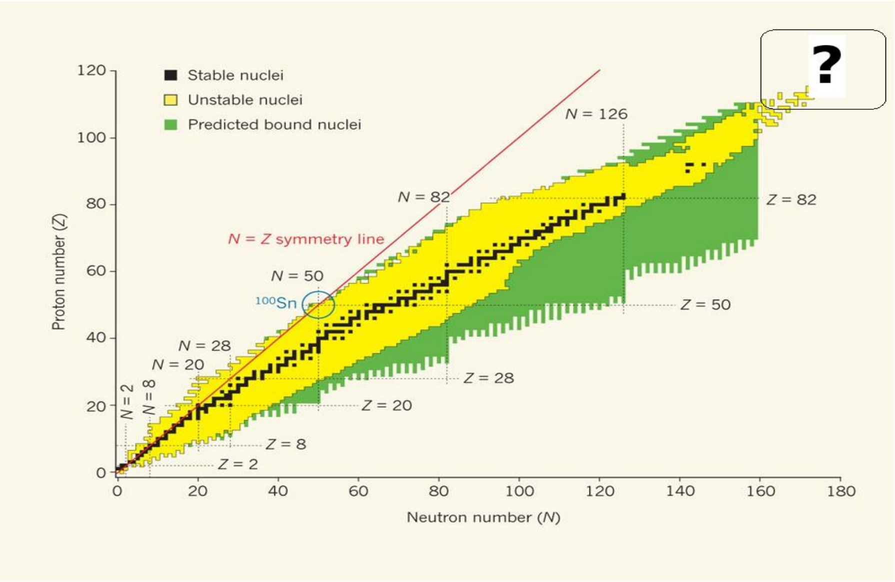
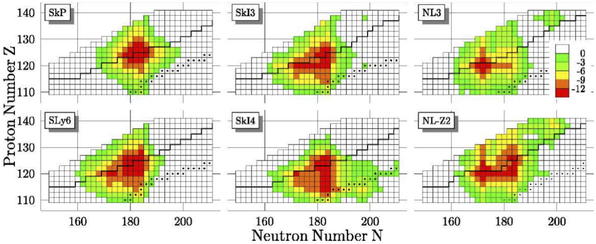
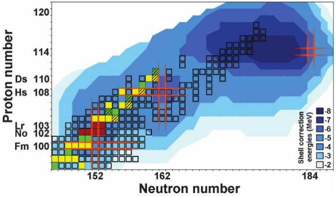


Where is the “island of stability”?



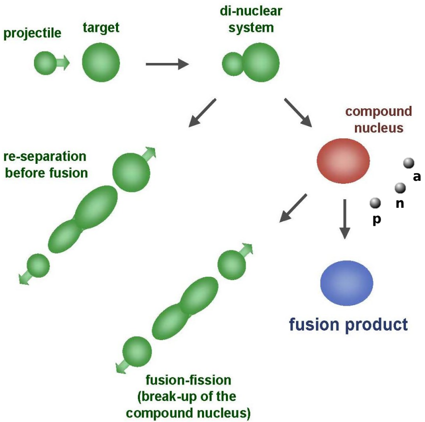
Mean-field calculations, M. Bender *et al.*, Phys. Lett. B 515 (2001) 42-48



Based on the FRDM, P. Möller *et al.*, At. Data Nucl. Data Tables 59 (1995) 185-381

Fusion-evaporation reaction

Schematic representation:



Fusion-evaporation reaction

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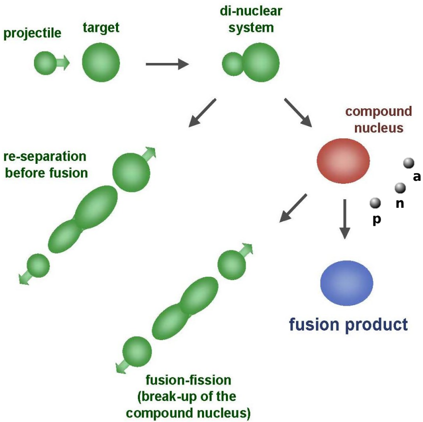
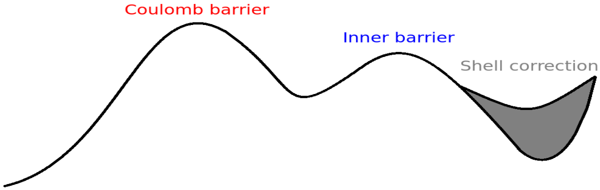


Typically, one has $B_f < S_n$ $\Rightarrow$ Nuclear fission is dominant!

Quasi-fission process only occurs in heavy binary systems $\Rightarrow$ fusion hindrance. Key factor for the synthesis of SHE!

Evaporation-residue (ER) cross-setion of SHE

$$\sigma_{\mathrm{ER}} \backsimeq \sigma_{\mathrm{cap}} \times P_{\mathrm{form}} \times W_{\mathrm{sur}}$$



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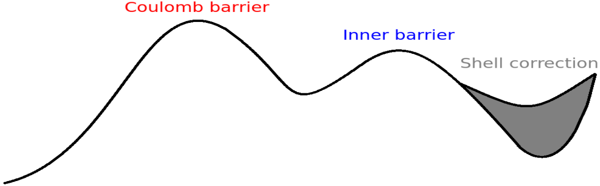
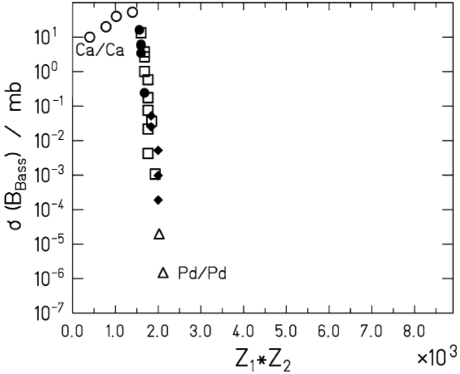


What is the height of the inner barrier? How to describe it, Langevin or DNS? Which degree of freedom is dominant? What is the strength of dissipation? ...

Fusion hindrance ... ?! An example

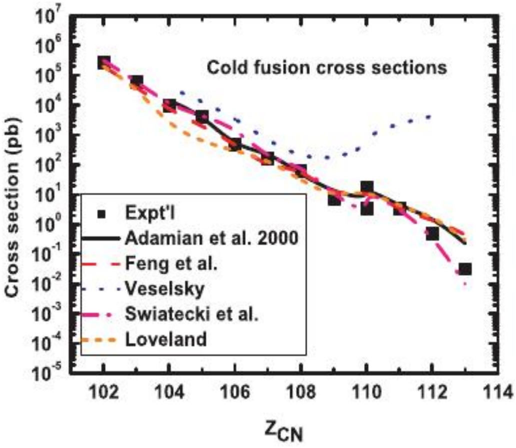
For light systems, $P_{\mathrm{form}} = 1$;

For heavy systems ($Z_t \cdot Z_p \gtrsim 1600 - 1800$), $P_{\mathrm{form}} \ll 1$.



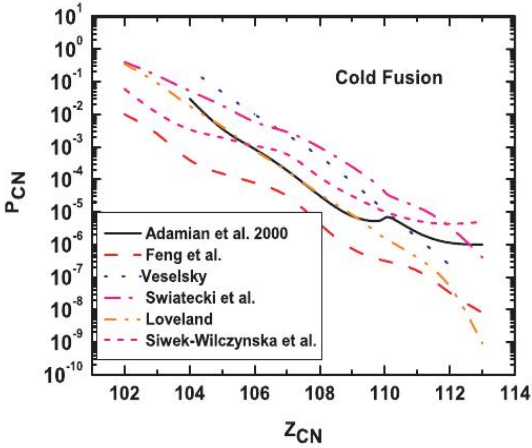
Total evaporation cross-sections, P. Armbruster, C. R. Phys. 4, 571 (2003)

Calculations highly consistent with data!



Naik, Loveland et al, Phys. Rev. C 76, 054604.

But, large discrepancies between P_{form} ...



Naik, Loveland et al, Phys. Rev. C 76, 054604.

What can we learn from this delicate situation?

- The better-known parts have the same discrepancies as the less-known part ($P_{\rm form}$)!
- Is it due to uncertainties associated with the better-known parts?

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- The better-known parts have the same discrepancies as the less-known part ($P_{\rm form}$)!
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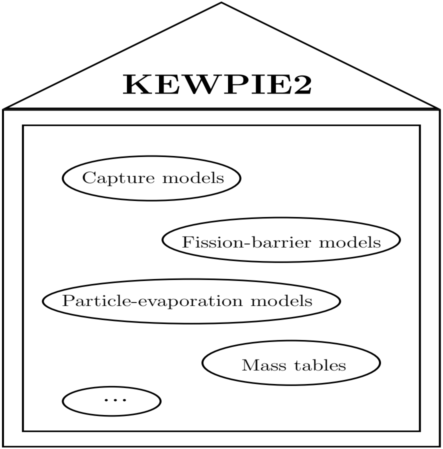
Solution:

- $$\overline{P}_{\rm form} = \frac{\sigma_{\rm ER}^{\rm 1n} \leftarrow \text{experimental}}{\sigma_{\rm cap}\overline{W}_{\rm sur} \leftarrow \text{theoretical}},$$
  where $\overline{W}_{\rm sur}$ is averaged due to energy loss in the target.

Experimental data + Computer code + Uncertainty propagation (MCM, proposed in GUM-S1)

How to calculate $\overline{W}_{\rm sur}$?



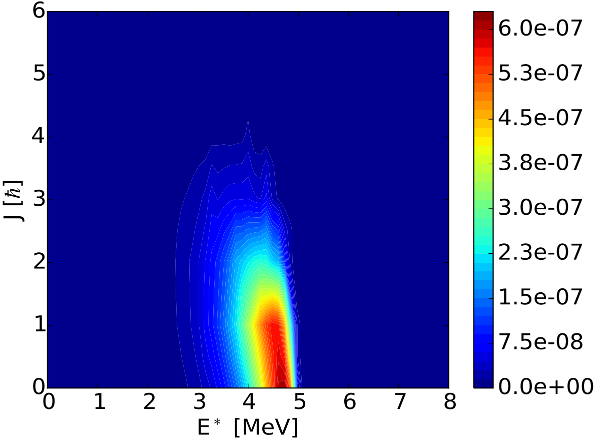
Basic features:

- Not a Monte-Carlo cascade code, but based on the discretization of population spectra. More efficient when dealing with extremely-low probability events.
- Single-barrier fission model, particle evaporation, improved state-density formula, γ-ray emission ...

A. Marchix, PhD thesis, Caen University (2007);
H. Lü *et al.*, paper submitted to CPC (2015)

Residual population spectrum

Example of the population spectrum calculated by KEWPIE2.



H. Lü *et al.*, paper submitted to CPC (2015)

Uncertainty sources: Parameters and Models

Parameters (input distributions):

- Experimental data (normally distributed);
- $\delta E^* \simeq 2 - 4$ MeV due to energy loss in the target;
- Reduced friction coefficient $\beta \simeq 1.0 - 9.0\ zs^{-1}$;
- Damping-shell energy $E_d \simeq 13.0 - 25.0$ MeV.

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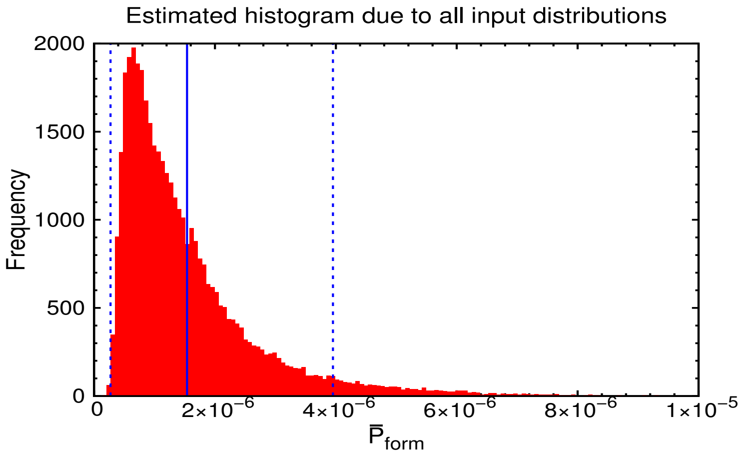
Models:

- Kramers-Strutinsky and collective enhancement factors;
- Level-density parameter models;
- Capture models;
- Fission-barrier models (model-dependent).

Impact of parameters

Example of the output distribution for ^{208}Pb(^{58}Fe,1n)^{265}Hs



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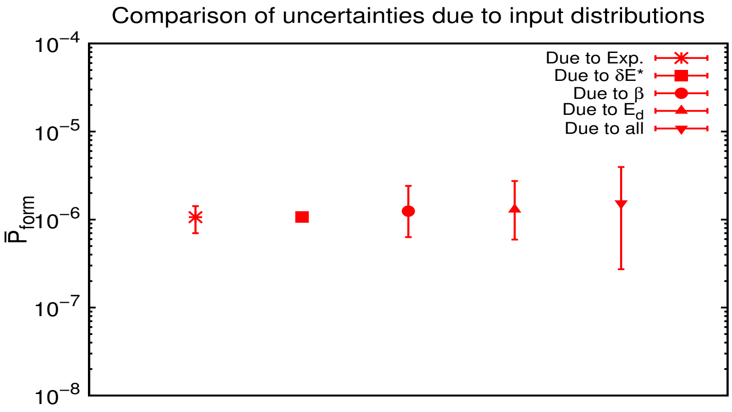
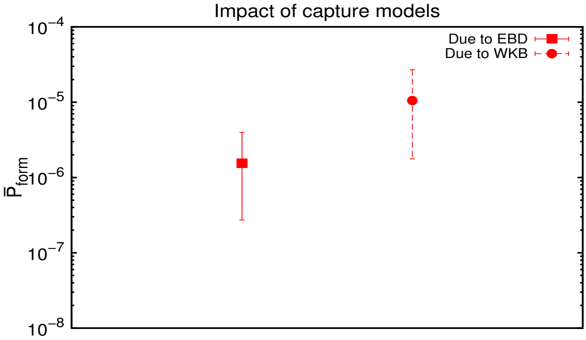
Example for ^{208}Pb(^{58}Fe,1n)^{265}Hs



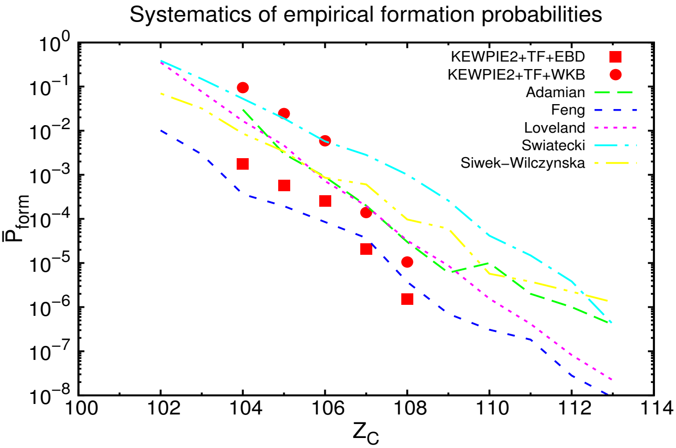
Impact of models (capture models)

Example for ^{208}Pb(^{58}Fe,1n)^{265}Hs



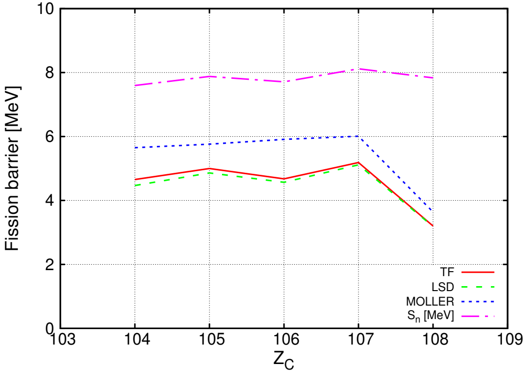
WKB approximation, W. Reisdorf, Z. Phys. A 300 (1981) 227, included in the HIVAP code;
Empirical barrier-distribution method, W. J. Swiatecki *et al.*, Phys. Rev. C 71, 014602 (2005).

Systematics (capture models)



Fortunately, capture cross-sections can be measured.

Impact of models (fission-barrier models)



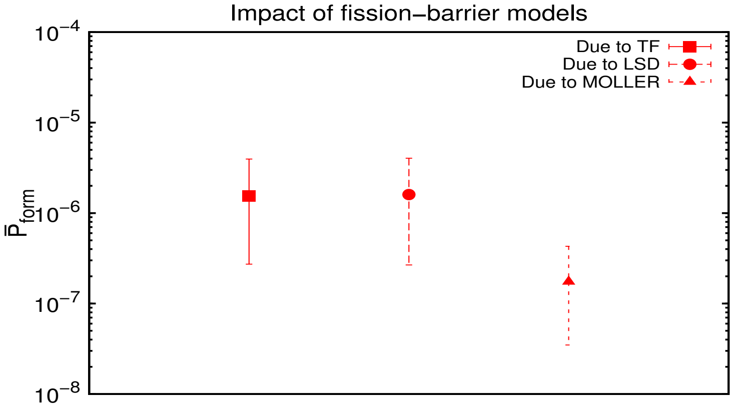
$B_f \simeq B_{\mathrm{LDM}} - \Delta E_{\mathrm{sh}}$. Thomas-Fermi, W. D. Myers *et al.*, Phys. Rev. C 62, 044610 (2000); Lublin-Strasbourg Drop, F. A. Ivanyuk *et al.*, Phys. Rev. C 79, 054327 (2009); Shell-correction energies from the FRDM.

Multidimensional calculation (5D), P. Mölle *et al.*, Phys. Rev. C 79, 064304 (2009).

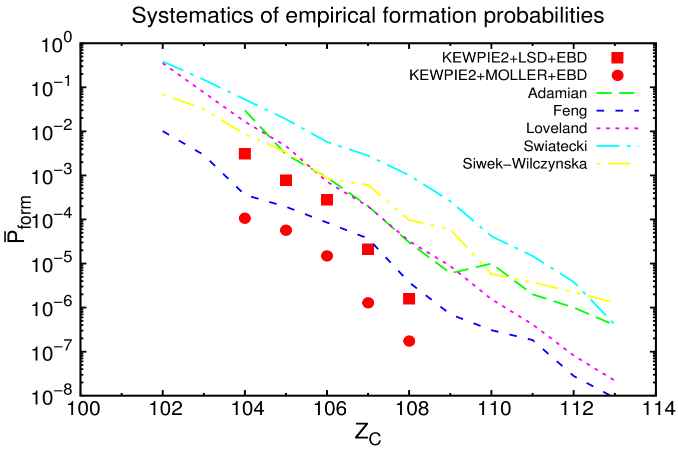
Generally differ by 1 MeV at most!

Impact of models (fission-barrier models)

Example for ^{208}Pb(^{58}Fe,1n)^{265}Hs



Systematics (fission-barrier models)



Unfortunately, uncertainties in fission barriers are not known ...

What can we conclude from this study?

Crucial points:

- Fission barrier is essential for uncertainty analysis.
- The resulting uncertainty due to the better-known parts is comparable to that of $P_{\rm form}$.
- How to constrain fission barriers and fusion models?

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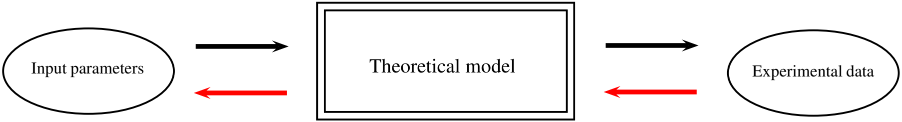
Crucial points:

- Fission barrier is essential for uncertainty analysis.
- The resulting uncertainty due to the better-known parts is comparable to that of P_{form}.
- How to constrain fission barriers and fusion models?

Without fusion hindrance, can we constrain fission barriers?
How to determine the uncertainty associated with B_f? Can we extract its probability distribution from data?

Bayesian inference (inverse problem)

$$P(\text{Parameters} \mid \text{Data}) \overset{?}{=} P(\text{Data} \mid \text{Parameters})$$



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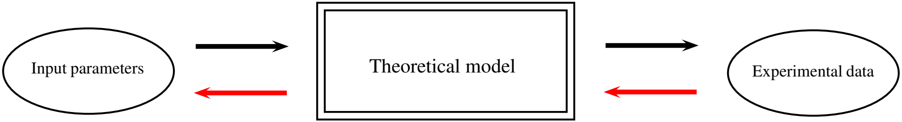

What is the relationship between conditional probabilities?
$P(\text{rain} \mid \text{cloud}) \neq P(\text{cloud} \mid \text{rain})$, etc.

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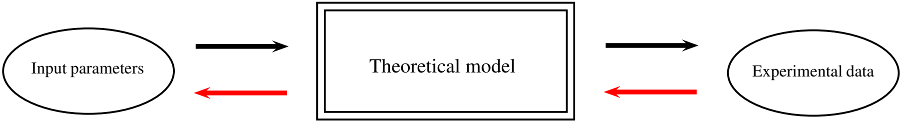

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We need Bayes rule!

Bayes rule

$$P(\text{Parameters} \mid \text{Data}) = \frac{P(\text{Data} \mid \text{Parameters}) \cdot P(\text{Parameters})}{P(\text{Data})}$$

Bayes rule

$$P(\text{Parameters} \mid \text{Data}) = \frac{P(\text{Data} \mid \text{Parameters}) \cdot P(\text{Parameters})}{P(\text{Data})}$$

Likelihood function $P(\text{Data} \mid \text{Parameters})$:

"Data" are observed survival probabilities and "Parameters" to B_f and other nuisance parameters (shell-damping energy, reduced friction coefficient, etc).

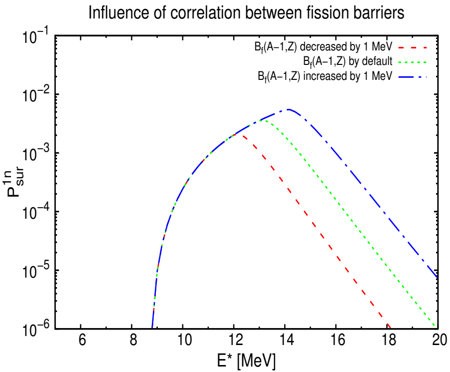
Prior distribution $P(\text{Parameters})$:

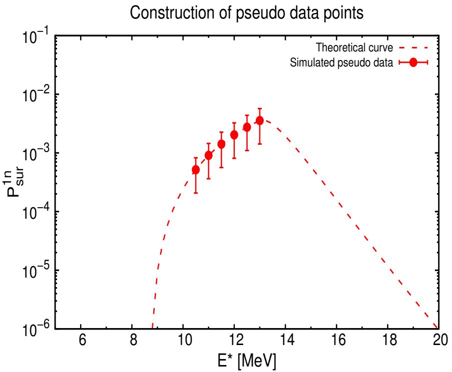
Representing our state of knowledge on B_f before seeing the data. Non-informative prior distribution is employed (maximum entropy).

Normalization factor $P(\text{Data})$:

Generally, it is extremely difficult to calculate because of multi-dimensional integrals.

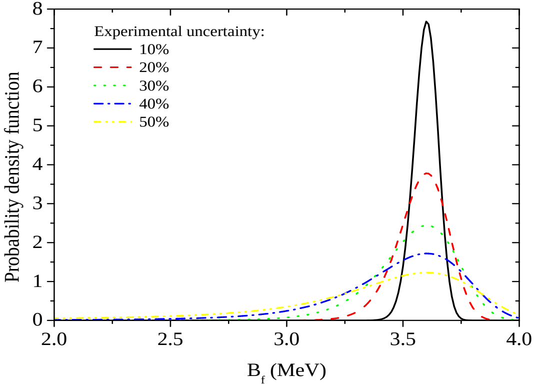
Pseudo-data for cold-fusion reaction





The increasing portion of the curve is dominated by the fission barrier of the mother nucleus, whereas the decreasing portion dominated by the fission barrier of the daugther nucleus.

Examples of $P(B_f)$



Based on a toy model for cold-fusion reaction, H. Lü et D. Boilley, EPJ Web of Conferences, 62 (2013) 03002.

Influences of the experimental uncertainty and the number of data

Table : Mean value of the fission barrier and the associated standard deviation. The true value of $B_f = 4.30$ MeV.

Number of data	Exp. uncertainty	$\langle B_f \rangle$ (MeV)	$u(B_f)$ (MeV)	$u(B_f)/\langle B_f \rangle$
2	20%	4.29	0.05	1.17%
2	40%	4.28	0.21	4.91%
2	60%	4.26	0.63	14.79%
4	60%	4.28	0.23	5.37%
6	60%	4.29	0.12	2.80%

The extracted relative uncertainty of B_f is much smaller than that of data.

Influence of the correlation between data points

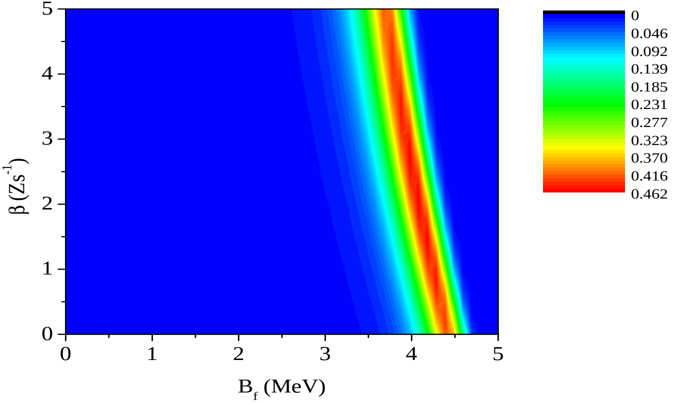
Table : Two pseudo-data points are considered.

Correlation coefficient	$\langle B_f \rangle$ (MeV)	$u(B_f)$ (MeV)	$u(B_f)/\langle B_f \rangle$
0.0	4.26	0.63	14.79%
0.1	4.20	0.71	16.90%
0.2	4.14	0.78	18.84%

Table : Six pseudo-data points are considered.

Correlation coefficient	$\langle B_f \rangle$ (MeV)	$u(B_f)$ (MeV)	$u(B_f)/\langle B_f \rangle$
0.0	4.29	0.12	2.80%
0.1	4.27	0.15	3.51%
0.2	4.26	0.18	4.23%

Can we extract two parameters?



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Conclusion and prospects

- In the present study, the impact of uncertainties on the formation probability has been investigated.
- The fission-barrier height plays a crucial role in fusion-evaporation reaction calculations.
- Bayesian inference can be used to extract information on the fission barrier from data.
- Neither fusion hindrance nor fission barrier is well known, but they are based on the same framework of the liquid-drop model. What is the correlation between them? How does it affect the production probability? Some follow-up work is underway ...

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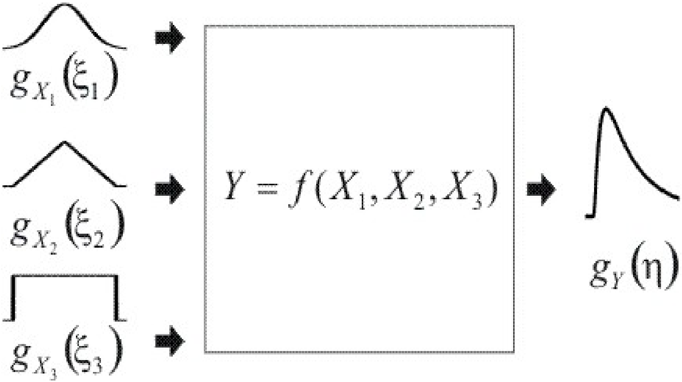
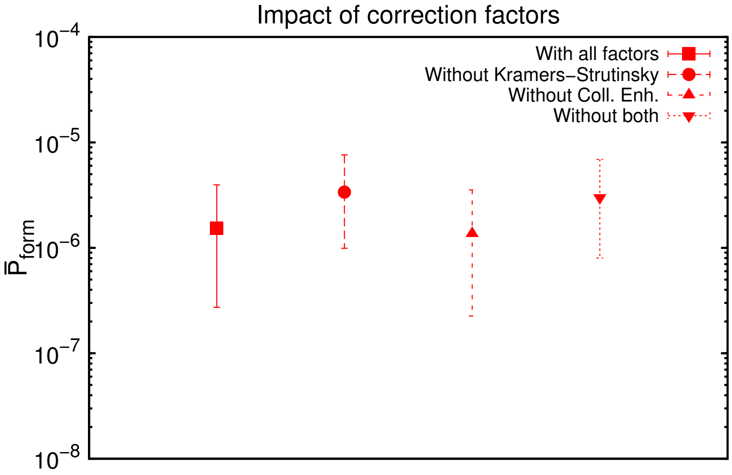
Thanks for your attention!

Uncertainty propagation (MCM)



The MCM determines numerically a probability density function (PDF) which encodes the knowledge about the quantity of interest. An estimate and its associated uncertainty are then determined by examining this PDF.

Models (correction factors)



Models (level-density parameters)