Nucleon elastic scattering off doubly closed-shell nuclei within HF+RPA with Gogny force

Guillaume Blanchon, Marc Dupuis

CEA, DAM, DIF

Understanding Nuclear Structure and Reactions Microscopically,
including the Continuum,
FUSTIPEN, March 17-21, 2014, GANIL Caen.

Nucleon elastic scattering

- Phenomenological optical potentials:
  - Valid in the range of the parametrization.
  - Most of the time local, $V(\mathbf{r})\delta(\mathbf{r}-\mathbf{r}')$, but some non-local developments, $V(\mathbf{r},\mathbf{r}')$.
    Perey & Buck, NP 32, 353 (1962), W. Dickhoff, PRC 82, 054306 (2010).
- Microscopic potentials: starting from NN interaction
  - Bare NN interaction: nuclear matter method, FRPA, coupled cluster.
    J. Hüfner and C. Mahaux, Ann. Phys. **73***, 525 (1972).*
    S. J. Waldecker et al. , Phys. Rev C **84***, 034616 (2011).*
    G. Hagen and N. Michel, Phys. Rev. C **86***, 021602(R) (2012).*
  - Effective NN interaction: nuclear structure method, cPVC.
    N. Vinh Mau, Theory of nuclear structure (IAEA, Vienna 1970) p. 931.
    K. Mizuyama et al., arXiv:210.2500v1 [nucl-th] 9oct 2012.

NUCLEAR STRUCTURE METHOD

N. Vinh Mau, Theory of nuclear structure (IAEA, Vienna 1970)
N. Vinh Mau, A. Bouyssy. NPA 257 (1976) 189-220
V. Bernard and N.V. Giai, NPA 327, 397 (1979)
F. Osterfeld, et al. PRC 23, 179 (1981)

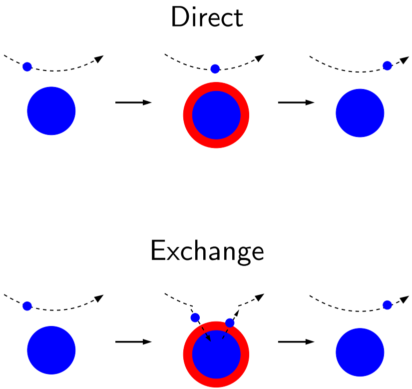
$$V = V_{HF} + \Delta V_{RPA}$$

Elastic scattering off a mean field

Direct　　　　Exchange

- V_{HF}
- Hartree-Fock with the NN Gogny effective interaction D1S
- Continuum HF

Elastic scattering with excitation of the target



- ΔV_{RPA}
- Excitations of the target described with RPA
- Spherical target nucleus
  Closed-shell nuclei
- HO basis 14 major shells

J. P. Blaizot, D. Gogny, et B. Grammaticos, NPA 265, 315 (1976).

J. F. Berger, M. Girod, et D. Gogny, Comp. Phys. Com. 63, 365 (1991).

Nuclear structure approach

"Optical potential for low-energy neutrons"

N. Vinh Mau, A. Bouyssy. NPA 257 (1976) 189-220.

Dyson equation & Hierarchy relation between G_0, G_1 and G_2

$$
\begin{aligned}
G_1(\mathbf{x},\mathbf{x'}) &= G_0(\mathbf{x},\mathbf{x'}) + \iint G_0(\mathbf{x},\mathbf{x}_0)\Sigma(\mathbf{x}_0,\mathbf{x}_1)G_1(\mathbf{x}_1,\mathbf{x'})d\mathbf{x}_0 d\mathbf{x}_1 \\
G_1(\mathbf{x},\mathbf{x'}) &= G_0(\mathbf{x},\mathbf{x'}) - i\iint G_0(\mathbf{x},\mathbf{x}_0)v(\mathbf{x}_0-\mathbf{x}_1)G_2(\mathbf{x}_0\mathbf{x}_1,\mathbf{x'}\mathbf{x}_1^+)d\mathbf{x}_0 d\mathbf{x}_1
\end{aligned}
$$

Self-energy

$$
\Sigma(\mathbf{x},\mathbf{x'}) = i\iint v(\mathbf{x}-\mathbf{x}_1)G_2(\mathbf{x}\mathbf{x}_1,\mathbf{x}_2\mathbf{x}_1^+)G_1^{-1}(\mathbf{x}_2,\mathbf{x'})d\mathbf{x}_1 d\mathbf{x}_2
$$

Hierarchy relation G_1, G_2 and G_3 & 3 approximations

- Three-body Green's functions can be replaced by a sum of products of one- and two-body Green's functions properly antisymmetrized
- Particle-particle propagators are treated in the ladder approximation, and particle-hole propagators in the bubble (RPA) approximations
- One-body Green's functions can be described by G_1^{HF} the HF propagators

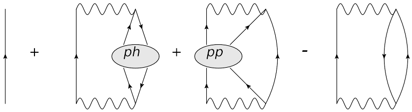
Nuclear structure approach

Bell & Squire

$$V(\mathbf{r},\mathbf{r}';E) = -\int_{-\infty}^{+\infty} e^{iE(t-t')}\Sigma(\mathbf{x},\mathbf{x'})d(t-t')$$

Optical potential

$$V = V^{HF} + V^{PP} + V^{RPA} - 2V^{(2)}$$



Use of EDF (Gogny interaction)

Density dependence comes, in the local density approximation, from the summation of ladder diagrams

$$V = V^{HF} + \mathrm{Im}\left[V^{(2)}\right] + V^{RPA} - 2V^{(2)}$$

Schrödinger integro-differential equation

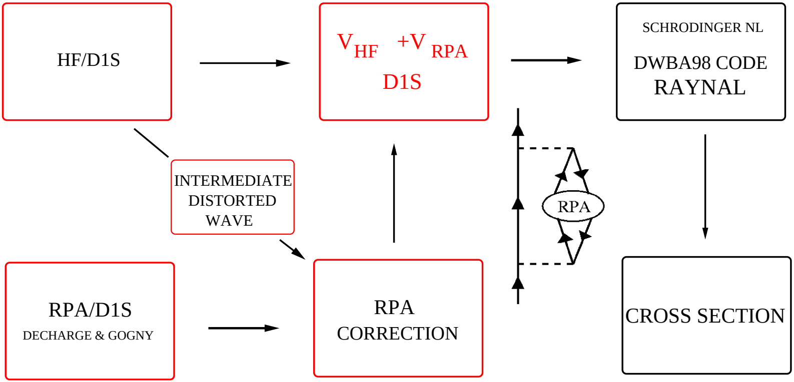
- n+A non-local and energy dependent potential

$$\left[\frac{d^2}{dr^2} + \frac{l(l+1)}{r^2} - k^2\right] f_{jl}(r) + r\int \nu_{jl}(r,r';E) f_{jl}(r') r' dr' = 0$$

$$V(\mathbf{r},\mathbf{r'};E) = \sum_{ljm} \mathcal{Y}_{ljm}(\hat{\mathbf{r}}) \nu_{lj}(r,r';E) \mathcal{Y}^{\dagger}_{ljm}(\hat{\mathbf{r}}')$$

- No localization of the potential
- Solved in a 15 fm box
  - Bound states
    R. H. Hooverman, NPA 189, 155 (1972).
  - Continuum
    J. Raynal, DWBA98, 1998, (NEA 1209/05).

Strategy: HF + RPA with D1S Gogny force

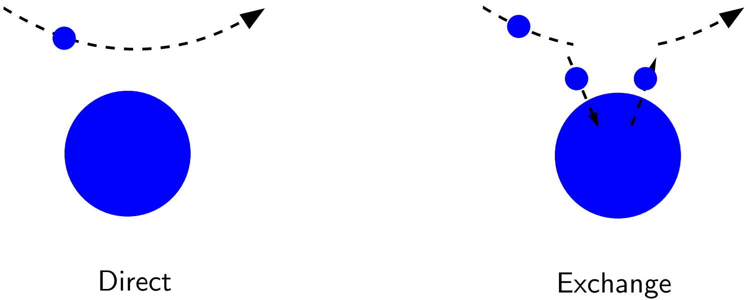


HARTREE-FOCK APPROXIMATION

$$V(\mathbf{r},\mathbf{r'},E) = V_{HF}(\mathbf{r},\mathbf{r'}) + \Delta V_{RPA}(\mathbf{r},\mathbf{r'},E)$$

C. B. Dover and N. V. Giai, NPA 190 (1972) 373
C. B. Dover and N. V. Giai, NPA 177 (1971) 559

Elastic scattering off a mean field



- Hartree-Fock with the NN Gogny effective interaction D1S
- HF with continuum D1S

HF approximation to the n+A potential

$$V_{HF}(\mathbf{r},\mathbf{r'}) = \int d\mathbf{r}_1 v(\mathbf{r},\mathbf{r}_1)\rho(\mathbf{r}_1)\delta(\mathbf{r}-\mathbf{r'}) - v(\mathbf{r},\mathbf{r'})\rho(\mathbf{r},\mathbf{r'})$$

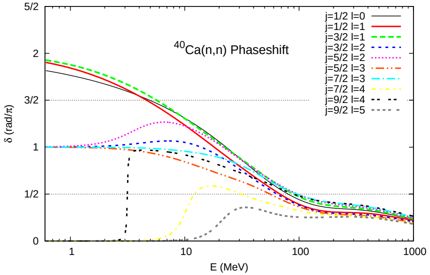
$$\rho(\mathbf{r}) = \sum_i n_i |\phi_i(\mathbf{r})|^2,$$

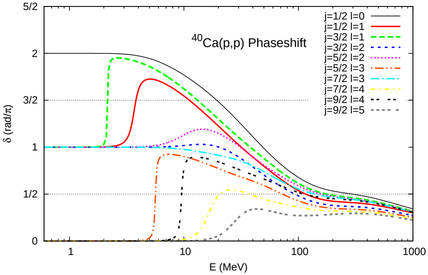
$$\rho(\mathbf{r},\mathbf{r'}) = \sum_i n_i \phi_i^*(\mathbf{r})\phi_i(\mathbf{r'})$$

- v: Gogny force
  Finite range NN interaction $\rightarrow$ V_{HF} non-local.
- v is real and energy independent
  V_{HF} is real and energy independent.
- HF in coordinate space
  $\rightarrow$ Good asymptotic behavior of the wave functions
  (not the case with HO basis).
  $\rightarrow$ Correct treatment of the continuum
  (Distorted Wave ϕ_λ, Resonances).

K. Davies, S. Krieger, and M. Baranger, Nuclear Physics 84, 545 (1966).

HF phase shift n/p+^{40}Ca



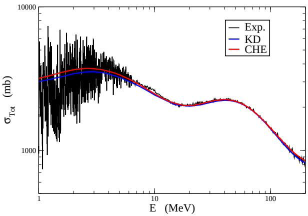


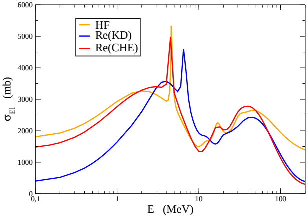
- Resonances when $\delta = n\pi/2$ (n odd).
- Correct DW treatment of the intermediate wave ϕ_λ.
- Impact on ΔV_{RPA}

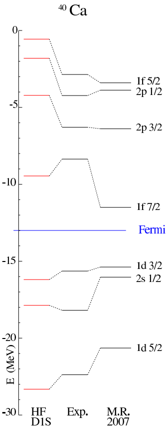
V^{HF} vs. $Re(V_{pheno})$

Total cross section n+^{40}Ca

Bound states HF/D1S Exp. CHE





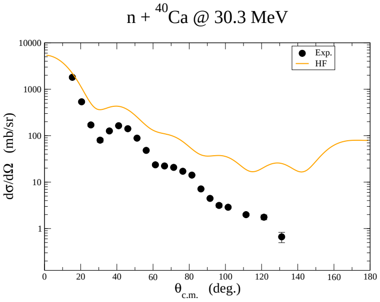


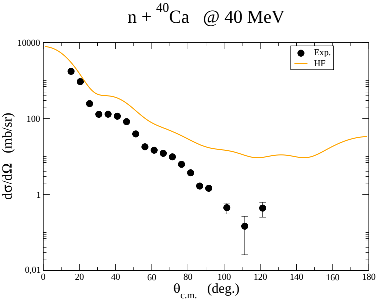
- V^{HF} gives the main contribution to the real part of the potential

(B. Morillon and P. Romain, Phys. Rev. C 70, 014601 (2004).) → dispersive potential

(A. J. Koning and J. P. Delaroche, Nuclear Physics A 713, 231 (2003).)

Elastic cross section n+^{40}Ca



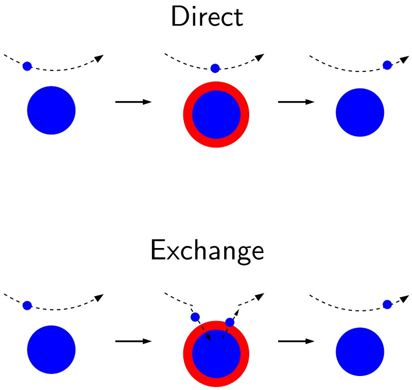


- HF potential is real... No absorption

RANDOM-PHASE APPROXIMATION

$$V(\mathbf{r}, \mathbf{r'}, E) = V_{HF}(\mathbf{r}, \mathbf{r'}) + \Delta V_{RPA}(\mathbf{r}, \mathbf{r'}, E)$$

Elastic scattering with excitation of the target



- ΔV_{RPA}
- Excitations of the target described with RPA
- Spherical target nucleus
- HO basis 14 major shells

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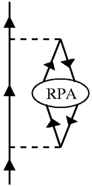
J. F. Berger, M. Girod, et D. Gogny, Comp. Phys. Com. 63, 365 (1991).

RPA potential

$$\Delta V_{RPA} = \mathrm{Im}\left[V^{(2)}\right] + V^{RPA} - 2V^{(2)}$$

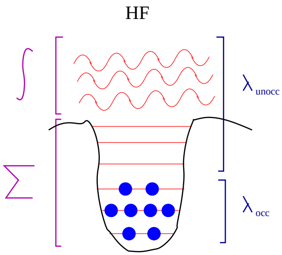
$$\begin{aligned} V^{RPA}(\mathbf{r},\mathbf{r}',E) &= \lim_{\eta\to 0^+} \sum_{N\neq 0, ijkl} \sum\!\!\!\!\!\!\int_{\lambda} \chi_{ij}^{(N)}\chi_{kl}^{(N)} \\ &\times \left(\frac{n_\lambda}{E-\epsilon_\lambda+E_N-i\eta} + \frac{1-n_\lambda}{E-\epsilon_\lambda-E_N+i\eta}\right) F_{ij\lambda}(\mathbf{r})F^*_{kl\lambda}(\mathbf{r}') \end{aligned}$$



with

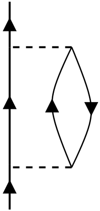
$$F_{ij\lambda}(\mathbf{r}) = \int d^3\mathbf{r}_1 \phi_i^*(\mathbf{r}_1) v(\mathbf{r},\mathbf{r}_1)[1-P]\phi_\lambda(\mathbf{r})\phi_j(\mathbf{r}_1)$$

- ϕ's are HF wave functions.
- We include both bound and continuum particles in constructing our intermediate state ϕ_λ.
- Finite sum: *compound-nucleus formation*
- Integral: *inelastic target excitations*



Second order potential

$$V^{(2)}(\mathbf{r},\mathbf{r}',E) = \lim_{\eta\to 0^+} \sum_{N\neq 0,ijkl} \sum\!\!\!\!\!\!\int_{\lambda} n_j(1-n_i)\delta_{ik}\delta_{jl} \times \left(\frac{n_\lambda}{E-\epsilon_\lambda+\epsilon_i-\epsilon_j-i\eta} + \frac{1-n_\lambda}{E-\epsilon_\lambda-\epsilon_i+\epsilon_j+i\eta}\right) F_{ij\lambda}(\mathbf{r})F^*_{kl\lambda}(\mathbf{r}')$$



- Potential built from pure particle-hole excitations
- Issue of the 1^- spurious contribution (translation mode)

Complex RPA potential

$$
\begin{aligned}
V_{RPA}(\mathbf{r},\mathbf{r}',E) &= \lim_{\eta\to 0^+} \sum_{N\neq 0,ijkl} \sum\!\!\!\!\!\!\int_{\lambda} \chi_{ij}^{(N)} \chi_{kl}^{(N)} \\
&\times \left(\frac{n_\lambda}{E-\epsilon_\lambda+E_N-i\eta} + \frac{1-n_\lambda}{E-\epsilon_\lambda-E_N+i\eta} \right) F_{ij\lambda}(\mathbf{r}) F^*_{kl\lambda}(\mathbf{r}')
\end{aligned}
$$

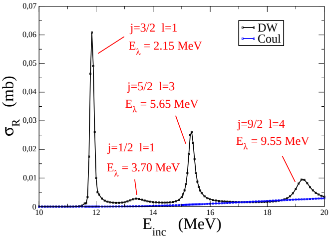
Plemelj formula $\lim_{\eta\to 0^+} \int_a^b \frac{f(x)}{x-x_0\mp i\eta}dx = \mathscr{P}\int_a^b \frac{f(x)}{x-x_0}dx \pm i\pi f(x_0)$

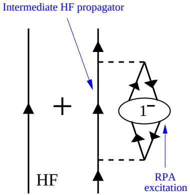
$$
\begin{aligned}
\lim_{\eta\to 0^+} \int \frac{-(1-n_\lambda) f^{(ijkl,N)}(\epsilon_\lambda)}{\epsilon_\lambda-(E-E_N)-i\eta} d\epsilon_\lambda &= \mathscr{P}\int \frac{-(1-n_\lambda) f^{(ijkl,N)}(\epsilon_\lambda)}{\epsilon_\lambda-(E-E_n)} d\epsilon_\lambda \\
&- (1-n_\lambda) i\pi \int f^{(ijkl,N)}(\epsilon_\lambda)\delta(\epsilon_\lambda-(E-E_N)) d\epsilon_\lambda
\end{aligned}
$$

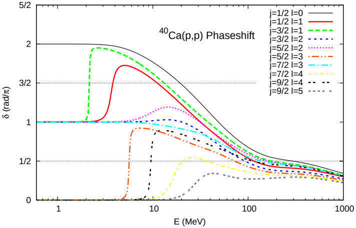
- When $\eta \to 0$, only $E_N < E$ excitations contribute to the imaginary part of the RPA potential.
- When $\eta \to 0$, no contribution from the *compound nucleus* terms to the absorption ($\sum_\lambda$).
- The determination of the real part requires all the excitations.

Effect of HF intermediate propagator

- p+^{40}Ca
- $V_{HF} + \mathrm{Im}(V_{RPA})$
- Coupling to the first 1^- $E_{1-} = 9.7 MeV$





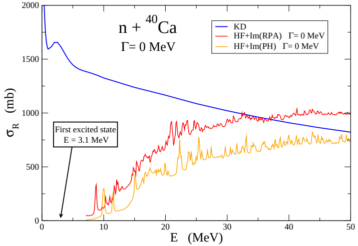


- Effect of resonances of the intermediate HF propagator.
- Enhancement of σ_R compared as with a Coulomb wave.

$$\lim_{\eta \to 0^+} \int Im\left(\frac{-(1-n_\lambda) f^{(ijkl,N)}(\epsilon_\lambda)}{\epsilon_\lambda - (E - E_n) - i\eta}\right) d\epsilon_\lambda \quad = \quad -(1-n_\lambda)\pi \int f^{(ijkl,N)}(\epsilon_\lambda)\delta(\epsilon_\lambda - (E - E_n)) d\epsilon_\lambda$$

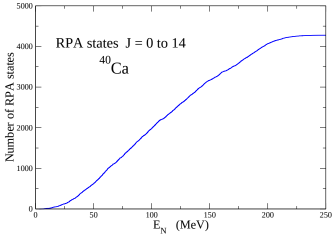
Effect of HF intermediate propagator

- σ_R from $V_{HF} + \mathrm{Im}(V_{RPA})$
- σ_R from $V_{HF} + \mathrm{Im}(V_{PH})$



→ Effect of the HF resonances on $\mathrm{Im}(V_{RPA})$

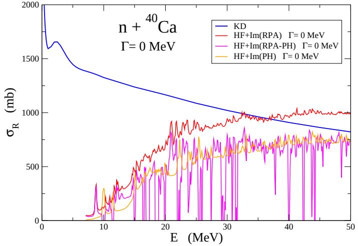
- Zero width calculation:
  - $\sigma_R = 0$ for incident energies below the energy of the first excited state of the target nucleus
- ^{40}Ca RPA states $J = 0 \to 8$



$$\lim_{\eta \to 0^+} \int \frac{-(1-n_\lambda) f^{(ijkl,N)}(\epsilon_\lambda)}{\epsilon_\lambda - (E - E_n) - i\eta} d\epsilon_\lambda = \mathscr{P} \int \frac{-(1-n_\lambda) f^{(ijkl,N)}(\epsilon_\lambda)}{\epsilon_\lambda - (E - E_n)} d\epsilon_\lambda - (1-n_\lambda) i\pi \int f^{(ijkl,N)}(\epsilon_\lambda) \delta(\epsilon_\lambda - (E - E_n)) d\epsilon_\lambda$$

Effect of HF intermediate propagator

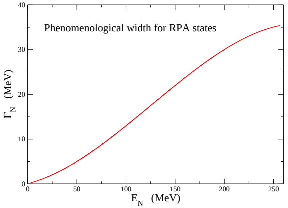
- σ_R from $V_{HF} + \mathrm{Im}(\Delta V_{RPA})$



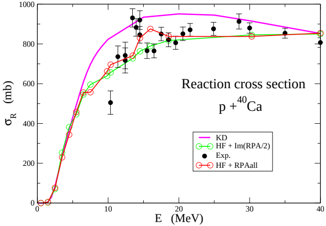
- In this work
  - Consistent scheme (Gogny interaction only)
  - Use of a phenomenological width (Harakeh and van der Woude)

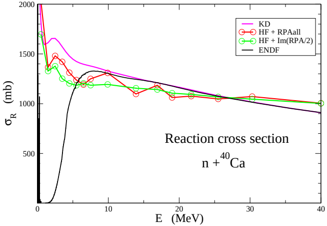
- Physical origin of width
  - Self-consistent scheme
  - $\eta \neq 0$ when HF propagator gets dressed by RPA
  - $E_N \rightarrow E_N + i\Gamma_N(E_N)$

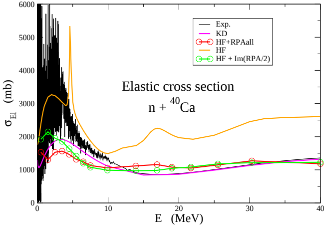
    Damping (doorway state) & continuum

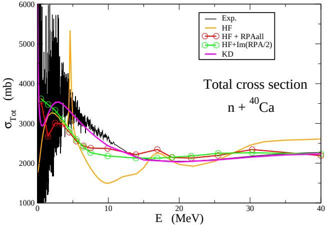


Integral cross sections n/p+^{40}Ca

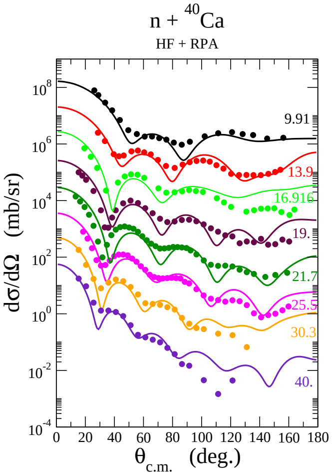


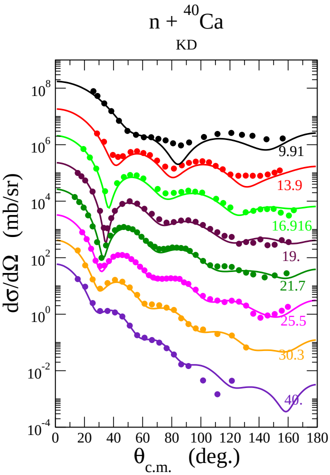




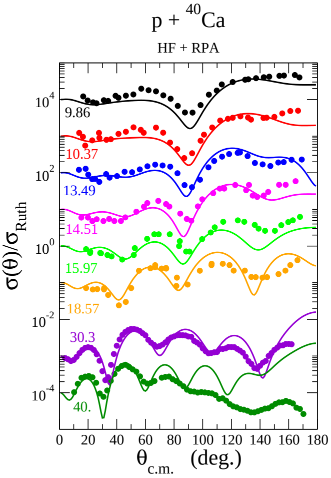


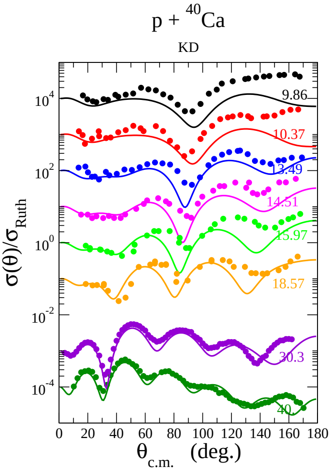
Elastic cross section n+^{40}Ca



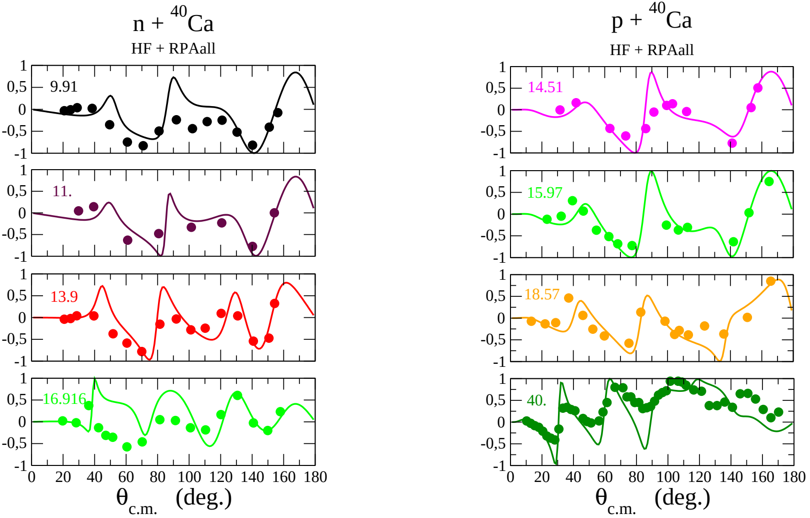


Elastic cross section p+^{40}Ca

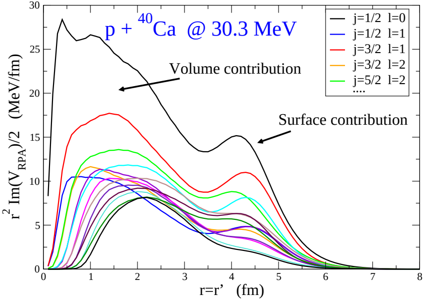


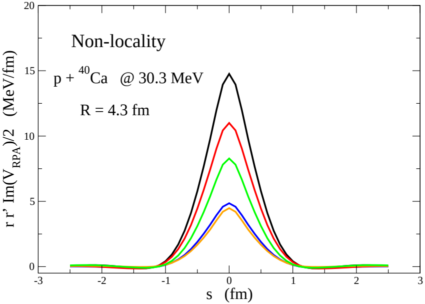


Analyzing powers n/p+^{40}Ca



Non locality of the imaginary part





Conclusion

- Summary:
  - We take into account absorption coming from the coupling to RPA states.
  - Consistent scheme.
  - Tools to deal with non-local potentials (bound and continuum states, HF in coordinate space).
  - Exact treatment of the intermediate state with resonances.
  - Good agreement with experiment (cross section, analyzing power) for ^{40}Ca
  - ^{48}Ca, ^{90}Zr, ^{132}Sn and ^{208}Pb in production
- Outlooks:
  - Bound single particle dressing.
  - Consistent width (N. Pillet mp-mh).
  - QRPA potential, deformed nuclei.
  - Inelastic scattering.