NUCLEAR PHYSICS FROM LATTICE QCD

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Outline

- QCD at Low Energies and the Lattice
- Nuclear Effective Field Theories
- EFT for Lattice Nuclei
- Outlook and Conclusion

Goal

Derivation of nuclear physics consistent with Standard Model (SM) of particle physics

- correct symmetries
- systematic

Why?

- Nucleus as the simplest complex system:
  quarks and gluons interacting strongly,
  yet exhibiting many regularities
  - QCD at large distances an unsolved part of the SM
  - tools for non-perturbative quantum (field) theories, *e.g.* cold atoms
- Nucleus as a laboratory:
  properties of the SM and beyond
  - nuclear matrix elements for symmetry tests
  - reaction rates for nucleosynthesis
  - equation of state for stellar structure
  - variation of parameters for cosmology
  - …

How?

Effective Field Theory

$$\begin{cases} T = T^{(\infty)}(Q \sim m \ll M) \sim N(M) \displaystyle\sum_{\nu=\nu_{\min}}^{\infty} \sum_i \tilde{c}_{\nu,i}(\Lambda) \left[\frac{Q}{M}\right]^{\nu} F_{\nu,i}\left(\frac{Q}{m};\frac{\Lambda}{m}\right) \\ \dfrac{\partial T}{\partial \Lambda} = 0 \end{cases}$$

normalization

"low-energy constants"

non-analytic, from loops

arbitrary regulator

"power counting"

counting index depending on properties of interactions

For $Q \sim m$, truncate … … consistently with RG invariance:

$$T = T^{(\bar{\nu})}\left[1 + \mathcal{O}\left(\frac{Q}{M}, \frac{Q}{\Lambda}\right)\right] \Rightarrow \frac{\Lambda}{T^{(\bar{\nu})}} \frac{\partial T^{(\bar{\nu})}}{\partial \Lambda} = \mathcal{O}\left(\frac{Q}{\Lambda}\right) \ll 1$$

controlled

model independent

<u>If so</u>
- want $\Lambda \gtrsim M$
- realistic estimate of errors comes from variation $\Lambda \in [M, \infty)$

QCD

$$\mathcal{L}_{QCD} = \bar{q}\left(i\partial\!\!\!/ + g_s G\!\!\!/\right)q - \frac{1}{2}\mathrm{Tr}\,G^{\mu\nu}G_{\mu\nu} \;+ \bar{m}\,\bar{q}\left(1-\varepsilon\tau_3\right)q \;+ \dots$$

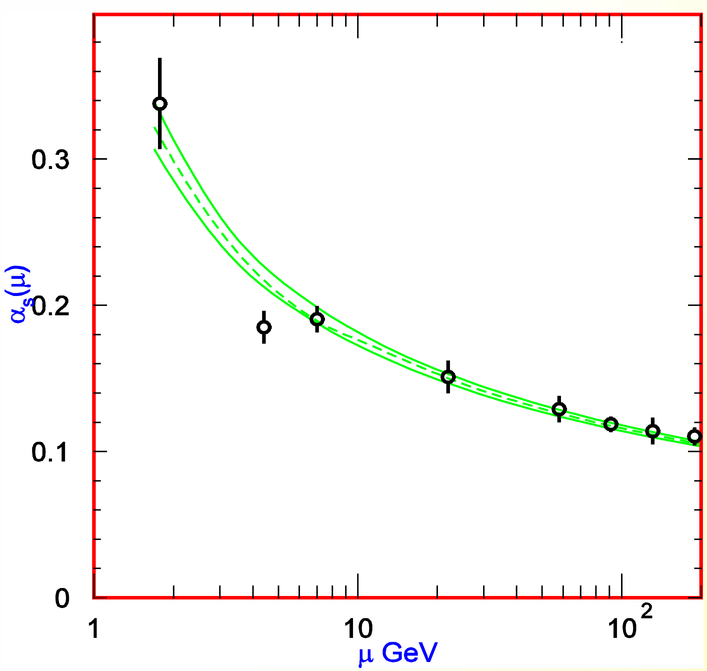
Basic mass scales

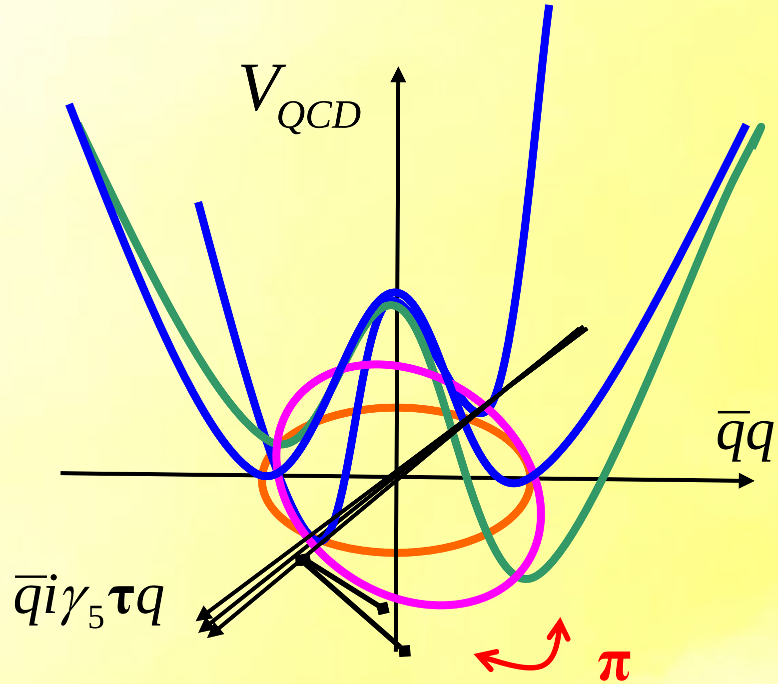
$$M_{QCD} \sim m_N, m_\rho, 4\pi f_\pi, \dots$$

$$\sim 1\ \mathrm{GeV}$$

$$m_\pi \sim \sqrt{\bar{m} M_{QCD}}$$

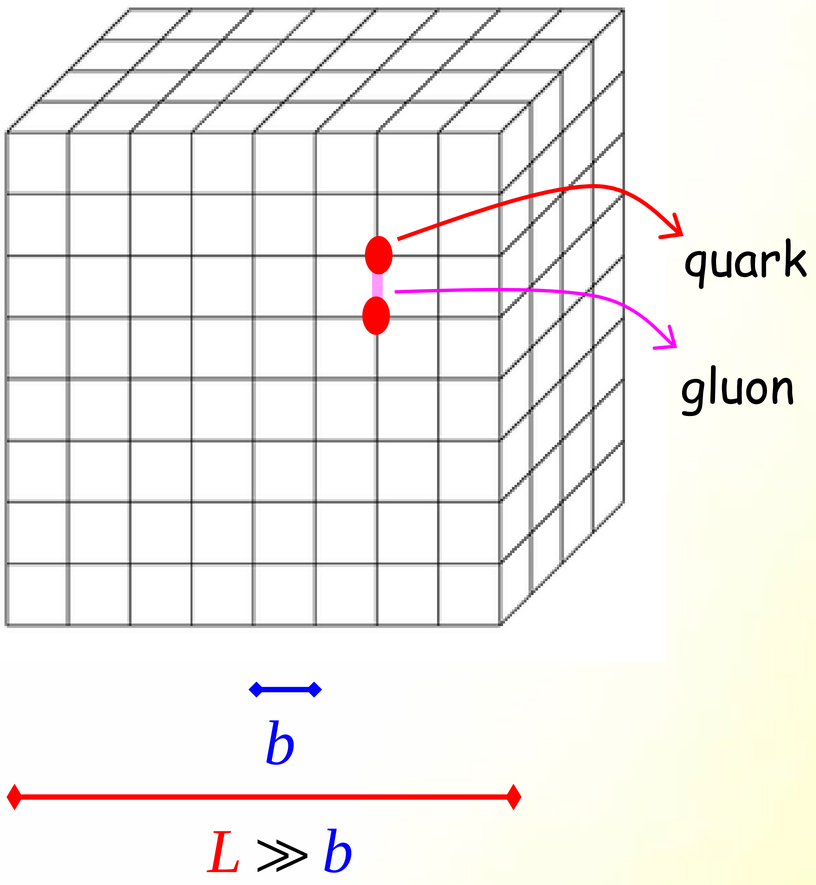
$$\simeq 140\ \mathrm{MeV}$$

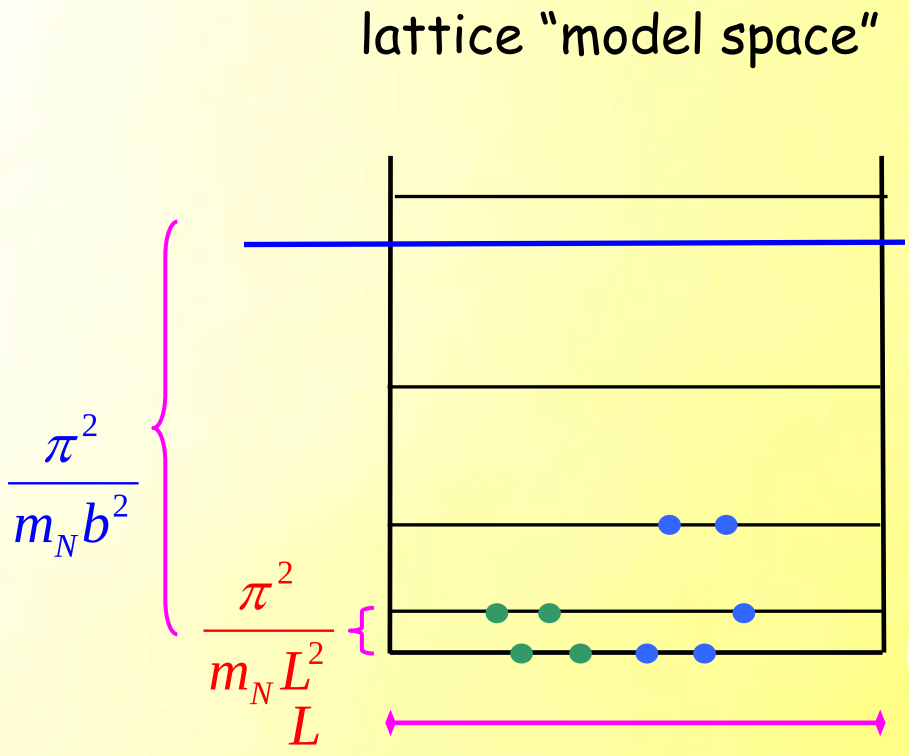




$$f_\pi \sim M_{QCD}/4\pi + \mathcal{O}(\bar{m}) \simeq 100\ \mathrm{MeV}$$

Lattice QCD





path integral solved with Monte Carlo methods, typically for unrealistically large quark masses

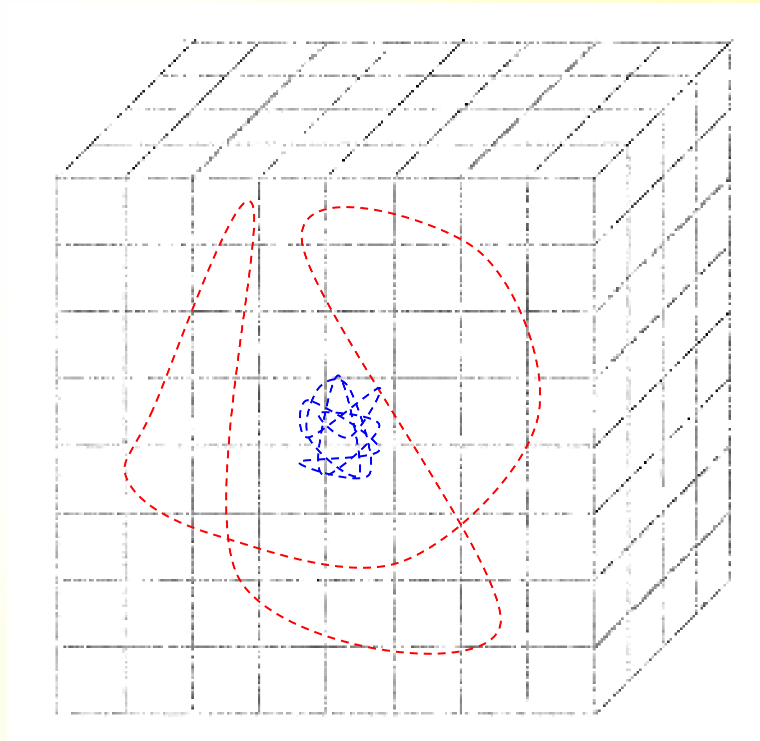
$$\cot\delta(E) = \frac{4}{\sqrt{m_N E b}}\left[\pi \frac{b}{L} \sum_{\mathbf{n}}^{|\mathbf{n}|<L/b} \frac{1}{(2\pi\mathbf{n})^2 - m_N E L^2} - 1\right]$$

Lüscher '91

nucleon

$$b \ll 1/M_{QCD}$$

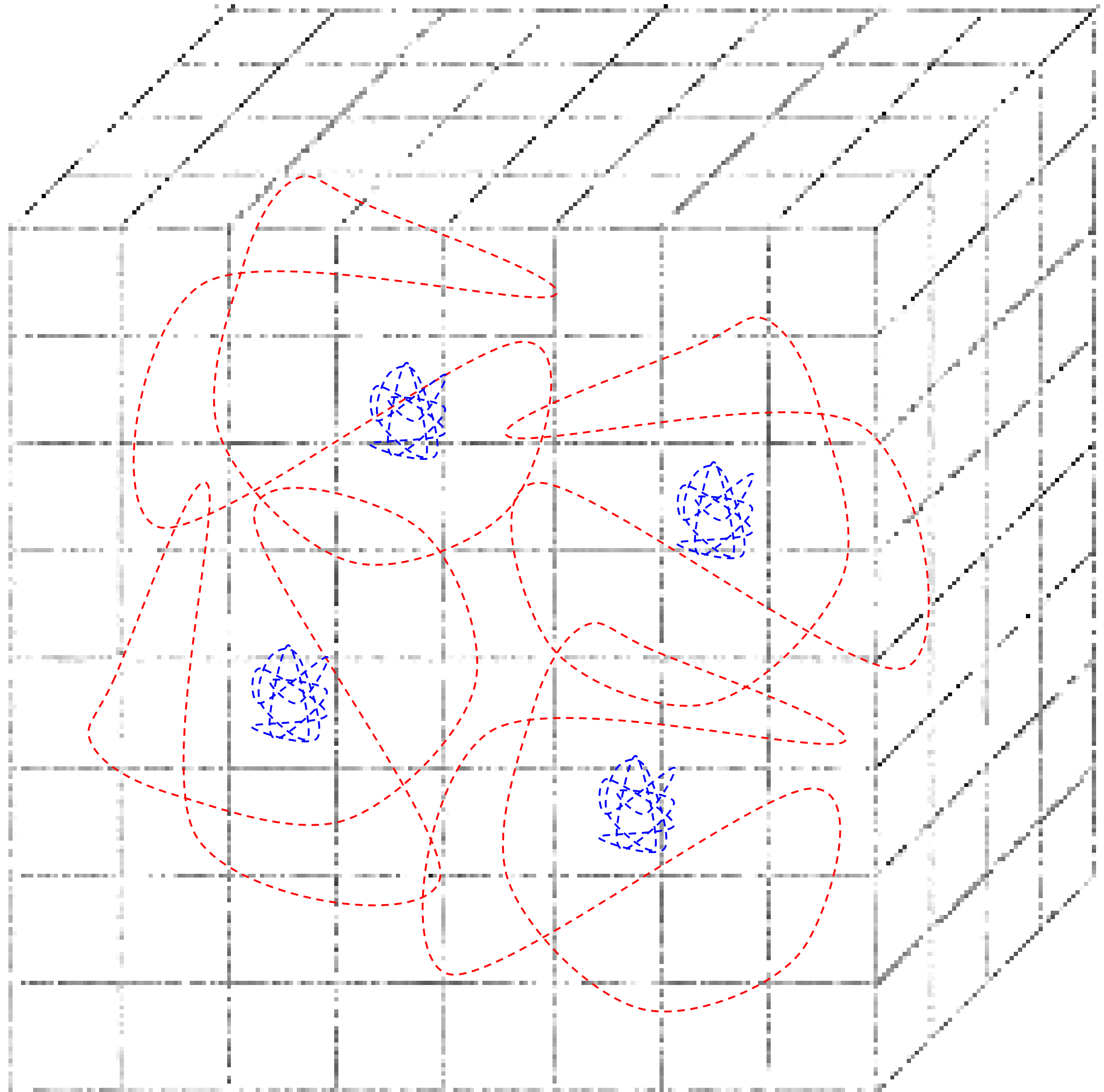
$$1/M_{QCD} \approx 0.3 \text{ fm}$$



$$1/m_\pi \cong 1.4 \text{ fm}$$

$$L \gg 1/m_\pi$$

nucleus



$b \ll 1/M_{QCD}$

$1/M_{QCD} \approx 0.3 \text{ fm}$

$R \sim \rho\left(m_\pi / f_\pi\right) A^{1/3}/m_\pi \approx 1.2\, A^{1/3}\text{fm}$

$L \gg \rho\, A^{1/3}/m_\pi$

two-step strategy

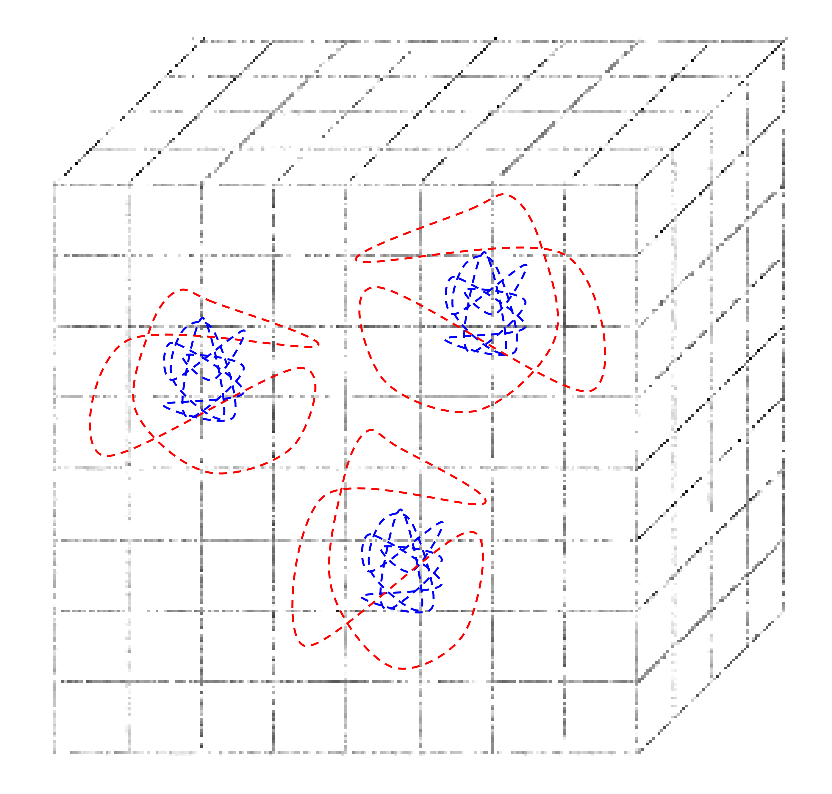
I) fit LECs for

$$A \le a \sim 3,4$$

$$m_\pi \ge M_\pi \sim 300, 400 \text{ MeV}$$

$$b \ll 1/M_{QCD}$$

$$1/M_{QCD} \approx 0.3 \text{ fm}$$



$$1/M_\pi$$

$$\rho(M_\pi/f_\pi)a^{1/3}/M_\pi$$

$$L \gg \rho(M_\pi/f_\pi)a^{1/3}/M_\pi$$

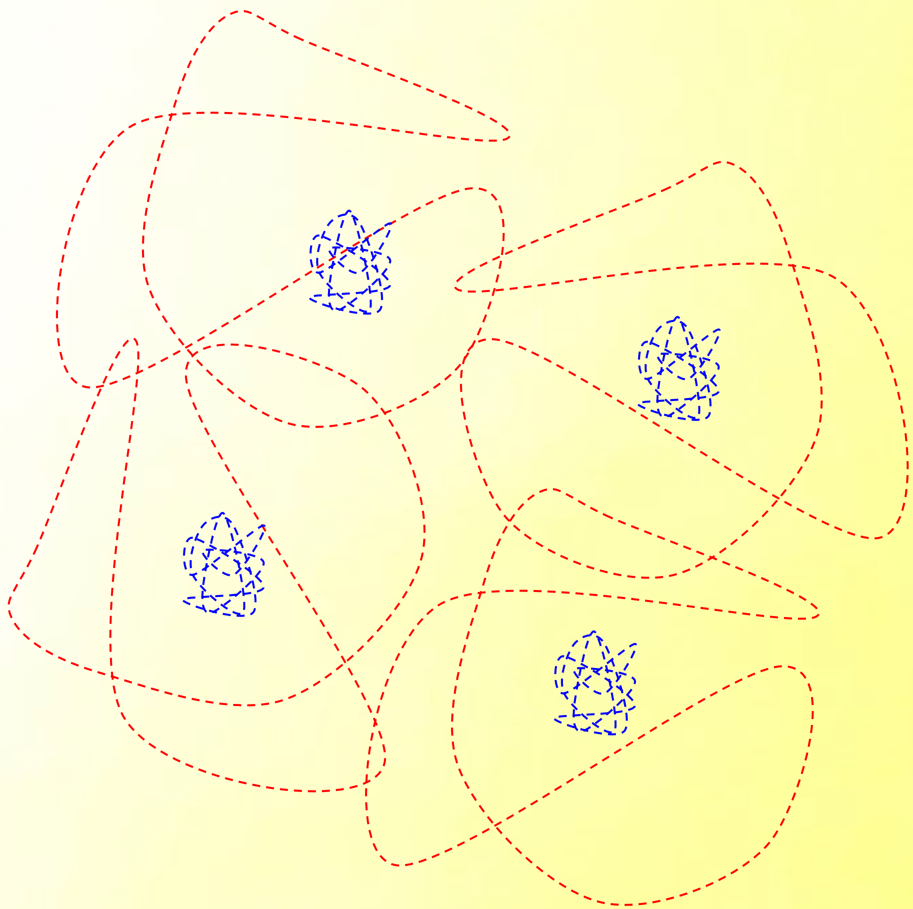
www.getcliparts.com

two-step strategy

II) solve EFT for

any A

any m_π



$1/M_{QCD} \approx 0.3\ \text{fm}$

$1/m_\pi$

$\rho(m_\pi / f_\pi)\, A^{1/3}/m_\pi$

Extrapolation in pion mass

Pionful (Chiral) EFT $\quad Q \sim m_\pi \ll M_{QCD}$

- degrees of freedom: nucleons, pions, Deltas (+ Roper + ?)

$$m_\Delta - m_N \sim 2m_\pi \quad \left(m_{N'} - m_N \sim 3m_\pi, \ldots\right)$$

- symmetries: Lorentz, ~~P~~, ~~T~~, ~~chiral~~

$$D_\mu = \frac{1}{1+\boldsymbol{\pi}^2/4f_\pi^2}\partial_\mu \qquad \mathcal{D}_\mu = \partial_\mu + \frac{i}{2f_\pi^2}\left(\boldsymbol{\pi}\times D_\mu\boldsymbol{\pi}\right)\cdot\mathbf{t}^{(I)}$$

$$\mathcal{L}_{EFT} = \frac{1}{2}D_\mu\boldsymbol{\pi}\cdot D^\mu\boldsymbol{\pi} - \frac{m_\pi^2}{2}\frac{\boldsymbol{\pi}^2}{1+\boldsymbol{\pi}^2/4f_\pi^2} + N^+\left(i\mathcal{D}_0 + \frac{\vec{\mathcal{D}}^2}{2m_N}\right)N + \frac{g_A}{2f_\pi}N^+\vec{S}\boldsymbol{\tau}N\cdot\cdot\vec{\mathcal{D}}\boldsymbol{\pi}$$

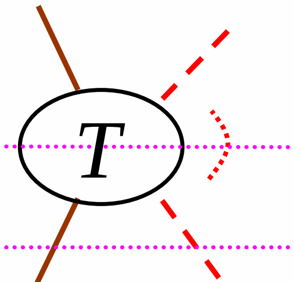
$$+C_0\, N^+N\, N^+N + C_2'\, N^+N\left(\vec{\mathcal{D}}N^+\right)\cdot\vec{\mathcal{D}}N + \ldots$$

other spin/isospin,
more derivatives,
powers of pion mass,
Deltas (Ropers, …),
etc.

- expansion in:

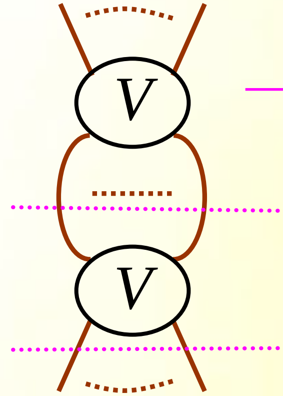
$$\frac{Q}{M_{QCD}} \sim \begin{cases} Q/m_N & \text{non-relativistic} \\ Q/m_\rho, \ldots & \text{multipole} \\ Q/4\pi f_\pi & \text{pion loop} \end{cases} \sim \frac{1}{5}$$

Weinberg '90, '92
Ordonez + v.K. '92
...



$$\frac{1}{\Delta E} \sim \frac{1}{Q}$$

vs.

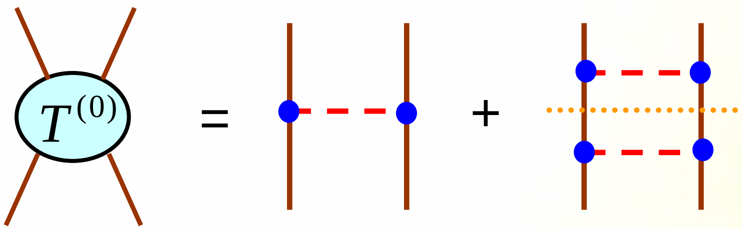


→ A-nucleon irreducible

A-nucleon reducible:

$$\frac{1}{\Delta E} \sim \frac{m_N}{Q^2}$$

infrared enhancement



$$= \mathcal{O}\left(\frac{1}{f_\pi^2}\right)\frac{1}{1-\mathcal{O}\left(\frac{Q}{\mu_\pi}\right)}$$

bound-state pole at

$$\sim \frac{1}{f_\pi^2} f_1\left(Q/m_\pi\right) \quad \sim \frac{1}{f_\pi^2}\frac{Q}{\mu_\pi} f_2\left(Q/m_\pi\right)$$

$$Q \sim \mu_\pi f\left(m_\pi/\mu_\pi\right)$$

$$\sim \frac{4\pi f_\pi}{m_N} f_\pi \approx f_\pi$$

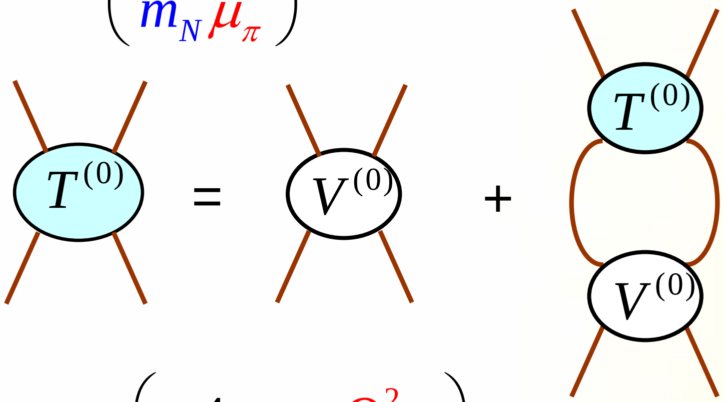
$$-E \sim \frac{\mu_\pi^2}{M_{QCD}} f^2\left(m_\pi/\mu_\pi\right)$$

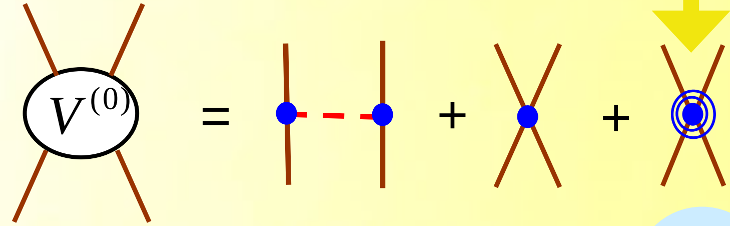
$$M_{nuc} = \mu_\pi \approx f_\pi \ll M_{QCD}$$

Nuclear scale arises in QCD due to spontaneous chiral symmetry breaking

Nogga, Timmermans + v.K. '05
Pavón + Ruiz-Arriola '06
...

LO $\mathcal{O}\left(\frac{4\pi}{m_N \mu_\pi}\right)$

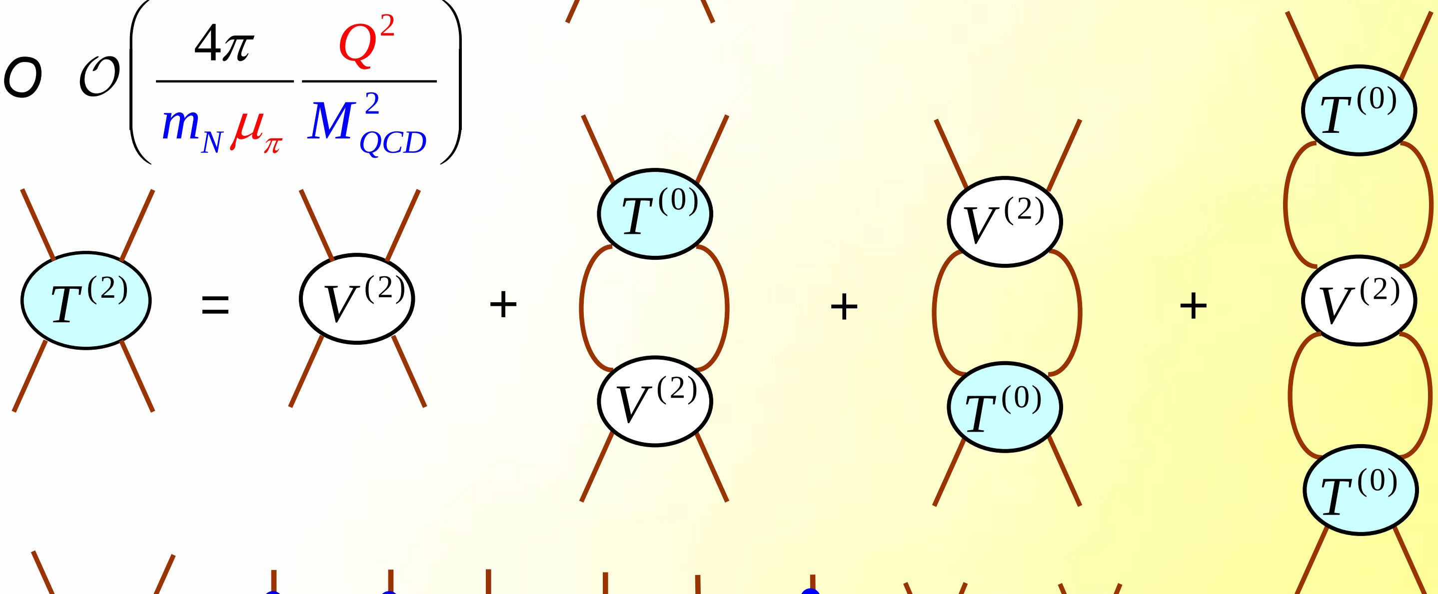


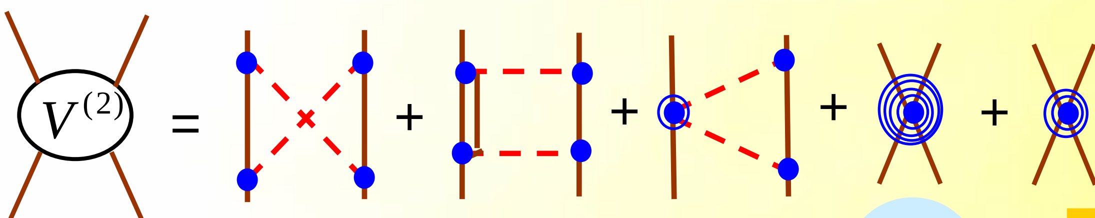


needed to renormalize OPE

$s = 1$
$l \leq 2$

NLO $\mathcal{O}\left(\frac{4\pi}{m_N \mu_\pi}\frac{Q^2}{M_{QCD}^2}\right)$





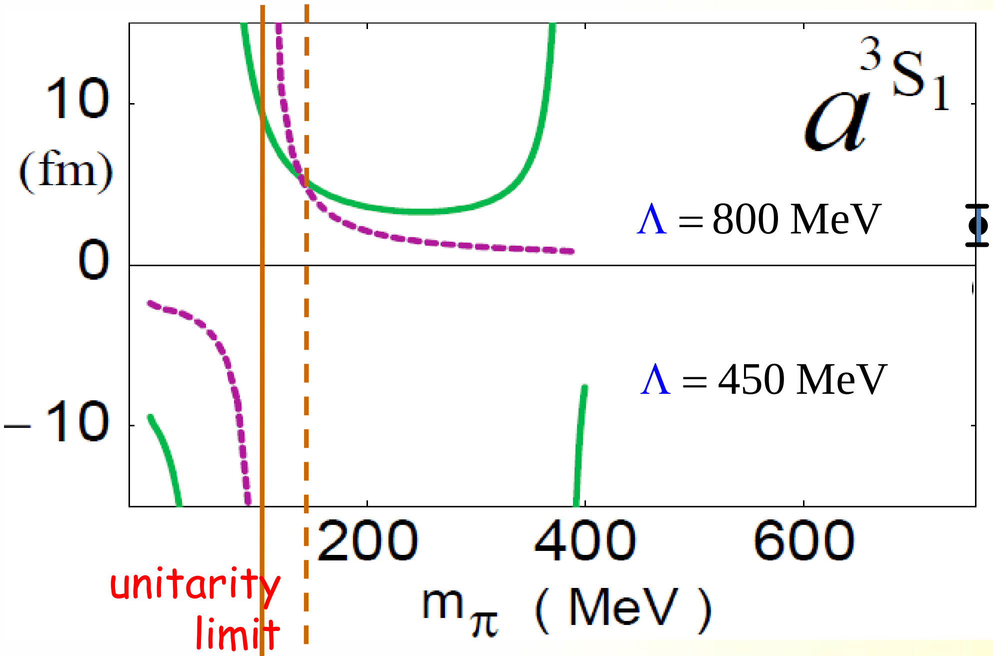
Long + v.K. '07
Pavón '11
...

$s = 1$
$l \leq 2$

enough to renormalize singular perturbations

etc.

Beane, Bedaque, Savage + v.K. '02
Beane + Savage '03
Epelbaum, Gloecke + Meissner '03
...

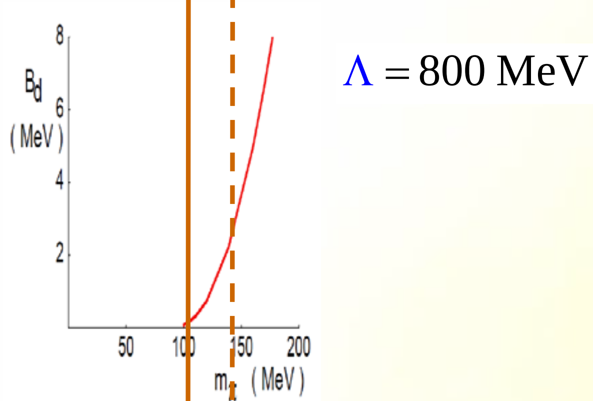


lattice
Beane *et al.* '13

incomplete NLO

square-well regularization

range $1/\Lambda$



$m_\pi^*(M_{QCD})$ $m_\pi \simeq 140$ MeV

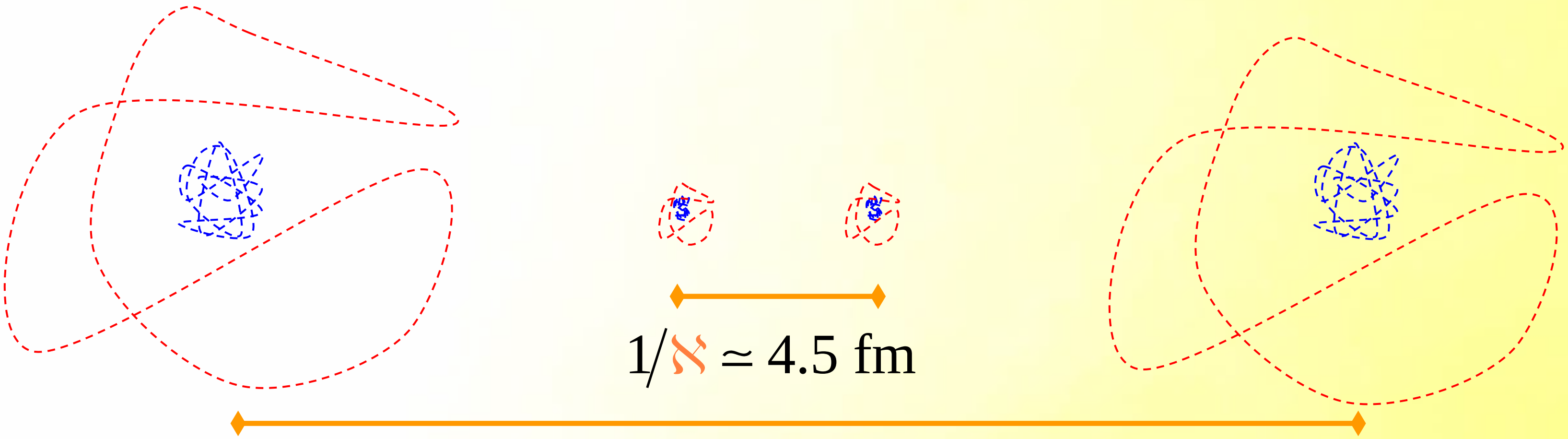
cf. atoms as magnetic field varies

QCD with $m_\pi \simeq 140$ MeV
near a Feshbach resonance
in pion mass

Scale $\aleph \sim \frac{m_\pi - m_\pi^*}{m_\pi^*} \mu_\pi \lesssim \mu_\pi$ emerges

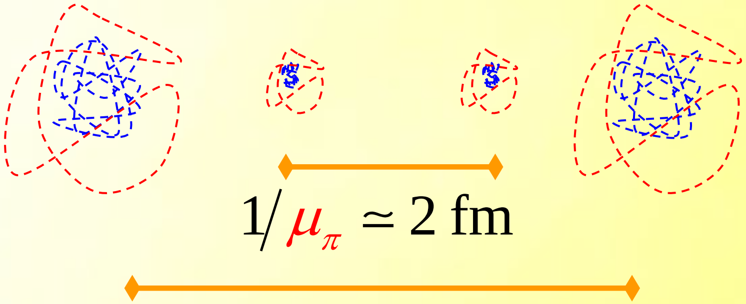
$$Q \sim \aleph \sim \frac{m_\pi - m_\pi^*}{m_\pi^*}\,\mu_\pi < \mu_\pi \lesssim m_\pi$$

e.g.
$m_\pi \simeq 140$ MeV



$$m_\pi > Q \sim \mu_\pi$$

e.g.
$m_\pi \sim 500$ MeV



Pionless EFT

$Q \sim \aleph \ll m_\pi$

- degrees of freedom: nucleons
- symmetries: Lorentz, $\not{P}$, $\not{T}$

$$\mathcal{L}_{EFT} = N^+ \left(i\partial_0 + \frac{\nabla^2}{2m_N} \right) N + C_0 N^+ N N^+ N + D_0 N^+ N N^+ N N^+ N$$

$$+ N^+ \frac{\nabla^4}{8m_N^3} N + C_2 N^+ N N^+ \nabla^2 N + \ldots$$

omitting spin, isospin

- expansion in: $\frac{Q}{M_\pi} = \begin{cases} Q/m_N & \text{non-relativistic} \\ Q/m_\pi, \ldots & \text{multipole} \end{cases}$

Universality: first orders apply also to neutral atoms

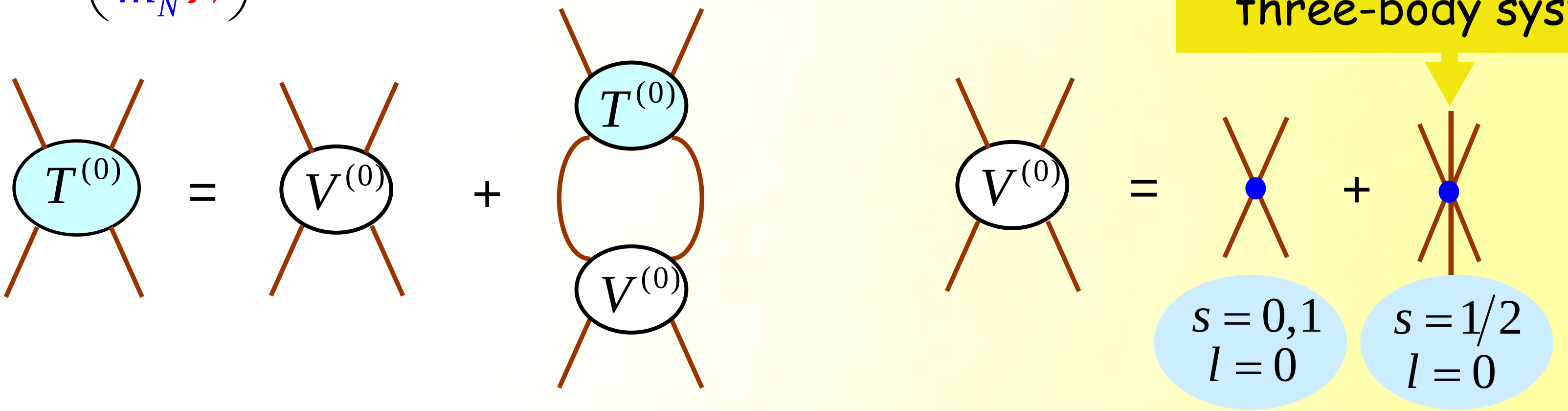
$M_\pi \rightarrow 1/l_{vdW}$ *where* $V(r) = -\frac{l_{vdW}^4}{2mr^6} + \ldots$

Bedaque, Hammer + v.K. '99 '00
Bedaque, Braaten + Hammer '01
...

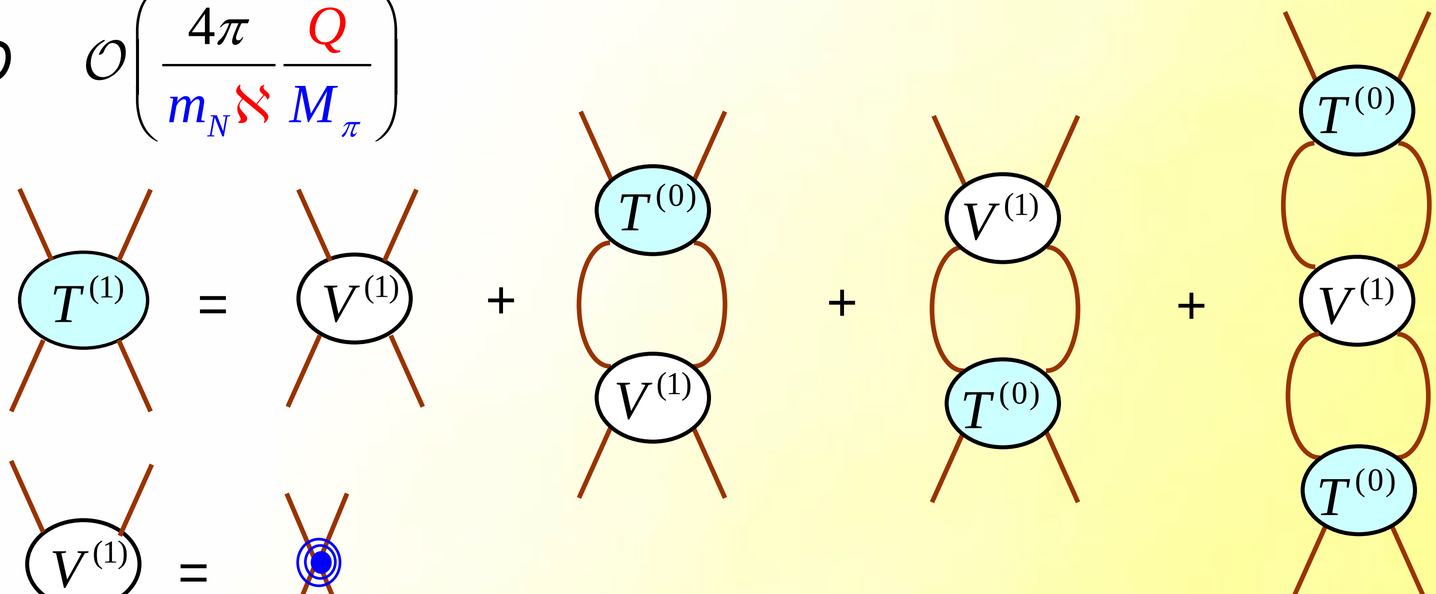
Bedaque + v.K. '97
...

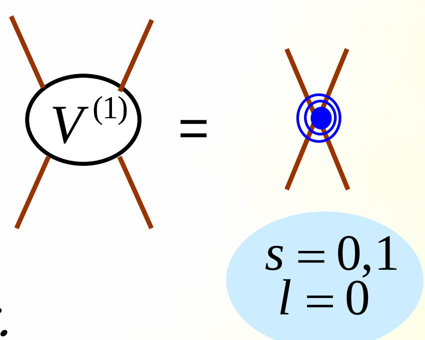
LO $\mathcal{O}\left(\frac{4\pi}{m_N \aleph}\right)$

needed to renormalize three-body system



NLO $\mathcal{O}\left(\frac{4\pi}{m_N \aleph}\frac{Q}{M_\pi}\right)$





enough to renormalize singular perturbations

Kaplan, Savage + Wise '98
v.K. '98
...

etc.

Extrapolation in nucleon number

$m_\pi \ll M_{QCD}$ — Pionful EFT, Pionless EFT

Pionless EFT — $m_\pi \sim M_{QCD}$

\+ <u>any</u> "exact" *ab initio* method

That is,

1) truncate EFT expansion at desired order
2) solve Schrödinger equation for low *A* at fixed cutoff (exactly for LO, subLOs in perturbation theory)
3) fit LECs to selected lattice input
4) solve Schrödinger equation for larger *A*
5) repeat steps 2-4 at other cutoffs
6) obtain observables at large cutoffs

Ab initio methods employed so far

- Effective-Interaction Hyperspherical Harmonics (EIHH) Barnea *et al.* '00' 01
  - ✓ hyperspherical coordinates: hyperradius + 3A-4 hyperangles
  - ✓ model space: hyperangular momentum $K \leq K_{max}$
  - ✓ wavefunction: expanded in antisymmetrized spin/isospin states
  - ✓ effective interaction: Lee-Suzuki projection to subspace "in medium"
  - ✓ extrapolation: K_{max} -> ∞

- Auxiliary-Field Diffusion Monte Carlo (AFDMC) Schmidt + Fantoni '99
  - ✓ integral equation for evolution of wavefunction in imaginary time τ in terms of Green's function
  - ✓ two- and more-body operators linearized by auxiliary fields (Hubbard-Stratonovich transformation)
  - ✓ trial wavefunction probed stochastically with weight given by free Green's function
  - ✓ lowest-energy state with symmetry of trial wavefunction projected into as τ -> ∞

LQCD data

m_π	140	510	805	805
Nucleus	[Nature]	[5]	[6]	[This work]
n	939.6	1320.0	1634.0	1634.0 *
p	938.3	1320.0	1634.0	1634.0
nn	-	7.4 ± 1.4	15.9 ± 3.8	15.9 ± 3.8 *
D	2.224	11.5 ± 1.3	19.5 ± 4.8	19.5 ± 4.8 *
^{3}n	-			-
^{3}H	8.482	20.3 ± 4.5	53.9 ± 10.7	53.9 ± 10.7 *
^{3}He	7.718	20.3 ± 4.5	53.9 ± 10.7	53.9 ± 10.7
^{4}He	28.30	43.0 ± 14.4	107.0 ± 24.2	
^{5}He	27.50			
^{5}Li	26.61			
^{6}Li	32.00			

[5] Yamazaki *et al.* '12
[6] Beane *et al.* '12
[This work] Barnea *et al.* '13

LO pionless fit:

$$m_N\,, C_{01}\,, C_{10}\,, D_1$$

Beane *et al.* '13

$$a^{(^1S_0)} = 2.33^{+0.19+0.27}_{-0.17-0.20}\ \text{fm} \quad , \quad r^{(^1S_0)} = 1.130^{+0.071+0.059}_{-0.077-0.063}\ \text{fm}$$

$$a^{(^3S_1)} = 1.82^{+0.14+0.17}_{-0.13-0.12}\ \text{fm} \quad , \quad r^{(^3S_1)} = 0.906^{+0.068+0.068}_{-0.075-0.084}\ \text{fm}$$

Barnea, Contessi, Gazit, Pederiva + v.K. '13

$$H^{(0)} = -\frac{1}{2m_N}\sum_i \nabla_i^2$$
$$+\frac{1}{4}\sum_{i<j}\left[\left(3C_{10}(\Lambda)+C_{01}(\Lambda)\right)+\left(C_{10}(\Lambda)-C_{01}(\Lambda)\right)\vec{\sigma}_i\cdot\vec{\sigma}_j\right]e^{-\Lambda^2 r_{ij}^2/4}$$
$$+\sum_{i<j<k}\sum_{cyc} D_1(\Lambda)\,\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j\, e^{-\Lambda^2\left(r_{ij}^2+r_{jk}^2\right)/4}$$

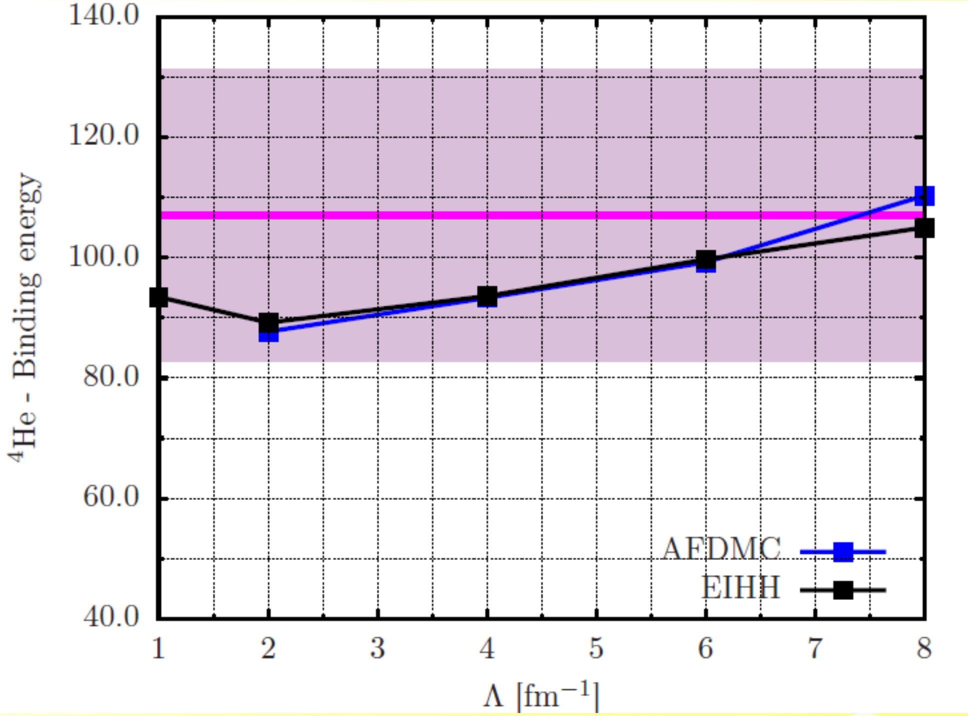
TABLE III. The LO LECs [GeV] for lattice nuclei at $m_\pi = 805$ MeV, as a function of the momentum cutoff Λ [fm^{-1}].

Λ	$C_{1,0}$	$C_{0,1}$	D_1
2	−0.1480	−0.1382	−0.07515
4	−0.4046	−0.3885	−0.3902
6	−0.7892	−0.7668	−1.147
8	−1.302	−1.273	−2.648

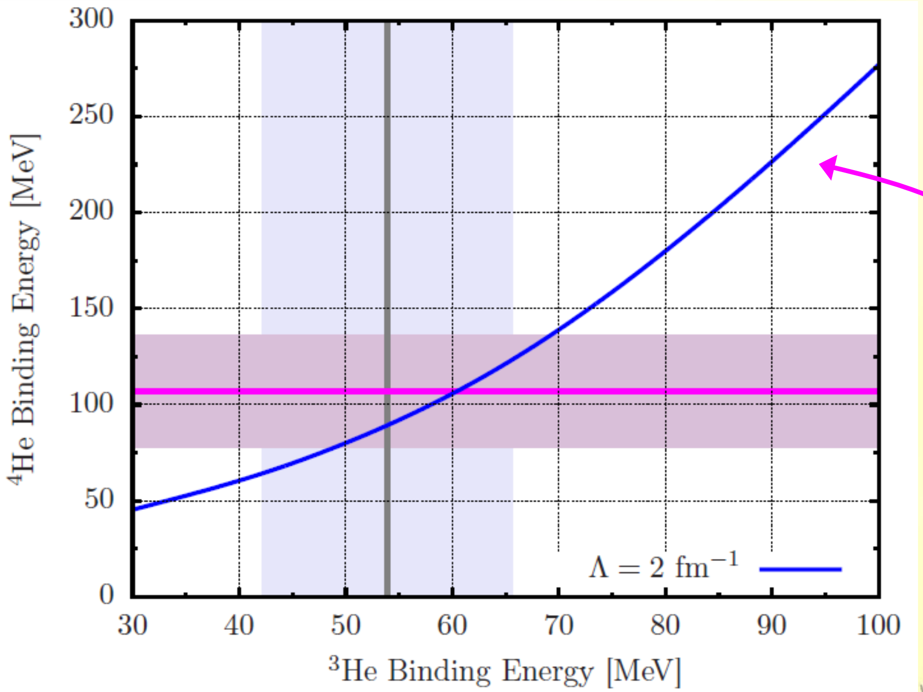
Barnea, Contessi, Gazit, Pederiva + v.K. '13

$$a^{(^3S_1)} = \underbrace{(1.2 \pm 0.5)}\ \text{fm}$$

cutoff variation 2 to 14 fm^{-1}



Tjon line



varying D_1
at fixed C_{SI}

- no excited states for *A* =2,3,4
- no ^{3}n droplet

m_π	140	510	805	805
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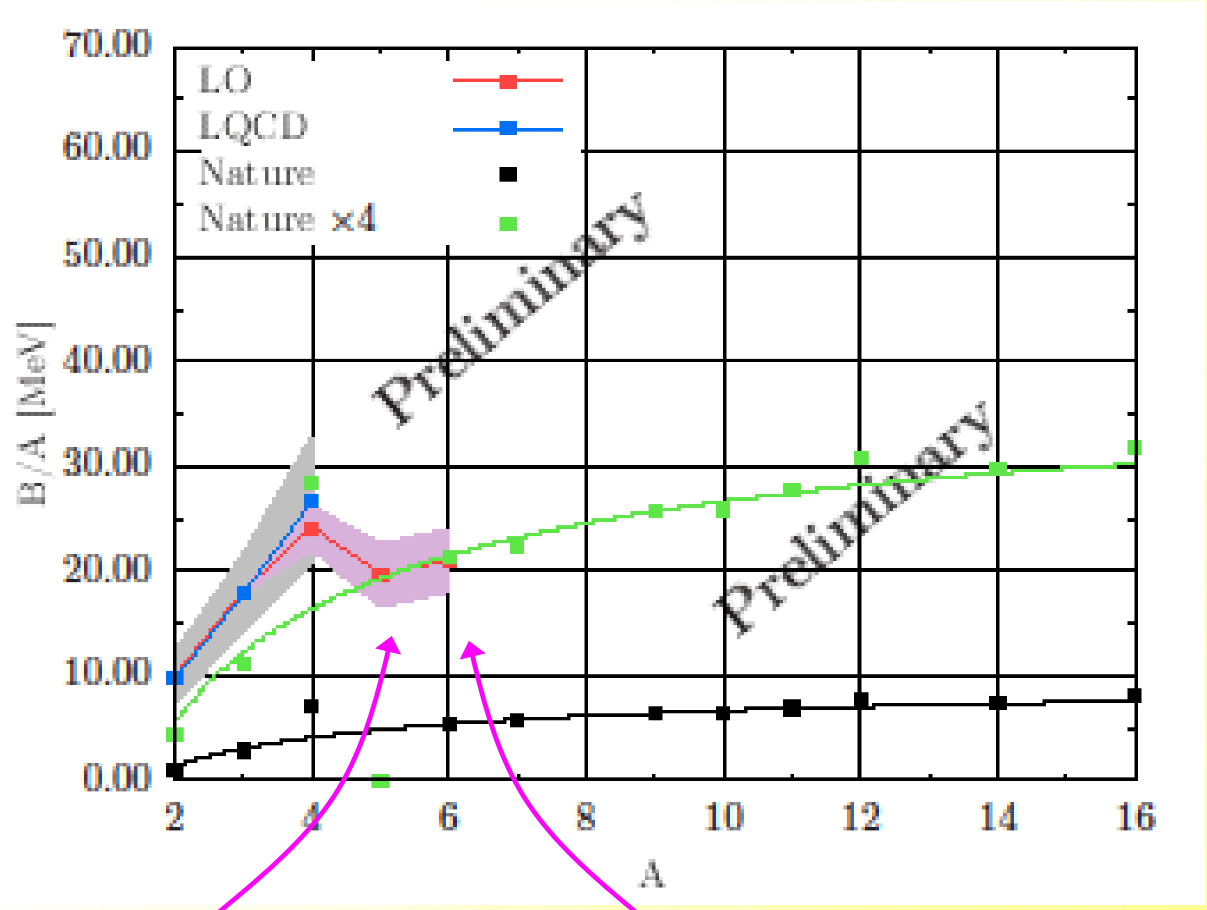
^{4}He	28.30	43.0 ± 14.4	107.0 ± 24.2	89 ± 36
^{5}He	27.50			98 ± 39
^{5}Li	26.61			98 ± 39
^{6}Li	32.00			122 ± 50

predictions

[5] Yamazaki *et al.* '12
[6] Beane *et al.* '12
[This work] Barnea *et al.* '13

Barnea, Contessi, Gazit, Pederiva + v.K. '13



$$B_5 \approx B_4$$

A=5 gap persists!?

$$\frac{B_6}{6} \approx \frac{B_4}{4}$$

nuclear saturation survives!?

What next?

- NLO at m_π = 805 MeV
- LO at m_π = 510 MeV
- larger A with AFDMC
- chiral EFT at lower pion masses when available
- …

Conclusion

- EFT is constrained *only* by symmetries and thus can be matched onto lattice QCD

- EFT allows controlled extrapolations in both pion mass *and* nucleon number

- World at large pion mass might be *just* a denser version of ours