Structure and composition of the outer crust

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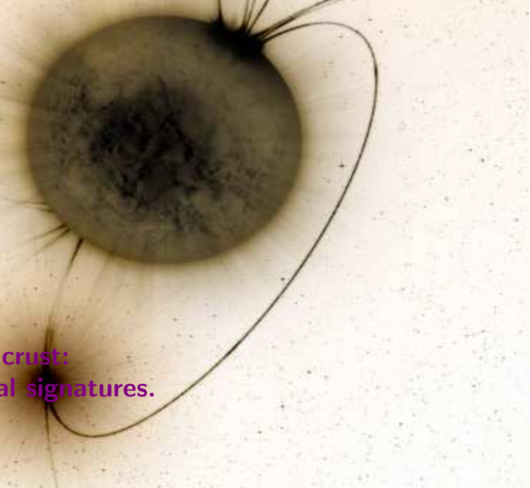


FUSTIPEN

Structure of the neutron star crust: experimental and observational signatures.

May 27th 2014.

“The composition of neutron stars is important (impacts on observations and predictions) **yet the description of these ultra-compact objects remains one of the biggest challenges facing nuclear and particle physics today”**

March 28th, 2013
David Lunney, Stéphane Goriely,
Susanne Kreim, and Robert Wolf

Relevance of the crust on the star evolution and dynamics

- The crust **separates** neutron star **interior** from the **photosphere** (X-ray radiation).
- The **thermal conductivity** of the crust is relevant for determining the **relation** between observed **X-ray flux** and the **temperature of the core**.
- **Electrical resistivity** of the crust might be important for the evolution of neutron star **magnetic** field.
- **Conductivity** and **resistivity** depend on the **structure and composition** of the crust.

Source: Pawel Haensel, 2001

Relevance of the crust on the star evolution and dynamics

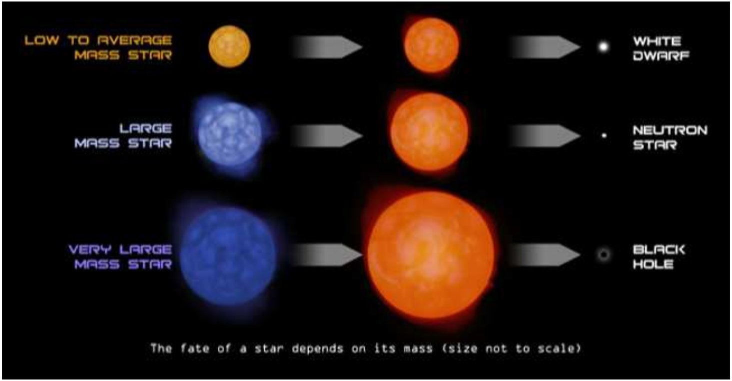
- **Neutrino emission** from the crust may significantly **contribute to total neutrino losses** from stellar interior (in some cooling stages).
- A **crystal lattice** (solid crust) is needed for modelling **pulsar glitches**, enables the excitation of **toroidal modes** of oscillations, can suffer elastic stresses...
- Mergers (binary systems that merge) may **enrich** the interstellar medium with **heavy elements**, created by a rapid neutron-capture process.
- In accreting neutron stars, instabilities in the **fusion light elements** might be responsible for the phenomenon of **X-ray bursts**.

Source: Pawel Haensel, 2001

Formation:

- Any star with $M > 10M_{\rm sun}$ may potentially become a **neutron star** in its final stage of evolution.
- When all **nuclear fuel is burned**, it produces an **iron-rich core** that collapses and can only be **supported by degeneracy pressure**.
- Core collapse and the **temperature raises** allowing for the **breaking of iron into alpha particles** (photo-disintegration).
- To decrease the energy, **electrons and protons** combine to form **neutrons** releasing energy in form of **neutrinos**.
- At nuclear densities (few times 10^{14} g cm^3), **neutron degeneracy** pressure compensates the **gravitational collapse**.
- **The remnant of this process is a neutron star**
- Angular momentum should be conserved $\rightarrow$ the **rotation period decreases in the process** as $\sim (R_{\rm ns}/R_{\rm pro})^2$
- Magnetic flux conservation $\rightarrow$ the **magnetic field increases** as $\sim (R_{\rm ns}/R_{\rm pro})^2$ **(not enough!!)**

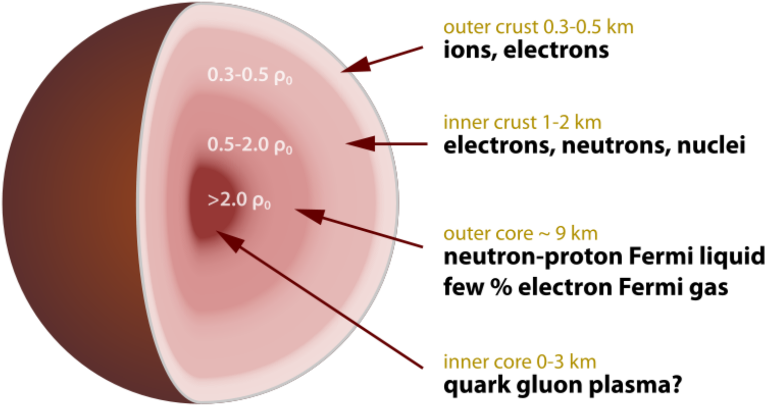
The fate of a star depends on its mass:



For a neutron star of mass $M \approx 1.4 M_{\text{Sun}}$

R (Km)	10	g/g_{Earth} (surface)	10^{11}
$\bar{\rho}$ (g/cm^3)	10^{14-15}	P (dyn/cm^2)	$0 - 10^{35}$
v_{scape}/c	0.5	B (G)	$10^{12} - 10^{15}$

Structure & Composition:



Crust: **"...cold, catalysed matter in which increasingly heavy and neutron-rich nuclides (resulting from electron capture) exist in a state of equilibrium for beta-decay processes..."**

Baym, Pethick and Sutherland, 1971

Outer crust

- $\rho_{e^-\text{ion.}} \sim 10^4 \text{g}/\text{cm}^3 \rightarrow \rho_{\text{drip}} \sim 4 \times 10^{11} \text{g}/\text{cm}^3$
- it is organized into a **Coulomb lattice** of **neutron-rich nuclei** embedded in a relativistic **uniform electron gas**
- $T \sim 10^6$ **K** $\rightarrow$ we can treat **nuclei and electrons at** $T = 0$ **K**
- At the **lowest densities**, the electronic contribution is negligible so the Coulomb lattice is populated by ^{56}**Fe** nuclei.
- As the **density increases**, the electronic contribution becomes important, it is **energetically advantageous** to lower its electron fraction by $e^- + (N, Z) \rightarrow (N + 1, Z - 1) + \nu_e$
- As the **density continues to increase**, the Coulomb lattice is made of more and **more neutron-rich nuclei** until the critical **neutron-drip** density is reached.

Model:

Penetrating from the atmosphere into the outer crust, a continuous value of the pressure is required to ensure **hydrostatic equilibrium** $\rightarrow$ optimise **Gibbs free energy** per baryon

$$g(A, Z; P) = \frac{E(A, Z; P) + PV}{A} = \varepsilon(A, Z; P) + \frac{P}{n}$$

$$\varepsilon(A, Z; P) = \varepsilon_n(A, Z) + \varepsilon_e(A, Z; P) + \varepsilon_l(A, Z; P)$$

- **Electron gas:** $\varepsilon_e(A, Z; P)$
- **Coulomb lattice:** $\varepsilon_l(A, Z; P)$
- **Nuclear masses:** $\varepsilon_n(A, Z)$

Model: electron energy ($B = 0$ G)

The electronic contribution at the densities of interest ($\rho \gtrsim 10^4 \mathrm{g/cm^3}$) can be modelled as a degenerate free Fermi gas

$$\varepsilon_e(A, Z; n) = \frac{\mathscr{E}_e}{n} = \frac{1}{n\pi^2} \int_0^{p_{\mathrm{F}e}} p^2 \sqrt{p^2 + m_e^2}\, dp$$

$$\varepsilon_e(A, Z; n) = \frac{m_e^4}{8\pi^2 n} \left[x_{\mathrm{F}} y_{\mathrm{F}} \left(x_{\mathrm{F}}^2 + y_{\mathrm{F}}^2 \right) - \ln(x_{\mathrm{F}} + y_{\mathrm{F}}) \right]$$

where dimensionless Fermi momentum and energies have been defined as follows:

$$x_{\mathrm{F}} \equiv \frac{p_{\mathrm{F}e}}{m_e} \quad \text{and} \quad y_{\mathrm{F}} \equiv \frac{\epsilon_{\mathrm{F}e}}{m_e} = \sqrt{1 + x_{\mathrm{F}}^2}$$

Model: electron energy (uniform B)

The electron motion is quantised into discrete Landau levels (ν) in the plane orthogonal to the magnetic field direction,

$$E(\nu, p_z)^2 = p_z^2 + m_e^2(1 + 2\nu B_\star)$$

where $B_\star = \frac{B}{B_c}$ and $B_c = m_e^2/e \sim 10^{13}$ G

The energy due to the interaction with the magnetic field cannot exceed the Fermi energy $\mu_e \to \nu_{\max}$ can be calculated as

$$m_e^2\,(1 + 2\nu_{\max} B_\star) = \mu_e^2 \implies \nu_{\max} = \frac{1}{2B_\star}\left(\frac{\mu_e^2}{m_e^2} - 1\right)$$

The energy density $n_e \varepsilon_e$,

$$n_e \varepsilon_e = \frac{B_\star m_e^4}{2\pi^2} \sum_{\nu=0}^{\nu_{\max}} g_\nu (1 + 2\nu B_\star)\, \tau_+\left[\frac{x_e(\nu)}{\sqrt{1+2\nu B_\star}}\right]$$

where $x_e(\nu) = \frac{p_e^F(\nu)}{m_e}$ and $\tau_\pm(x) = \frac{1}{2}x\sqrt{1+x^2} \pm \frac{1}{2}\ln\left(x + \sqrt{1+x^2}\right)$

Model: lattice energy ($B = 0$ G)

The calculation of the potential energy of the Coulomb lattice consists of divergent contributions that must be cancelled as required by the overall charge neutrality of the system.

It has been shown that the most energetically favourable configuration is a crystallisation into a body-centred cubic lattice, with an energy

$$\varepsilon_l(A, Z, n) = -(1.81962)\frac{(Ze)^2}{a} = -C_{\rm bcc}\frac{Z^2}{A^{4/3}}p_F$$

where

- $n_e = nZ/A$ (neutrality) and $p_F \equiv (3\pi^2 n)^{1/3}$
- a is the lattice constant, for a bcc $a/2R_{\rm N} = (\pi/3)^{1/3}$
- $C_{\rm bcc} = 3.40665 \times 10^{-3}$

Model: lattice energy

Some considerations

- The Bohr-van Leeuwen theorem[1] ensures that the lattice energy is not affected by the magnetic field.
- bcc lattice may not be the body-centred cubic one[2]: intense magnetic fields cause an anisotropic screening of the Coulomb force by the electron gas leading to Friedel oscillations in the ion-ion potential.
- In some circumstances, it has been shown that finding interpenetrating cubic lattices made of different ions is favourable with respect to a body-centred-cubic lattice of any other single ion[3].

Since the lattice contribution to the energy is small, we neglected such effects.

1) The theorem applies to an isolated system that cannot rotate. If there is only one state of thermal equilibrium in a given temperature and field, and the system is allowed to return to equilibrium after a field is applied, then there will be no magnetisation.

2) P. F. Bedaque, S. Mahmoodifar, 2013

3) C. J. Jog, R. A. Smith, 1982

Model: nuclear masses ($B = 0$ G)

Example: the liquid-drop mass formula may be written in the absence of pairing correlations

$$\varepsilon_n(x, y) = m_p y + m_n(1 - y) - a_{\rm v} + \frac{a_{\rm s}}{x} + a_{\rm c} x^2 y^2 + a_{\rm a}(1 - 2y)^2$$

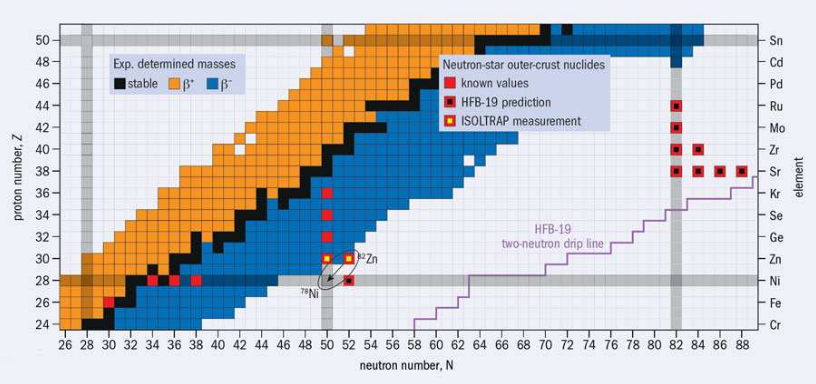
where $x \equiv A^{1/3}$ and $y \equiv Z/A$ is the proton (or electron) fraction.

While simple, the liquid-drop mass formula captures the essential physics of the outer crust: **a competition between an electronic density that drives the system towards more neutron-rich nuclei and a nuclear symmetry energy that opposes such a change.**

However, it is a macroscopic model **unable to describe** important shell effects such as **magicity**.

Magic numbers

Realistic model extrapolations predict that nuclides present in the crust cluster around the magic neutron numbers, $N = 50$ and 82, only high-precision measurements in these exotic nuclei can confirm this (R N Wolf *et al.*, 2013)



Are then nucleides in the region of Sr and Zr with *N* larger than 82 magic? 82 keeps its magicity? (Difficult to be directly measured)

Model: nuclear masses ($B = 0$ G)

For this reason, the crust should be ultimately computed using sophisticated mass formulas in which will be desirable to also propagate model uncertainties. In our case,

- Two of the models (Möller&Nix, 1997 and Duflo&Zucker, 1995) are based on sophisticated mass formulas calibrated to thousands of available experimental masses throughout the periodic table. **Most accurate to date.**
- Two microscopic models with very different behaviours of the symmetry energy contribution to the nuclear masses. Both are based on an effective covariant Lagrangian (Lalazissis *et al.*, 1999 and Todd-Rutel *et al.*, 2005). **To analyse the dependence on the symmetry energy.**

Model: nuclear masses (uniform B)

A covariant effective formulation with nucleons and mesons as the effective degrees of freedom embedded in a uniform magnetic field have been solved consistently, assessing in a realistic way the effect of intense magnetic fields on the nuclear structure (D. Peña Arteaga *et. al.*, 2011).

The new terms: **the coupling of the proton orbital motion with the external magnetic field**,

$$\mathcal{L}_{BO} = -e\bar{\psi}\gamma^{\mu}A_{\mu}^{(e)}\psi,$$

and the **coupling of protons and neutrons intrinsic dipole magnetic moments with the external magnetic field**

$$\mathcal{L}_{BM} = -\bar{\psi}\chi_{\tau_3}^{(e)}\psi,$$

where $\chi_{\tau_3}^{(e)} = \kappa_{\tau_3}\mu_N \frac{1}{2}\sigma_{\mu\nu}F^{(e)\mu\nu}$, $\sigma_{\mu\nu} = \frac{i}{2}\left[\gamma_{\mu},\gamma_{\nu}\right]$ and $\mu_N = e\hbar/2m$ is the nuclear magneton and $\kappa_n = g_n/2$, $\kappa_p = g_p/2 - 1$ with $g_n = -3.8263$ and $g_p = 5.5856$ are the intrinsic magnetic moments of protons and neutrons. Interactions with the external magnetic field are marked the superscript (e).

Model: nuclear masses (uniform B)

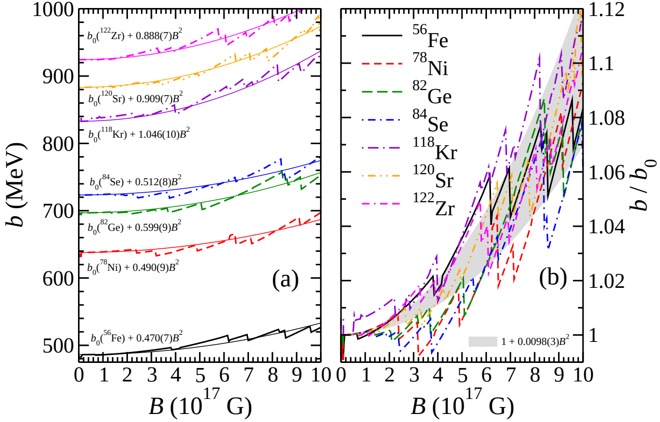
The effects that the coupling of protons and neutrons to an external magnetic field can be classified as follows:

- *Neutron paramagnetism*: induces a relative shift of levels with neutron spins directed along the magnetic field. Since g_n is negative, configurations with the spin anti-parallel to B are energetically favoured.
- *Proton paramagnetism*: since g_p is positive, configurations where the proton spin is parallel to B are favoured.
- *Proton orbital magnetism*: favours configurations where the proton angular momentum projection is oriented along the direction of the external magnetic field.

It is thus expected that the magnetic field effect on the single-particle structure is more pronounced for protons than for neutrons.

DD-ME2: nuclear masses (uniform B)

Qualitatively, **the total binding energy of nuclei present in the outer crust is not predicted to increase by more than a 10%**.



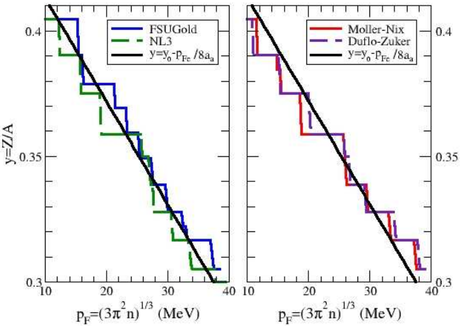
Note the large value of B[1] to produce a macroscopic effect on the binding energies!!

1) Virial theorem shows that the highest value allowed is found around 10^{18} G (Lai&Shapiro, 1991).

Results

Results: toy model for the outer crust

Assumptions: Liquid-drop model with no pairing nor B is used to describe nuclear masses and $m_e/p_{\mathrm{F}e} \to 0$



A good approx to the first order solution in p_{F_e}: $y(p_{F_e}) = y_0 - \frac{p_{F_e}}{8a_\mathrm{a}}$ where y_0 is the solution at $n = 0 \to {}^{56}$Fe.

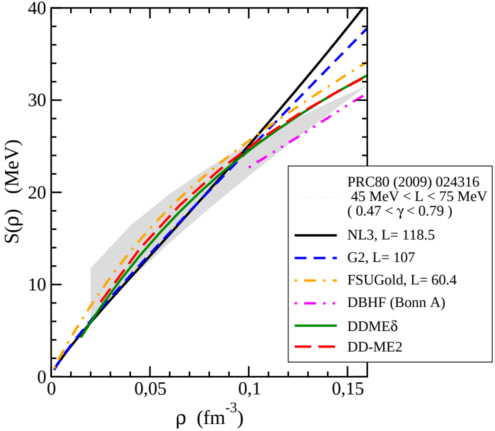
Results: toy model for the outer crust

- the optimal value of y_0 emerges from a **competition** between **Coulomb and asymmetry** terms ($n = n_e/y \sim 0$, no electronic contribution)
- Coulomb drives Z to zero ($y_0 \to 0$) and asymmetry drives $N = Z$ ($y_0 \to 1/2$)
- The evolution of y with the density is controlled by $p_{\mathrm{F}e}/a_a \to$ **the larger the (a)symmetry energy, the slower the evolution away from y_0**
- $8a_\mathrm{a} \approx 100$ MeV, significantly larger than the electronic Fermi momentum over the entire region of interest (first-order approximation reasonable as seen in the previous figure)

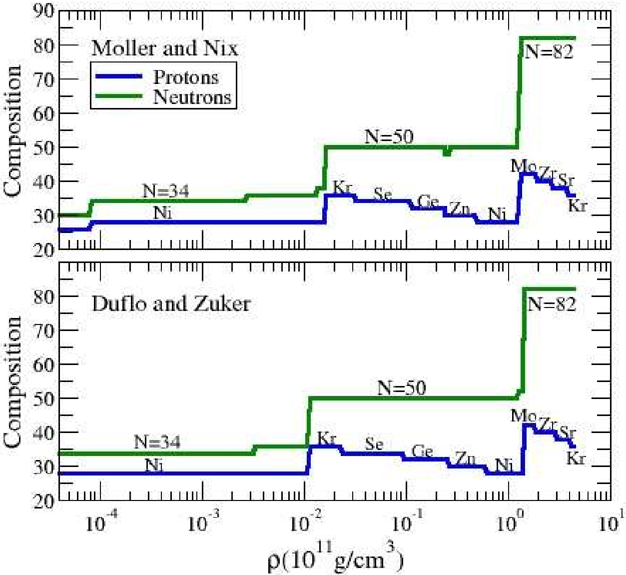
Essential physics: competition between an electronic contribution (favours neutron-rich nuclei) and the nuclear symmetry energy (favours symmetric nuclei)

Symmetry energy within realistic models

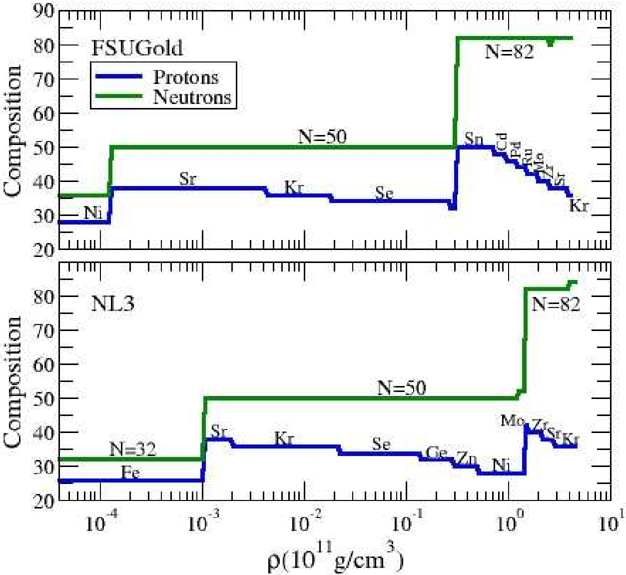
While the electronic contribution well known, the density dependence of the symmetry energy is not:



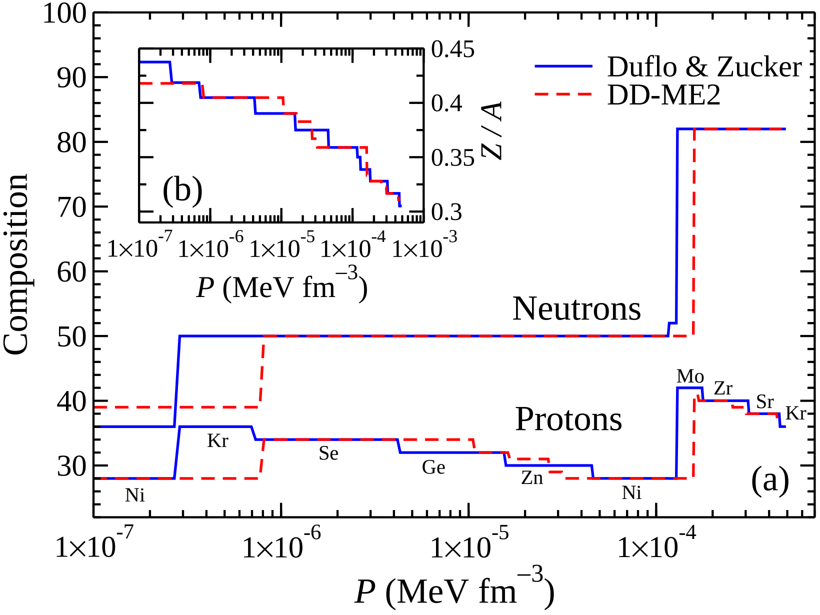
Results: realistic mass models ($B = 0$ G)



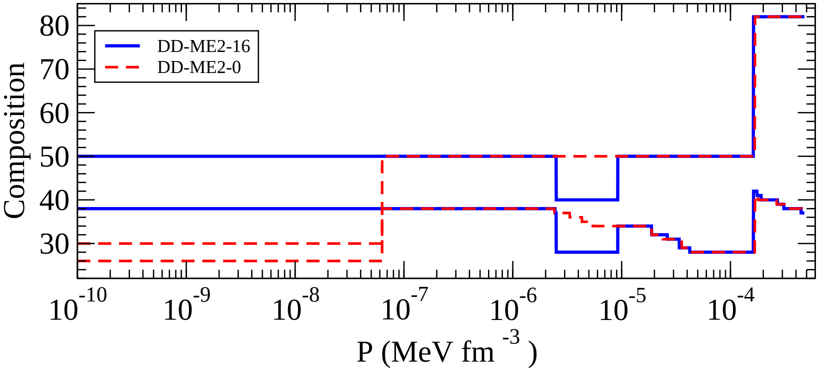
Results: impact of the symmetry energy ($B = 0$ G)



Results: comparison between models ($B = 0$ G)



Results: impact of a uniform magnetic field of 10^{16} G

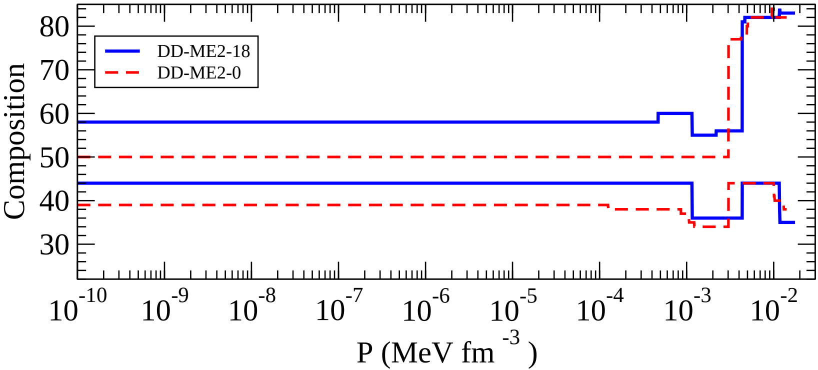


DD-ME2-16 (B effects on nuclei and electrons included)

DD-ME2-0 (B effects ONLY on electrons included)

PRELIMINARY: ^{88}Sr stable nucleus, $b = 768.468(1)$ **MeV**

Results: impact of a uniform magnetic field of 10^{18} G

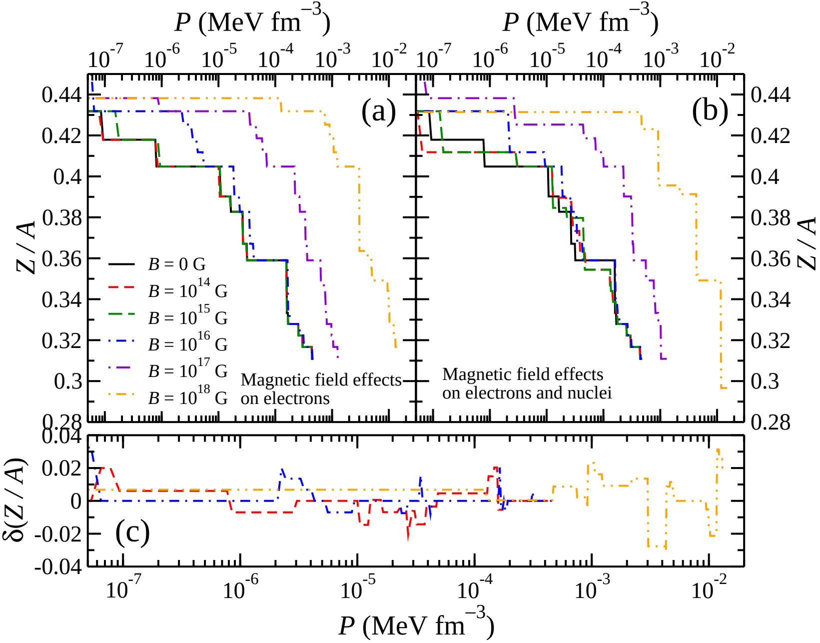


DD-ME2-18 (B effects on nuclei and electrons included)

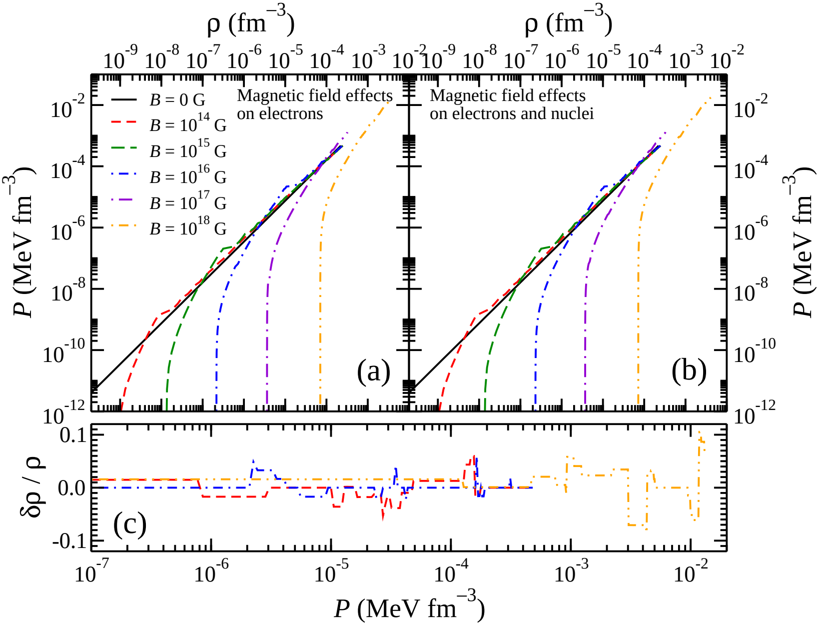
DD-ME2-0 (B effects ONLY on electrons included)

PRELIMINARY ^{102}Ru stable nucleus, $b = 877.954(1)$ **MeV**

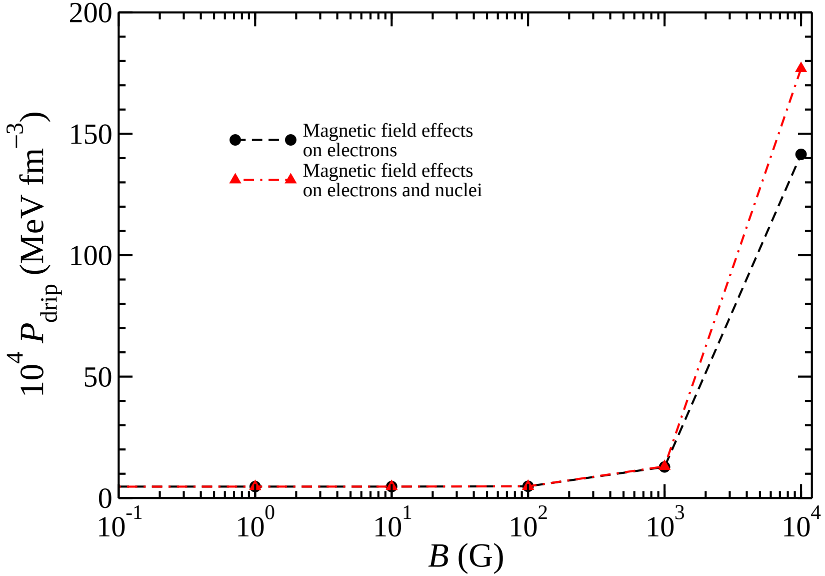
Results: impact of a uniform magnetic field



Results: impact of a uniform magnetic field



Results: impact of a uniform magnetic field



Conclusions

- The outer crust may provide observational information related with the properties of exotic nuclei and of the nuclear equation of state at sub-saturation densities.
- Nuclear physics input is crucial for an accurate understanding of the structure and composition of the outer crust.
- Electron gas contributions to the Gibbs free energy of the outer crust neglecting and accounting for a uniform B are well under understood
- Possible anisotropies in the electron gas may produce Friedel oscillations in the ion-ion potential. This may favour a different lattice configuration. In certain conditions may be favourable to mix nucleids in the the lattice. The changes in energy are thought not to qualitatively affect the composition and structure of the outer crust. If accurate predictions are needed (assuming nuclear input has no extrapolation errors) one would need to include such an effect.

Conclusions

- The nuclear symmetry energy is not well determined at sub-saturation energies, this implies that some changes might be expected in the composition and structure of the outer crust depending on the model. May observational data shed some light on that?
- The most accurate mass models available in the literature, predict very similar outer crusts compositions and structure pointing to the robustness of those predictions (at least for modest extrapolations).
- The possible existence of an intense magnetic field clearly change the composition and structure of the outer crust: on electrons, it delays the appearance of plateaus in the composition shifting also the neutron drip line and, on nuclei, it may change the most stable species in the lattice. Intense magnetic fields favours a uniform crusts, mostly populated by one or two nuclear species. Surprisingly, the predicted nuclei are very close to the valley of stability!!

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Thank you for your attention!