Spontaneous fission: impact of pairing and the least action approach

Samuel A. Giuliani [†‡], L. M. Robledo[‡], R. Rodríguez-Guzmán[§]

[†]Technische Universität Darmstadt, Darmstadt, Germany
[‡]Universidad Autónoma de Madrid, Madrid, Spain
[§]Rice University, Houston, Texas, USA

October 13-14, 2014

Challenges in the microscopic description of nuclear large amplitude collective dynamics
GANIL – Caen

Outline

1. Introduction
   - Microscopic approach
   - Fission observables
2. Impact of pairing
   - The energy-density functionals
   - Pairing and spontaneous fission lifetimes
3. Dynamic fission path
   - Minimizing the action
   - Dynamic vs static approach
4. Conclusions

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The fission process of heavy and superheavy nuclei

- Important for the r-process nucleosynthesis: fission cycling is a mechanism to obtain a robust r-process.

- Useful to study the influence of magic numbers in nuclear structure.

- Hypothetical island of stability?

- Nuclei far from stability: theoretical models required!

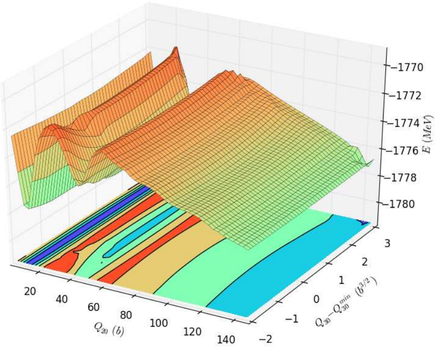
A microscopic approach: the Density Functional Theory

Two main ingredients:

- Evolution of the energy from the ground state to the scission point:
  - HFB theory with constrained field,
  - effective interactions (Skyrme, Gogny, RMF, others EDF. . .).
- Collective inertias associated to the fission path:
  - several theories (ATDHFB vs GCM),
  - different approximations (exact, cranking approximation, perturbative cranking approximation. . .)

Fission observables

- Spontaneous fission lifetimes:
  - computed using the WKB formula.
- Parameters defining the potential energy surface:
  - inner and outer fission barrier heights (model dependent),
  - isomer excitation energy.
- Fission fragments distribution:
  - phenomenological description



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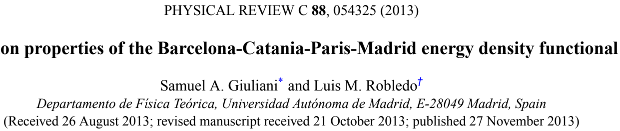
The energy-density functionals



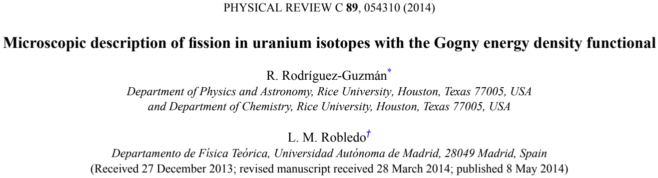
PHYSICAL REVIEW C **88**, 054325 (2013)

Fission properties of the Barcelona-Catania-Paris-Madrid energy density functional

Samuel A. Giuliani* and Luis M. Robledo†
Departamento de Física Teórica, Universidad Autónoma de Madrid, E-28049 Madrid, Spain

(Received 26 August 2013; revised manuscript received 21 October 2013; published 27 November 2013)

- Density functional inspired in microscopic EoS,
- nuclear matter properties mapped onto finite nuclei models using LDA,
- good reproduction at masses (rms $\sim$ 1.6 MeV for even–even nuclei).



PHYSICAL REVIEW C **89**, 054310 (2014)

Microscopic description of fission in uranium isotopes with the Gogny energy density functional

R. Rodríguez-Guzmán*
Department of Physics and Astronomy, Rice University, Houston, Texas 77005, USA
and Department of Chemistry, Rice University, Houston, Texas 77005, USA

L. M. Robledo†
Departamento de Física Teórica, Universidad Autónoma de Madrid, 28049 Madrid, Spain

(Received 27 December 2013; revised manuscript received 28 March 2014; published 8 May 2014)

- Finite range density dependent interaction,
- several fits including fission data (D1S) or even-even masses (D1M).

Spontaneous fission lifetimes

Semiclassical approach given by the WKB formalism:

$$t_{\mathsf{sf}} = 2.86 \times 10^{-21}(1 + \exp(2S))\,.$$

Action along the (multidimensional) fission path:

$$S = \int_a^b ds \sqrt{2 \times B(s)\left[\,V(s) - (E_{\mathrm{GS}} + E_0)\right]}\,.$$

Collective inertias:

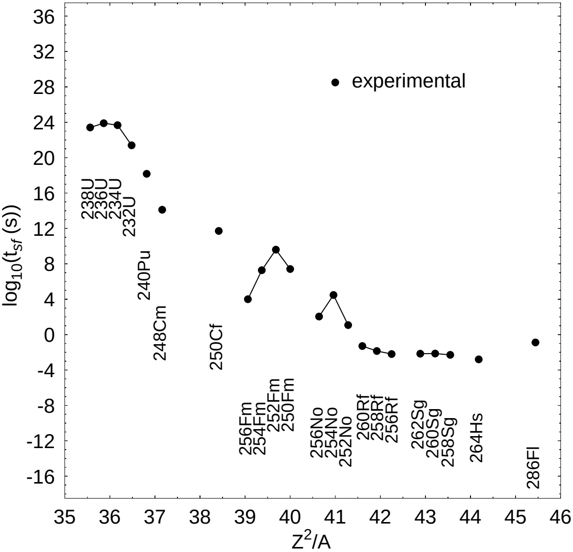
$$B(s) = \sum_{i,j} B_{ij}\frac{dq_i}{ds}\frac{dq_j}{ds}\,.$$

pairing correlations strengths: impact on t_{sf}?

η: multiplicative factor of the pairing gap field

Spontaneous fission lifetimes (BCPM results)

● Exp: Xiaojun Bao et al., Nucl. Phys. **A906**, 1–13 (2013).



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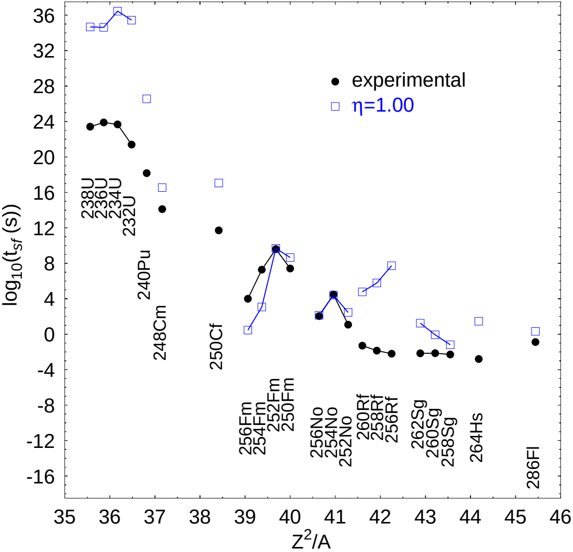

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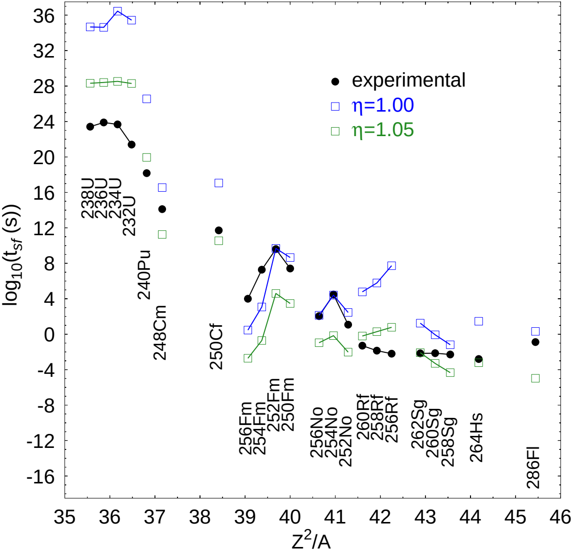

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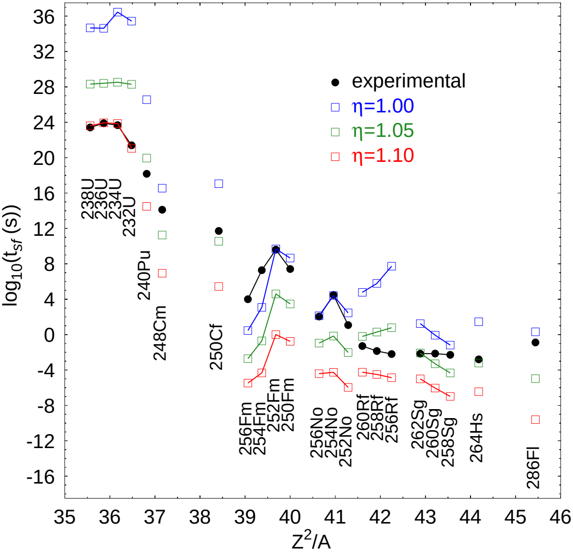
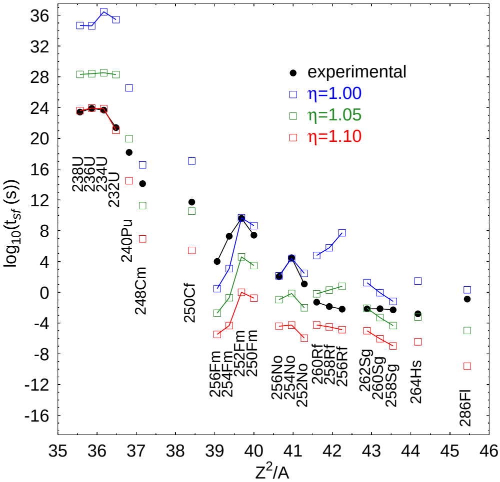


Pairing and spontaneous fission lifetimes



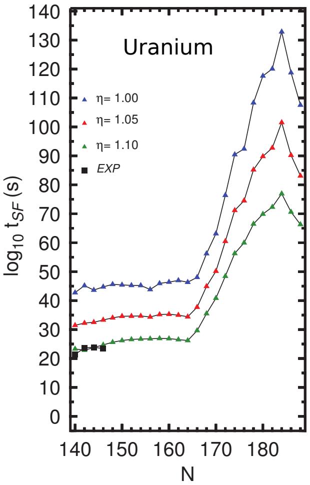
Pairing strength η

- Strong impact on collective inertias $B(Q_{20})$.
- Large t_{SF} variation (up to 13 OM).

Model predictions

Experimental trend with mass number well reproduced: confidence for extrapolation to exotic nuclei.

Pairing and spontaneous fission lifetimes



Pairing strength η

- Strong impact on collective inertias $B(Q_{20})$.
- Large t_{SF} variation (up to 13 OM).
- Similar results obtained with the Gogny force.

Model predictions

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The origins of the dynamic approach

- 1972 – Funny Hills paper (Brack et al.): spontaneous fission lifetimes computed using the least action principle,

$$S = \int_a^b ds \sqrt{2 \times B(s) \left[E_{\mathsf{rot}}(s) - (E_{\rm GS} + E_0)\right]} \, .$$

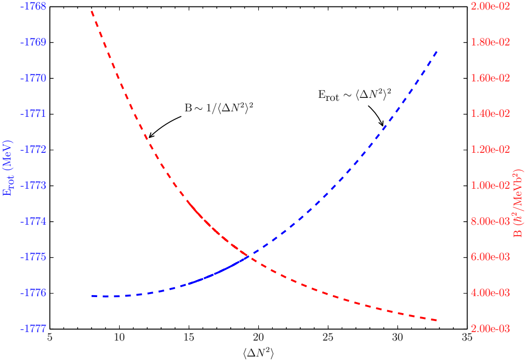
- 1974 – L.G. Moretto and R.P. Babinet: pairing gap Δ as degree of freedom of a simple fission model,

$$B \sim \frac{1}{\Delta^2}; \quad V(s) = V_0(s) + 2g(\Delta - \Delta_0)^2 \, .$$

- Our idea: find the minimum of the action using $\Delta N^2 = N^2 - \langle N^2 \rangle$ as collective degree of freedom.

Minimizing the action: $B(s)$ vs E_{rot}

$$S = \int_a^b ds \sqrt{2 \times B(s) \left[E_{\text{rot}}(s) - (E_{\text{GS}} + E_0) \right]}$$



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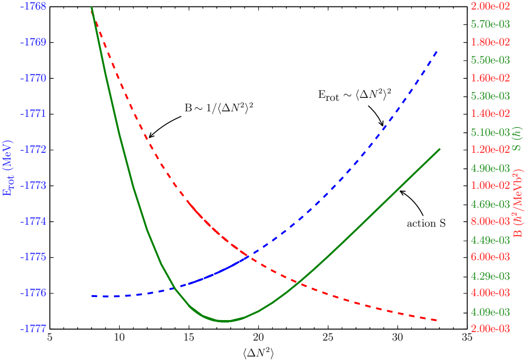

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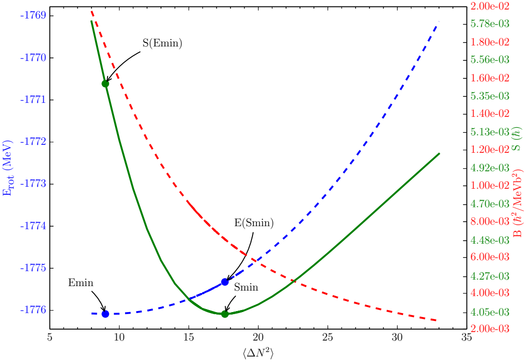

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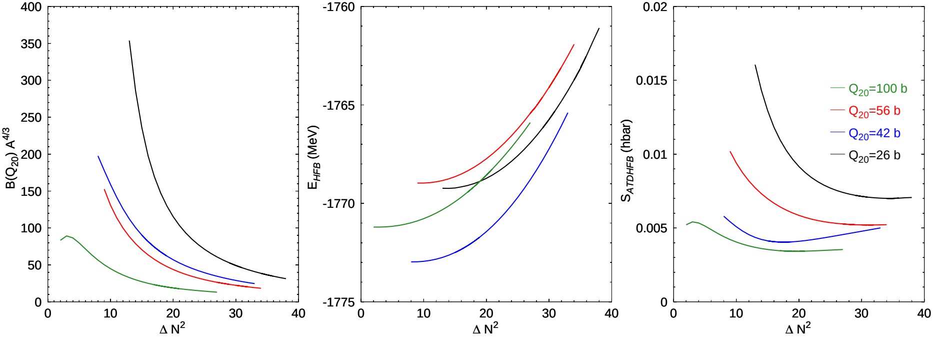
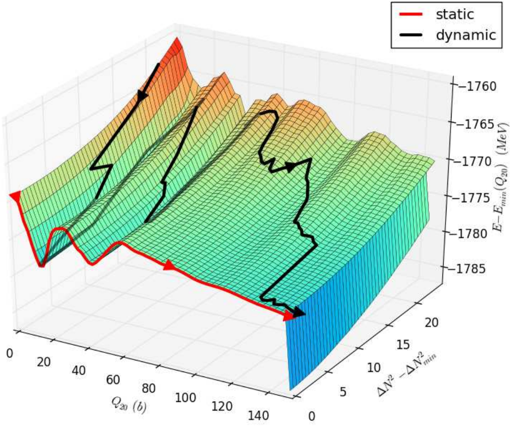


S. G., L. M. Robledo and R. Rodríguez-Guzmán, arXiv:1408.6940[nucl-th] (2014)

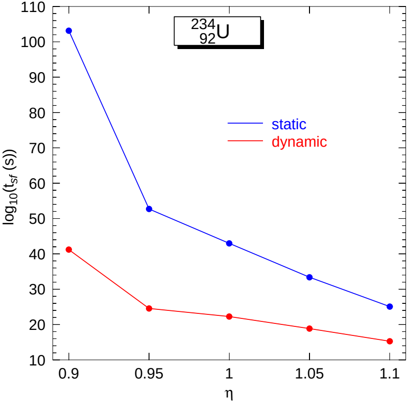
- S_{min} strongly differ from $S(E_{\text{min}})$ (selfconsistent value).

The least action path



- The least action path (black) strongly differ from the least energy one (red)!

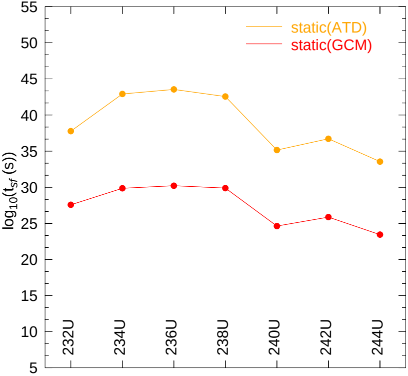
dynamic *vs* static approach



- Large quenching of the spontaneous fission lifetimes.
- Results more robust against changes in the pairing strength η!

Theory of collective masses $B(s)$: GCM vs ATDHFB

$$t_{\mathsf{sf}} = t_0 \exp\left(\tfrac{2}{\hbar}\int_a^b ds\sqrt{2\cdot B(s)[V(s)-E_0]}\right)$$



- ATDHFB inertias roughly two times larger than GCM.

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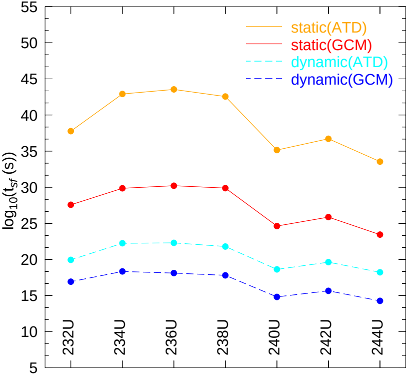
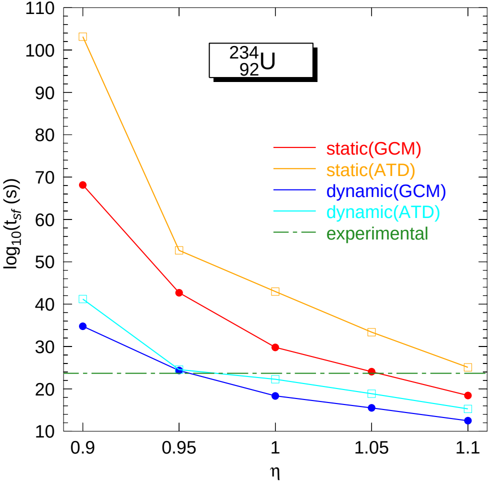


- ATDHFB inertias roughly two times larger than GCM.
- Results more robust against the collective inertias computations!

Summarizing. . .

dependence with η and B



Method	t_{sf} ATD (s)	t_{sf} GCM (s)
E_{min}	0.81×10^{43}	0.70×10^{30}
$S_{\text{min}}(Q_{20}, Q_{30})$	0.44×10^{42}	0.64×10^{29}
$S_{\text{min}}(Q_{20}, Q_{40})$	0.12×10^{43}	0.10×10^{29}
$S_{\text{min}}(Q_{20}, \Delta N^2)$	0.18×10^{23}	0.21×10^{19}

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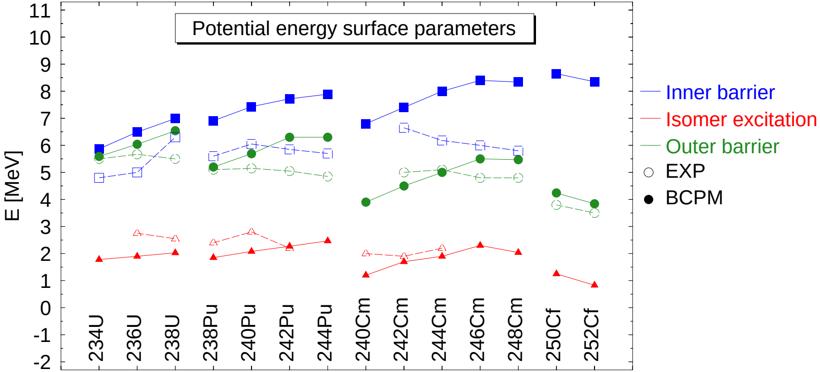
Conclusions

- DFT gives a good qualitative description of the fission process.
- But there are several uncertainties:
  - pairing strength, collective inertias and something else (quantal fluctuations, BMF effects. . .).
- The least action principle is a more robust approach: less sensitivity to pairing strengths and collective inertia computations.
- But we are still dealing with 3-4 OM of uncertainties.
- **Future work:**
  - Implementation of triaxiality.
  - Exact computation of the collective masses (50% larger than cranking!).
  - A theory beyond HFB is demanded.

THANK YOU!

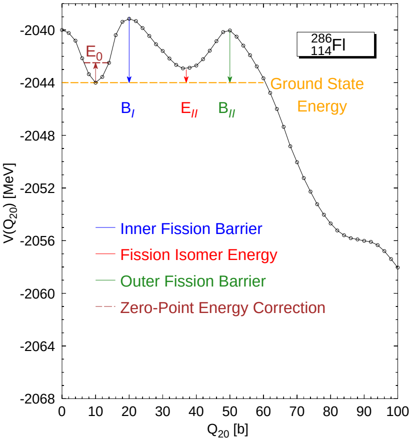
BCPM barrier heights and isomer energy

Exp: B. Sing et al., Nucl. Data Sheets **97**, 241 (2002); R. Capote et al., Nucl. Data Sheets **110**, 3107 (2009).



- Less sensitive to pairing correlations, but values are model dependent.
- Outer barrier and isomer energy values quite well reproduced for all nuclei.
- Inner barriers are reduced when triaxiality is allowed (Guzmán et al., arXiv:1312.7229).

Mean-field quantities



Constraining fields

- Quadrupole moment operator: $Q_{20} = z^2 - \frac{1}{2}(x^2 + y^2)$.
- Octupole and hexadecapole explorations.

Fission observables

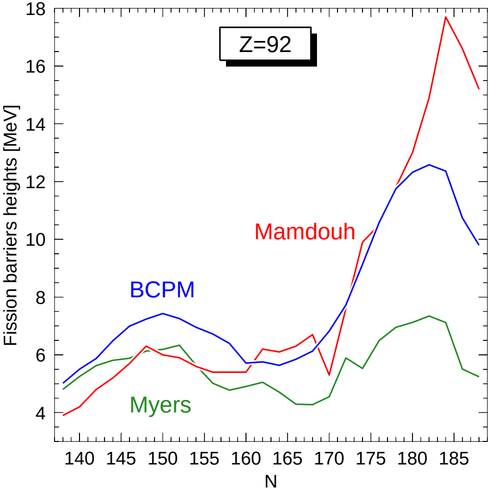
- Spontaneous fission half-lives:

  $t_{\mathrm{sf}} = 2.86\times10^{-21}(1+\exp(2S))$ (s)

- Isomer energy, inner and outer fission barrier heights.

$$S = \int_a^b dQ_{20}\sqrt{2 \times B(Q_{20})\left[V(Q_{20}) - (E_{\mathrm{GS}} + E_0)\right]}$$

Uranium fission barrier heights theoretical predictions

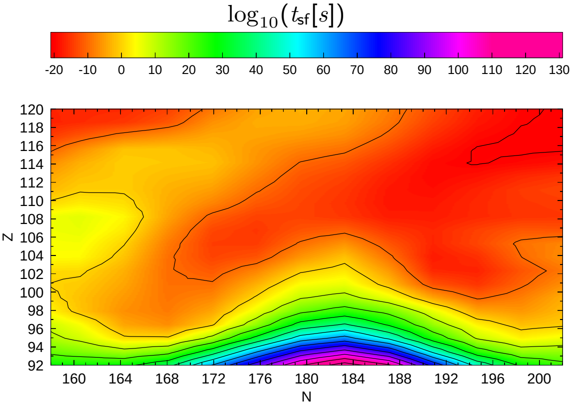


Enhancement around $N = 184$ also predicted by other models!

Mamdouh: Mamdouh et al., Nucl. Phys. **A679**, 337–358 (2001).

Myers: Myers et al., Phys. Rev. **C60**, 014606 (1999).

The superheavy nuclear landscape: fission properties



- For $Z \leq 106$ peak of stability at $N = 184$ (predicted magic number!).
- Lightest nuclei: neutron-rich isotopes $\sim$ stable against spontaneous fission.
- Heaviest nuclei: increase of fission barrier heights in $176 \leq N \leq 184$.

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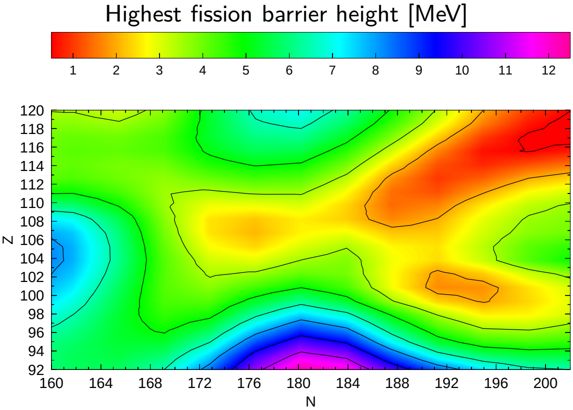

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The BCPM functional

The energy of a finite nucleus is given by

$$E = T_0 + E_{int}^{\infty} + E_{int}^{FR} + E^{s.o.} + E_C + E_{pair}$$

$$E_{int}^{\infty}[\rho_p, \rho_n] = \int d\vec{r} \Big[P_s(\rho)(1-\beta^2) + P_n(\rho)\beta^2 \Big] \rho$$

with $\rho(\vec{r}) = \rho_n(\vec{r}) + \rho_p(\vec{r})$ and $\beta(\vec{r}) = (\rho_n(\vec{r}) - \rho_p(\vec{r}))/\rho(\vec{r})$.

P_s and P_n are polynomial fits to reproduce microscopic EoS in nuclear matter.

- Phenomenological surface contribution

$$E_{int}^{FR}[\rho_n, \rho_p] = \frac{1}{2} \sum_{t,t'} \iint d\vec{r} d\vec{r'} \rho_t(\vec{r}) v_{t,t'}(\vec{r} - \vec{r'}) \rho_{t'}(\vec{r'})$$

with $v_{t,t'}(r) = V_{t,t'} \, e^{-r^2/r_{0\,tt}{}^2}$; $V_{n,n} = V_{p,p} = V_L = 2\tilde{b}_1/(\pi^{3/2} r_{0L}^3 \rho_0)$;
$V_{n,p} = V_{p,n} = V_U = (4a_1 - 2\tilde{b}_1)/(\pi^{3/2} r_{0U}^3 \rho_0)$.

M.Baldo et al. Phys. Lett. B663 (2008) 390; Phys. Rev. C87 064305 (2013)

Remaining contributions to the EDF

- Coulomb

  Direct $E_C^H = (1/2)\iint d\vec{r}d\vec{r'}\rho_p(\vec{r})|\vec{r}-\vec{r'}|^{-1}\rho_p(\vec{r'})$

  Exchange: $E_C^{ex} = -(3/4)(3/\pi)^{1/3}\int d\vec{r}\rho_p(\vec{r})^{4/3}$
- Spin-Orbit

$$\hat{v}_{ij}^{so} = iW_{LS}(\vec{\sigma}_i + \vec{\sigma}_j)\cdot[\vec{k}' \times \delta(\vec{r}_i - \vec{r}_j)\vec{k}]$$

Free parameters

W_{LS} and r_{0L}, r_{0U}

- Pairing Correlations (E. Garrido et al. Phys. Rev. C **60**, 064312 (1999))

  Zero-range interaction,

$$v^{pp}(\rho(\vec{r})) = \eta \times \frac{v_0}{2}\left[1 - \gamma\left(\frac{\rho(\vec{r})}{\rho_0}\right)^{\alpha}\right], \qquad \rho_0 = \frac{2}{3\pi^2}k_F^3 .$$

$\eta \equiv$ multiplicative parameter setting the pairing strength. . .