Shell-model derivation of the shears mechanism

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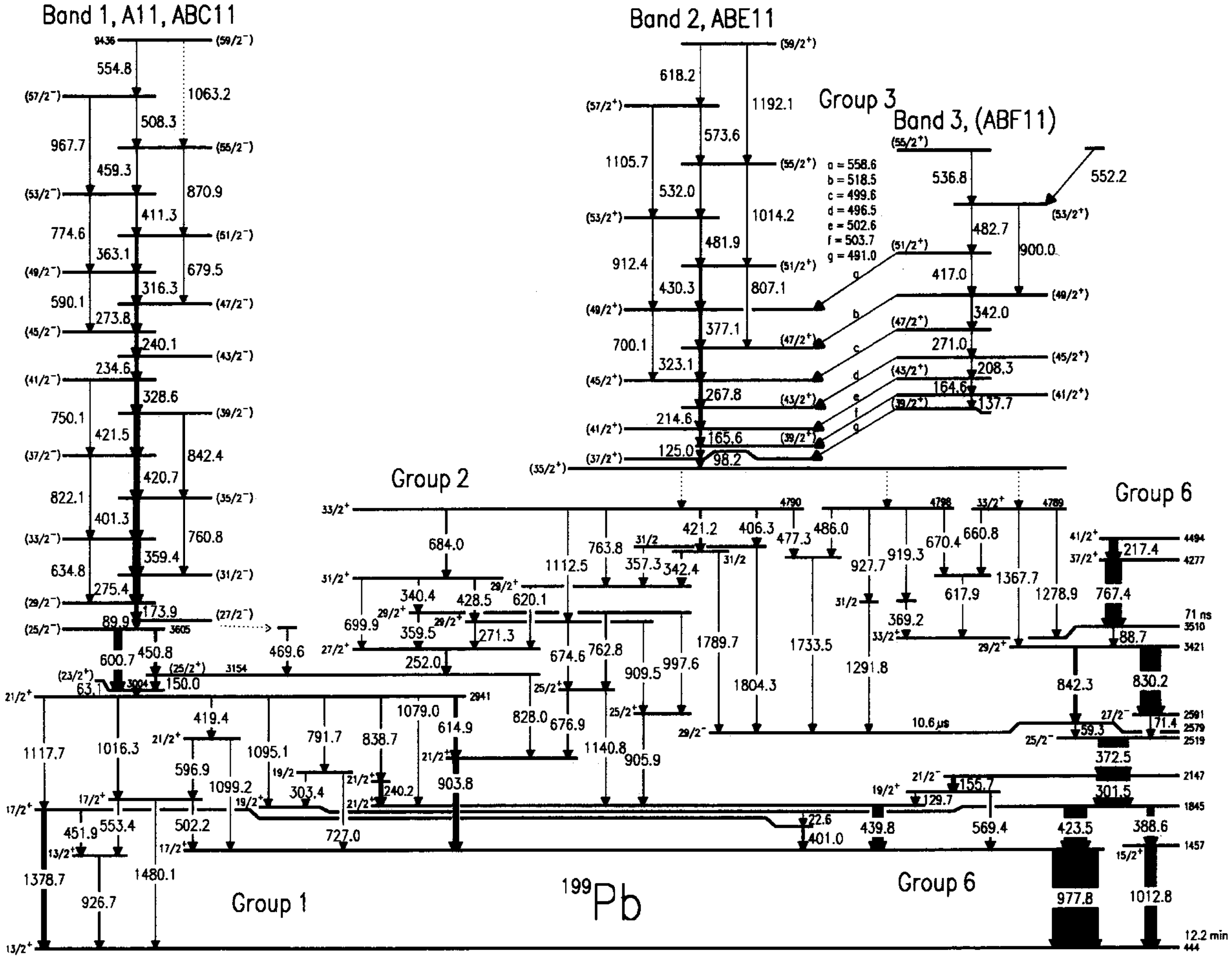
Dipole workshop, GANIL, October 2013

Bands without deformation

Regular sequences of levels (bands) are usually associated with nuclear collective behaviour.

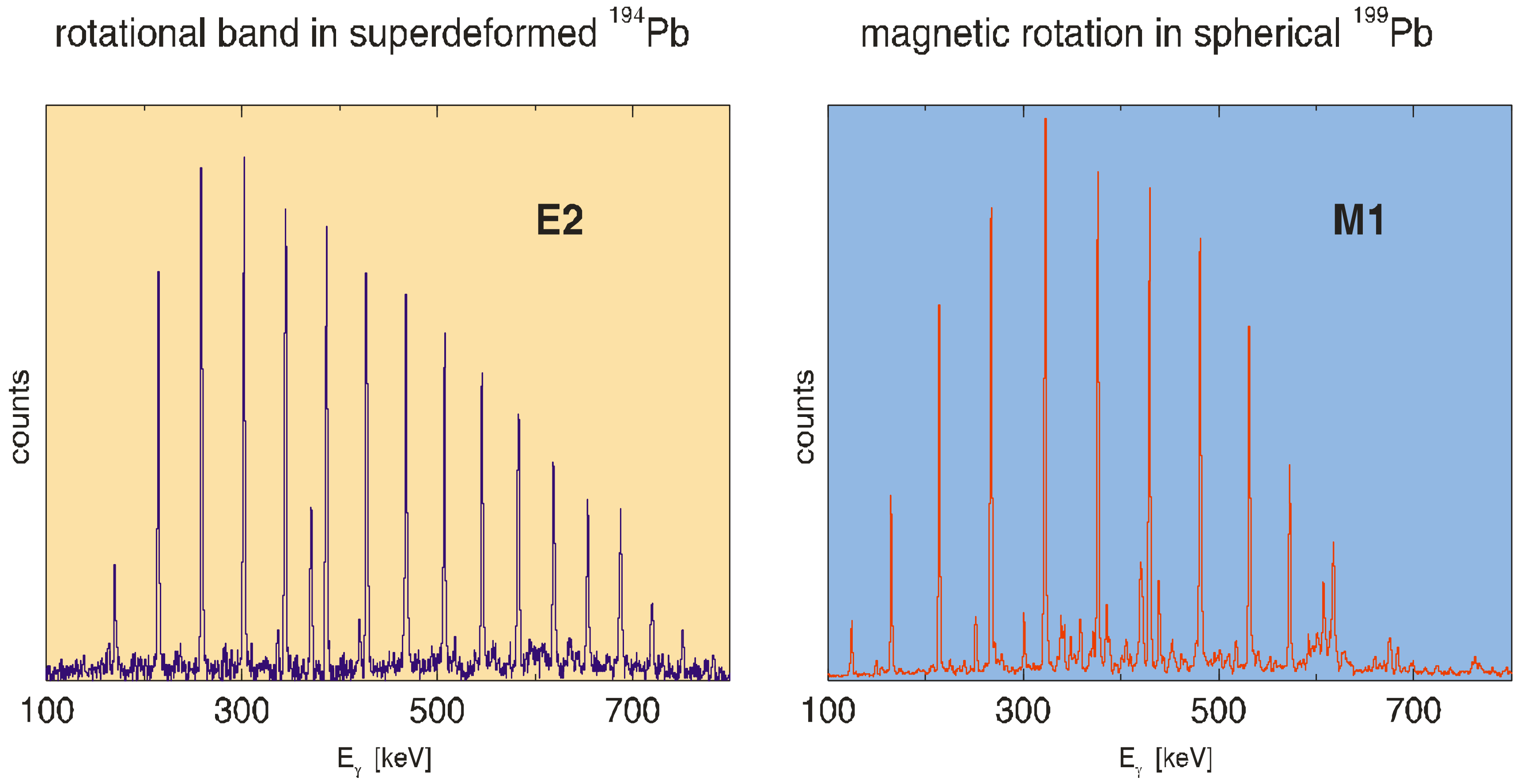
In several regions of the nuclear chart in the neighbourhood of closed-shells nuclei regular bands are observed.

An example: ^{199}Pb



G. Baldsiefen *et al.*, Nucl. Phys. A **574** (1994) 521

Regular sequences



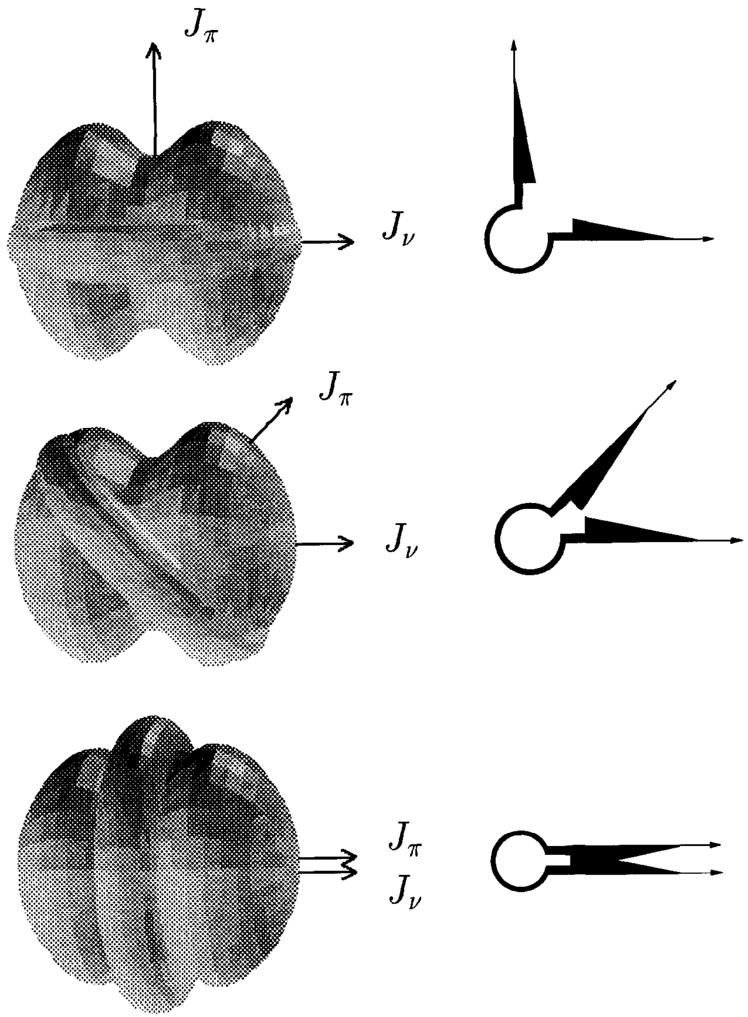
Shears bands

Question: How can sequences of levels appear rotational when deformation is weak?

Answer: Through the shears mechanism. This implies strong in-band M1 transitions.

S. Frauendorf, Nucl. Phys. A **557** (1993) 259c

The shears mechanism



S. Frauendorf *et al.*, Nucl. Phys. A **601** (1996) 41

Semi-classical interpretation

Schematic model in terms of the coupling of two vectors J_ν and J_π and a 'shears' angle

$$\theta_{\nu\pi} = \arccos \frac{J(J+1) - J_\nu(J_\nu+1) - J_\pi(J_\pi+1)}{2\sqrt{J_\nu(J_\nu+1)J_\pi(J_\pi+1)}}$$

An effective interaction of the form

$$V(\theta_{\nu\pi}) = V_0 + V_2 P_2(\cos\theta_{\nu\pi}) + \cdots$$

➜ Can this geometry of the shears mechanism be derived from the spherical shell model?

A.O. Macchiavelli *et al.*, Phys. Rev. C **57** (1998) R1073
A.O. Macchiavelli *et al.*, Phys. Rev. C **58** (1998) R621

A shell-model configuration

Assume a shears band in terms of two neutron particles and two proton holes:

$$\left|N\right\rangle \equiv \left|j_\nu j'_\nu ;J_\nu\right\rangle \quad \& \quad \left|P^{-1}\right\rangle \equiv \left|j_\pi^{-1} j'^{-1}_\pi ;J_\pi\right\rangle \Rightarrow \left|NP^{-1};J\right\rangle$$

How do the energies of these states evolve as a function of J ?

How does this evolution depends on the angular momenta of the single-particle orbits and on the angular momenta of the 'blades'?

Take $j_\nu=j'_\nu$ and $j_\pi=j'_\pi$.

The shears matrix element

Consider a hamiltonian of the generic form

$$\hat{H} = \hat{H}_\nu + \hat{H}_\pi + \hat{V}_{\nu\pi}$$

The relative shears-band energies depend only on the neutron-proton interaction:

$$\frac{\langle NP^{-1};J|\hat{V}_{\nu\pi}|NP^{-1};J\rangle}{(2J_\nu+1)(2J_\pi+1)}$$

$$= -4\sum_R (2R+1) V^R_{j_\nu j_\pi, j_\nu j_\pi} \begin{Bmatrix} j_\nu & j_\pi & J_\pi & J_\nu \\ R & j_\pi & J & j_\nu \\ j_\nu & j_\pi & J_\pi & J_\nu \end{Bmatrix}$$

The non-shears matrix element

The corresponding matrix element for a particle-particle or hole-hole configuration:

$$\frac{\langle NP;J|\hat{V}_{\nu\pi}|NP;J\rangle}{(2J_\nu+1)(2J_\pi+1)} = \frac{\langle N^{-1}P^{-1};J|\hat{V}_{\nu\pi}|N^{-1}P^{-1};J\rangle}{(2J_\nu+1)(2J_\pi+1)}$$

$$= 4\sum_R (2R+1) V^R_{j_\nu j_\pi, j_\nu j_\pi} \begin{bmatrix} j_\nu & j_\pi & J_\pi & J_\nu \\ R & j_\pi & J & j_\nu \\ j_\nu & j_\pi & J_\pi & J_\nu \end{bmatrix}$$

The classical limit

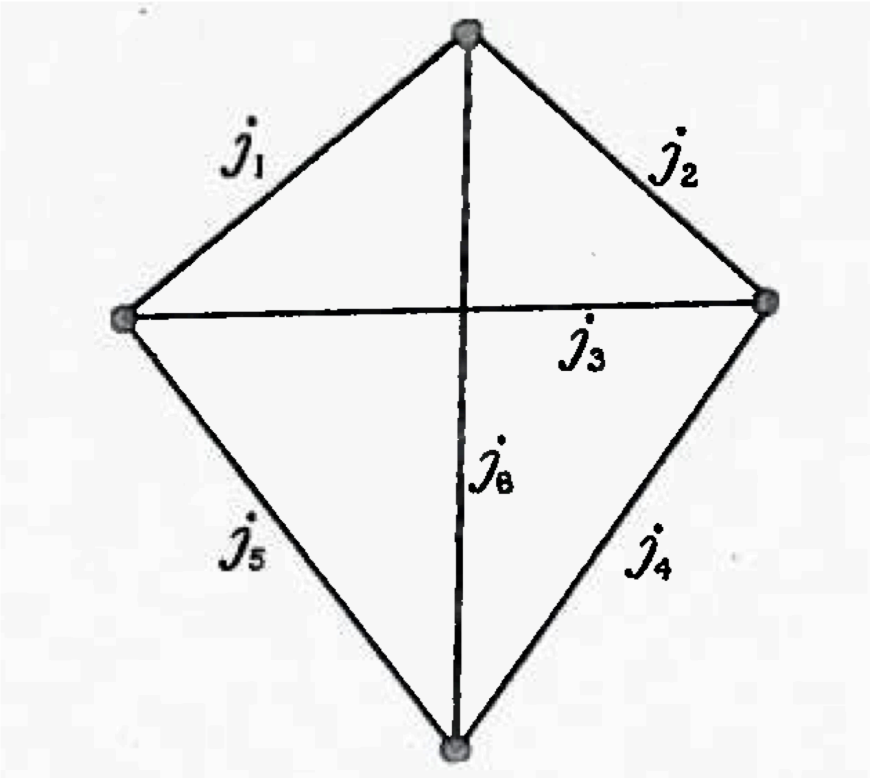
Study the shears matrix element in the classical limit, i.e. for large angular momenta.

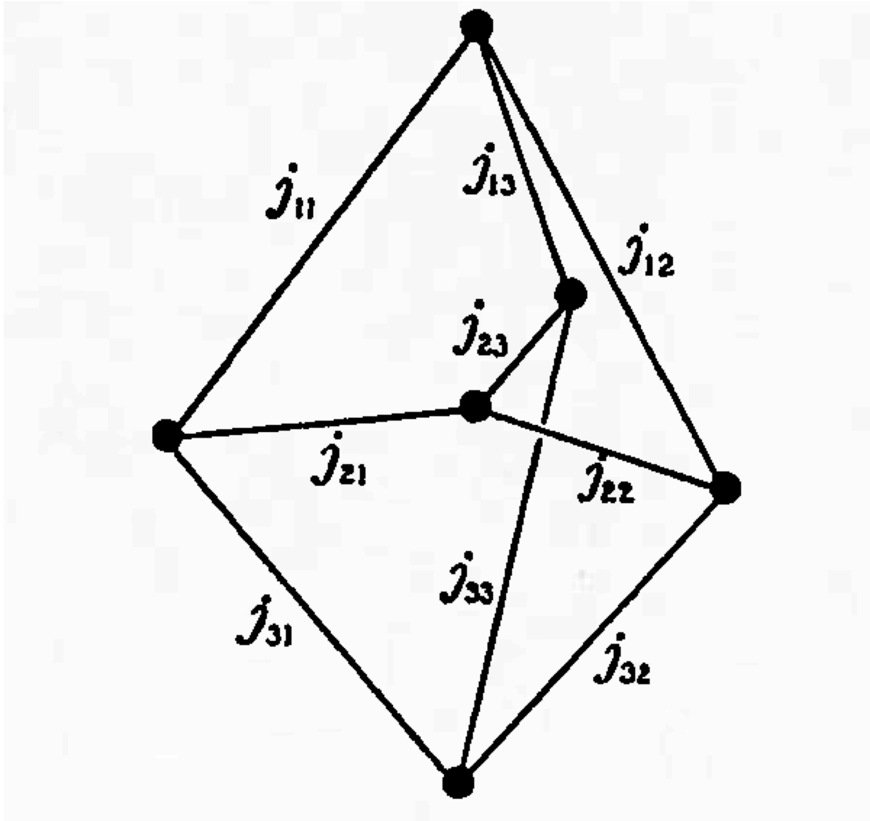
Semi-classical expressions are known for Wigner ($3j$) and Racah ($6j$) coefficients but not for $3nj$ coefficients with $n > 2$.

A field of active mathematical research with connections to graph theory, quantum gravity, spin networks...

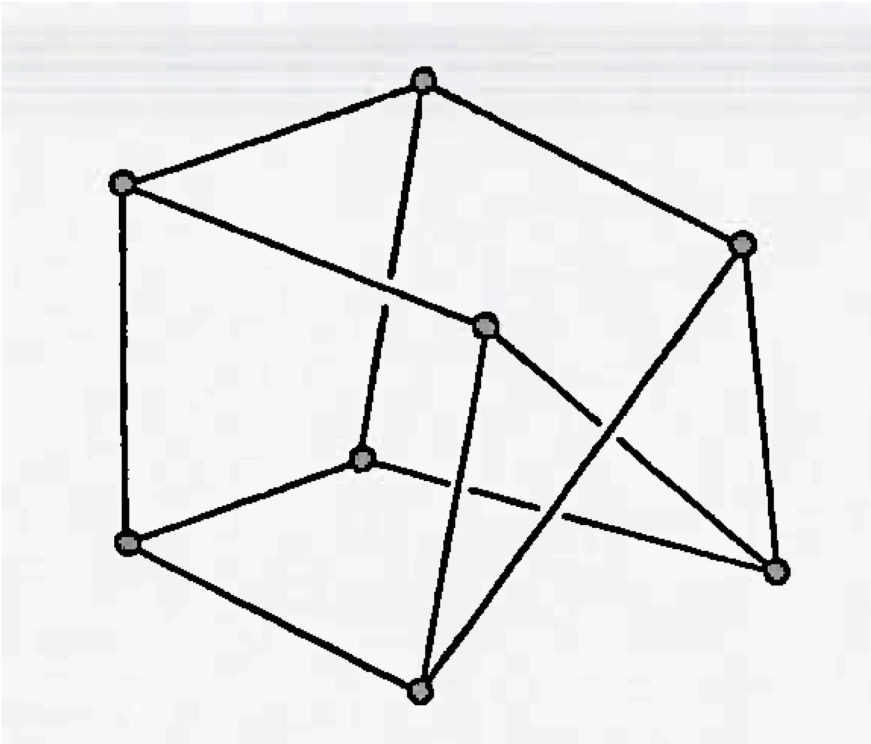
E.P. Wigner, *Group Theory* (1959)
G. Ponzano & T. Regge, *Group Theoretical Methods in Physics* (1968)

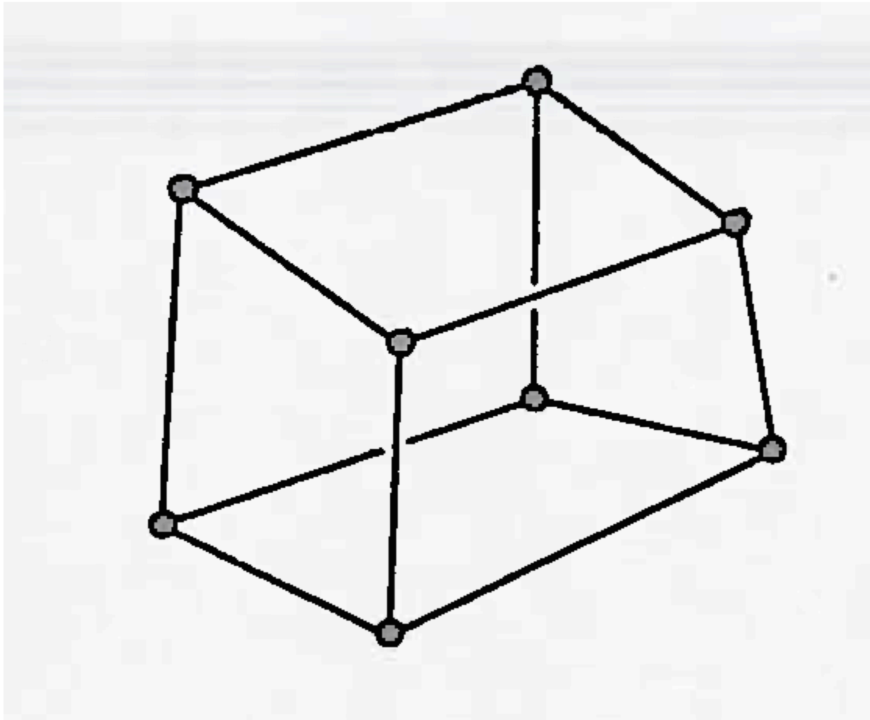
$3nj$ coefficients as graphs





Two kinds of $12j$ symbols

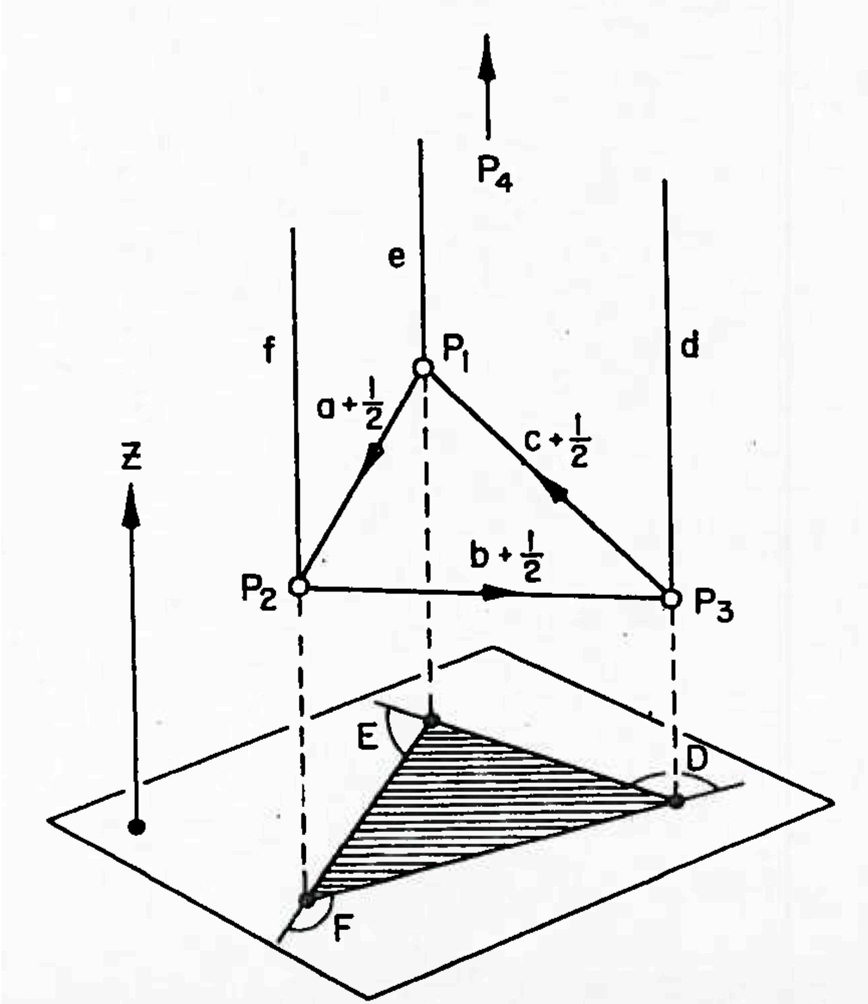




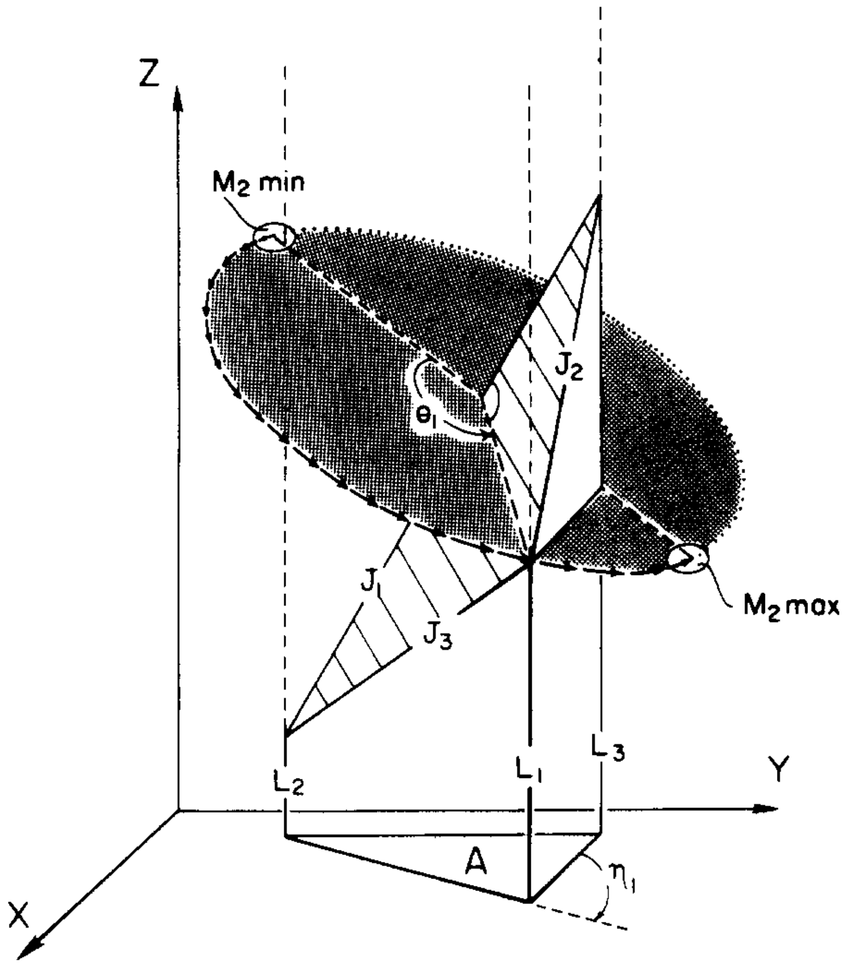
3j as the limit of 6j

The asymptotic formula:

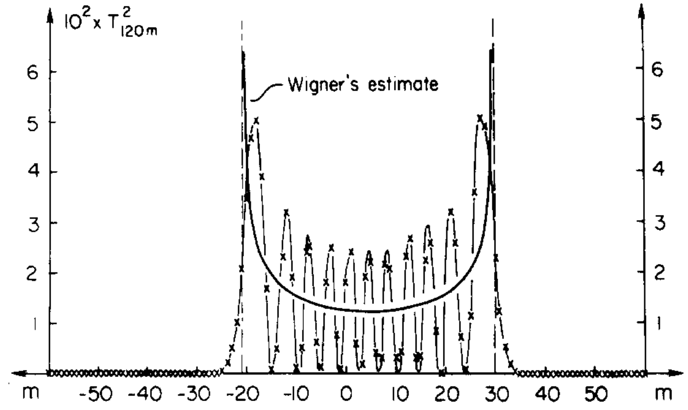
$$\begin{Bmatrix} a & b & c \\ d+R & e+R & f+R \end{Bmatrix} \xrightarrow{R\to\infty} \frac{(-)^{a+b+c+2(d+e+f)}}{\sqrt{2R}} \times \begin{pmatrix} a & b & c \\ e-f & f-d & d-e \end{pmatrix}$$



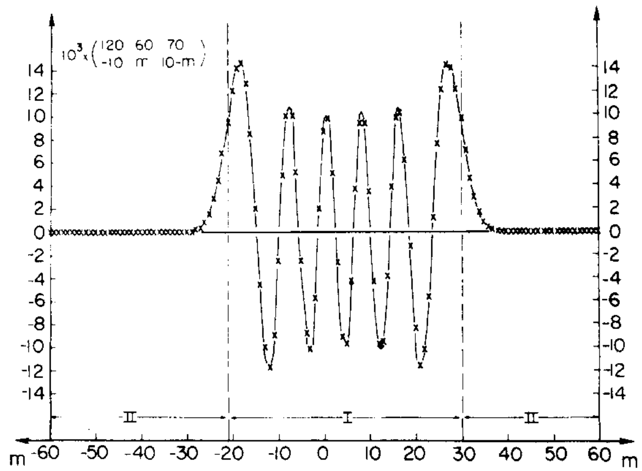
(Semi-)classical approximations



(a)



(b)



(c)

K. Schulten & R.G. Gordon, J. Math. Phys. **16** (1975) 1961 & 1971

Matrix elements of MSDI

Modified surface delta interaction:

$$\hat{V}^{\mathrm{MSDI}}(i,j) = -4\pi a'_T \delta(\bar{r}_i - \bar{r}_j)\delta(r_i - R_0) + b'\bar{\tau}_i \cdot \bar{\tau}_j + c'$$

Its matrix elements are

$$-\frac{(2j_\nu+1)(2j_\pi+1)}{2}\left[a_{01}\begin{pmatrix} j_\nu & j_\pi & R \\ ½ & -½ & 0 \end{pmatrix}^2 + a_0\begin{pmatrix} j_\nu & j_\pi & R \\ ½ & ½ & -1 \end{pmatrix}^2\right] - b + c$$

with

$$a_{01} = \frac{a_0 + a_1}{2} - (-)^{\ell_\nu + \ell_\pi + R}\frac{a_0 - a_1}{2}$$

$$a_T = a'_T C(R_0), \quad b = b'C(R_0), \quad c = c'C(R_0)$$

P.J. Brussaard & P.W.M. Glaudemans, *Shell-Model Applications* (1977)

A simple sum

An exact result:

$$\sum_R (2R+1)\begin{Bmatrix} j_\nu & j_\pi & J_\pi & J_\nu \\ R & j_\pi & J & j_\nu \\ j_\nu & j_\pi & J_\pi & J_\nu \end{Bmatrix} = \frac{1}{(2J_\nu+1)(2J_\pi+1)}$$

A.P. Yutsis *et al.*, *The Theory of Angular Momentum* (1962)

A more complicated sum (1)

An exact result:

$$S_n = \sum_R (2R+1) \begin{pmatrix} j_\nu & j_\pi & R \\ \frac{1}{2} & n-\frac{1}{2} & -n \end{pmatrix}^2 \begin{Bmatrix} j_\nu & j_\pi & J_\pi & J_\nu \\ R & j_\pi & J & j_\nu \\ j_\nu & j_\pi & J_\pi & J_\nu \end{Bmatrix}$$

$$= \sum_{\substack{m_\nu M_\nu \\ m_\pi M_\pi}} \begin{pmatrix} j_\nu & j_\nu & J_\nu \\ \frac{1}{2} & m_\nu & M_\nu \end{pmatrix}^2 \begin{pmatrix} j_\pi & j_\pi & J_\pi \\ -n+\frac{1}{2} & m_\pi & M_\pi \end{pmatrix}^2 \begin{pmatrix} J_\nu & J_\pi & J \\ M_\nu & M_\pi & M \end{pmatrix}^2$$

A more complicated sum (2)

An approximate result (Wigner):

$$\begin{pmatrix} J_\nu & J_\pi & J \\ M_\nu & M_\pi & M \end{pmatrix}^2 \mapsto \frac{1}{4\pi A}$$

Therefore

$$S_n \approx \frac{1}{4\pi A} \sum \begin{pmatrix} j_\nu & j_\nu & J_\nu \\ ½ & m_\nu & M_\nu \end{pmatrix}^2 \begin{pmatrix} j_\pi & j_\pi & J_\pi \\ -n + ½ & m_\pi & M_\pi \end{pmatrix}^2$$

$$\approx \frac{1}{4\pi(2j_\nu + 1)(2j_\pi + 1)A}$$

A is the area of a triangle with sides of lengths $J_\nu+1/2$, $J_\pi+1/2$ and $J+1/2$.

In terms of the shears angle

The shears angle is the angle between the angular momentum vectors of neutron particles and the proton holes:

$$\theta_{\nu\pi} = \arccos \frac{J(J+1) - J_\nu(J_\nu+1) - J_\pi(J_\pi+1)}{2\sqrt{J_\nu(J_\nu+1)J_\pi(J_\pi+1)}}$$

We have

$$S_n \approx \frac{2}{\pi(2j_\nu+1)(2j_\pi+1)(2J_\nu+1)(2J_\pi+1)\sin\theta_{\nu\pi}}$$

Another sum

Another approximate result:

$$\bar{S}_0 = \sum_R (-)^R (2R+1) \begin{pmatrix} j_\nu & j_\pi & R \\ 1/2 & -1/2 & 0 \end{pmatrix}^2 \begin{Bmatrix} j_\nu & j_\pi & J_\pi & J_\nu \\ R & j_\pi & J & j_\nu \\ j_\nu & j_\pi & J_\pi & J_\nu \end{Bmatrix}$$

$$\approx -(-)^{j_\nu + j_\pi} \frac{2}{\pi(2j_\nu+1)(2j_\pi+1)(2J_\nu+1)(2J_\pi+1)\tan\theta_{\nu\pi}}$$

Classical shears matrix element

We obtain for a MSDI the following classical approximation of the shears matrix element:

$$\left\langle NP^{-1};J\left|\hat{V}_{\nu\pi}^{\mathrm{MSDI}}\right|NP^{-1};J\right\rangle \approx 4\left(b-c\right)+\frac{\alpha_{\mathrm{s}}}{2\pi\sin\theta_{\nu\pi}}+\frac{\alpha_{\mathrm{t}}}{2\pi\tan\theta_{\nu\pi}}$$

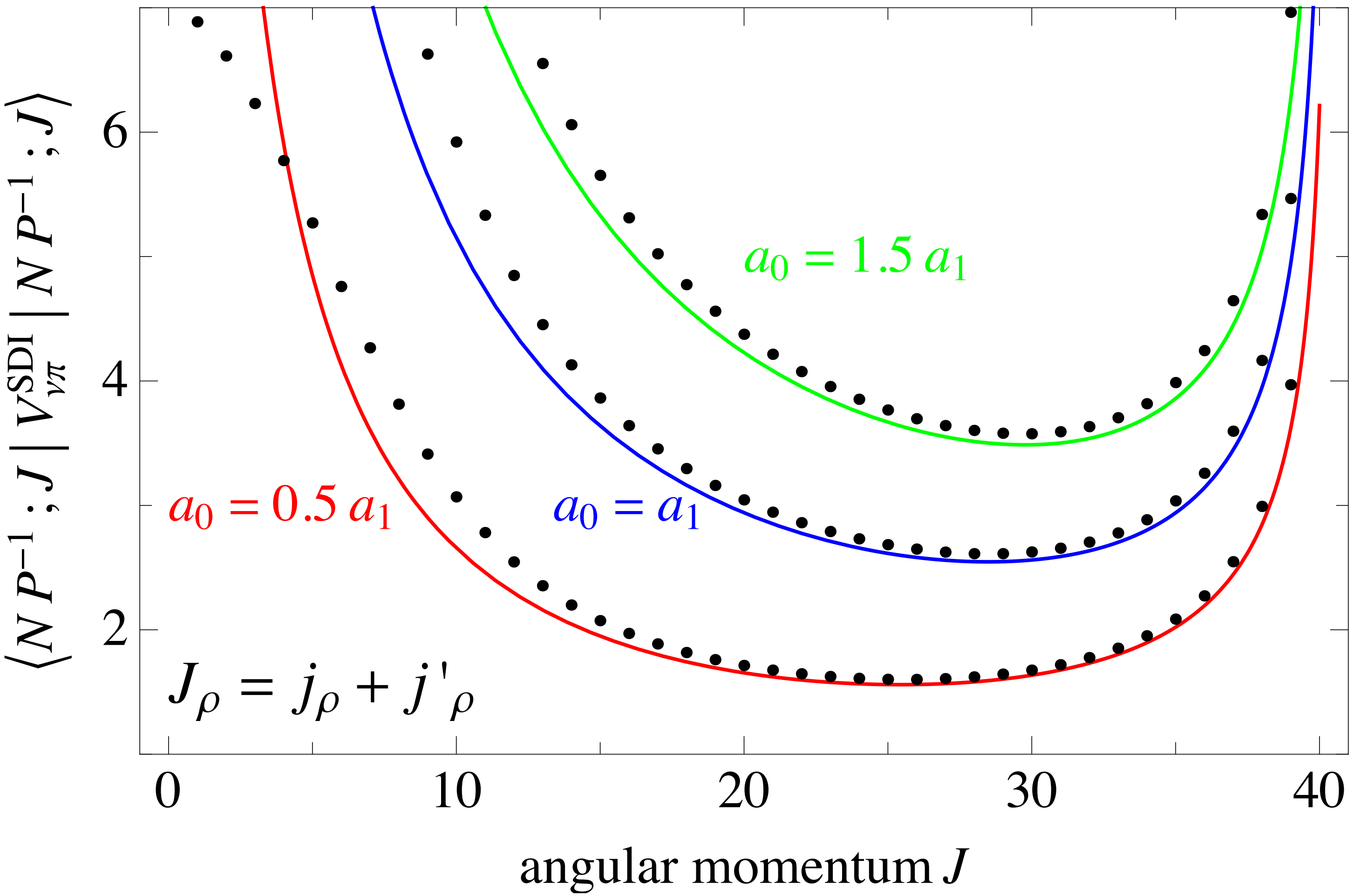
with

$$\alpha_{\mathrm{s}}=4\left(3a_0+a_1\right),\quad \alpha_{\mathrm{t}}=4\left(a_0-a_1\right)\varphi$$

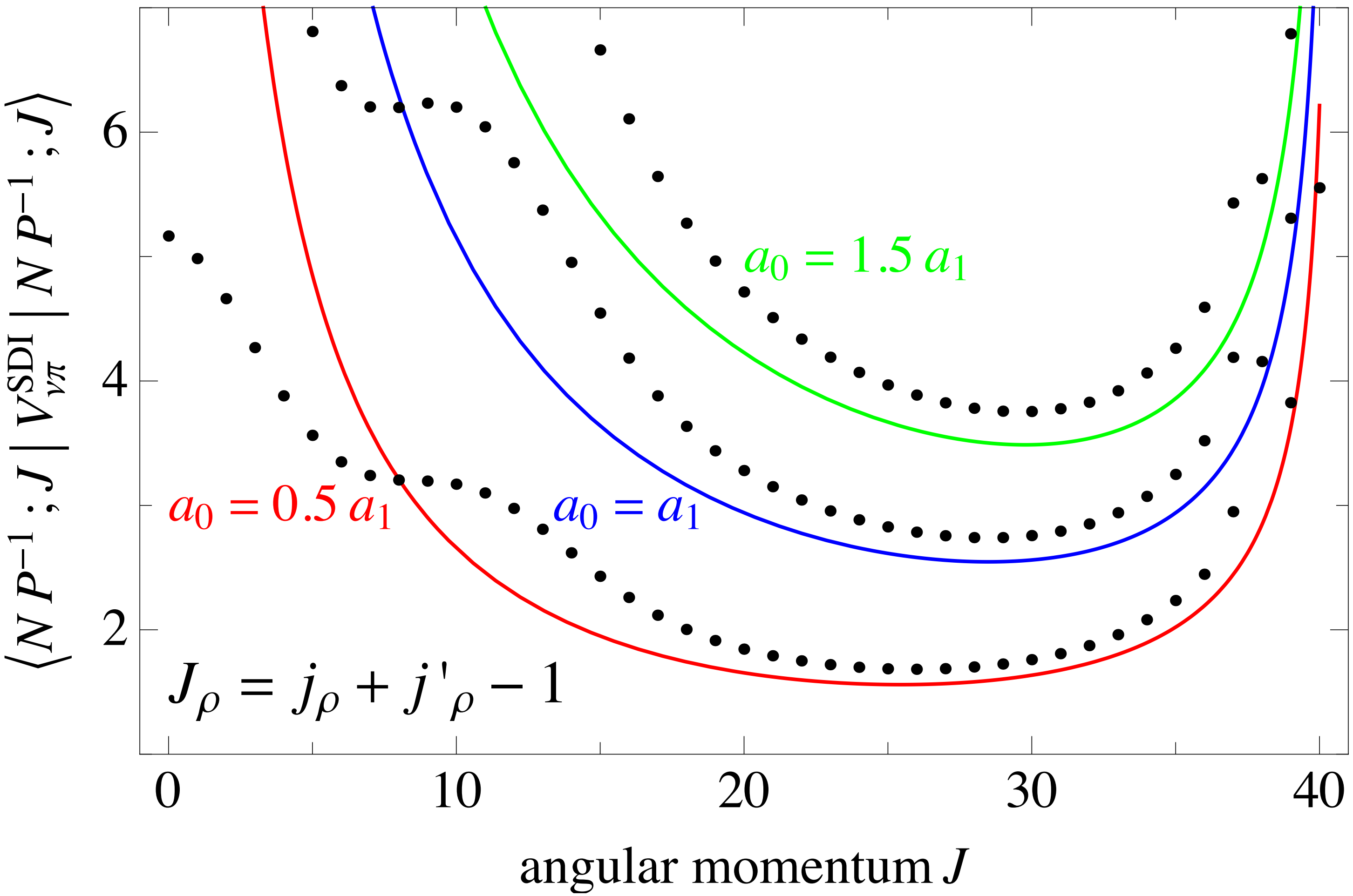
$$\varphi=\frac{1}{4}\left(\varphi_\nu\varphi_\pi+\varphi_\nu\varphi'_\pi+\varphi'_\nu\varphi_\pi+\varphi'_\nu\varphi'_\pi\right)$$

$$\varphi_\rho=\left(-\right)^{l_\rho+j_\rho},\quad \varphi'_\rho=\left(-\right)^{l'_\rho+j'_\rho}$$

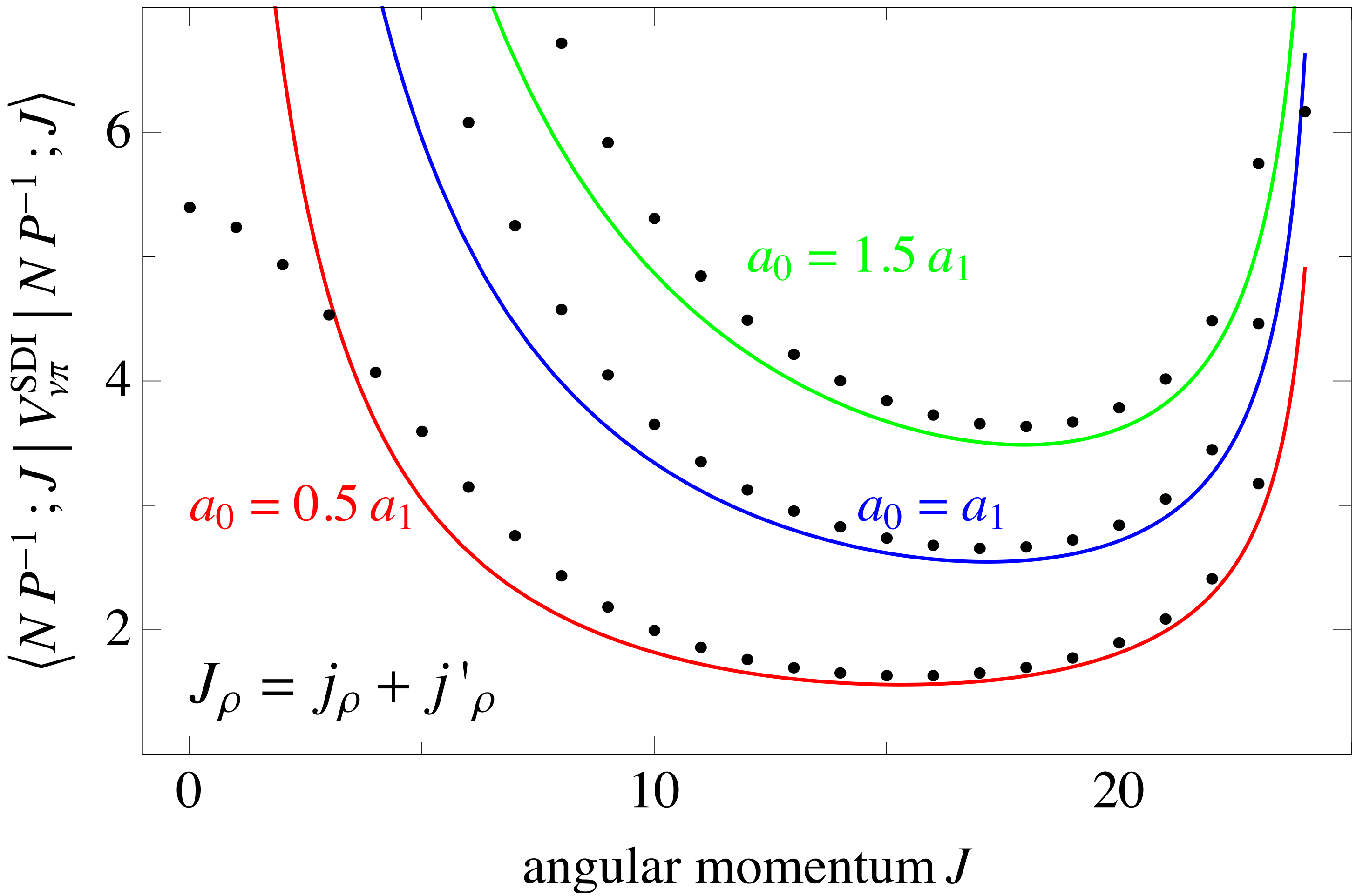
j_ϱ=19/2 & j'_ϱ=21/2 & J_ϱ=20



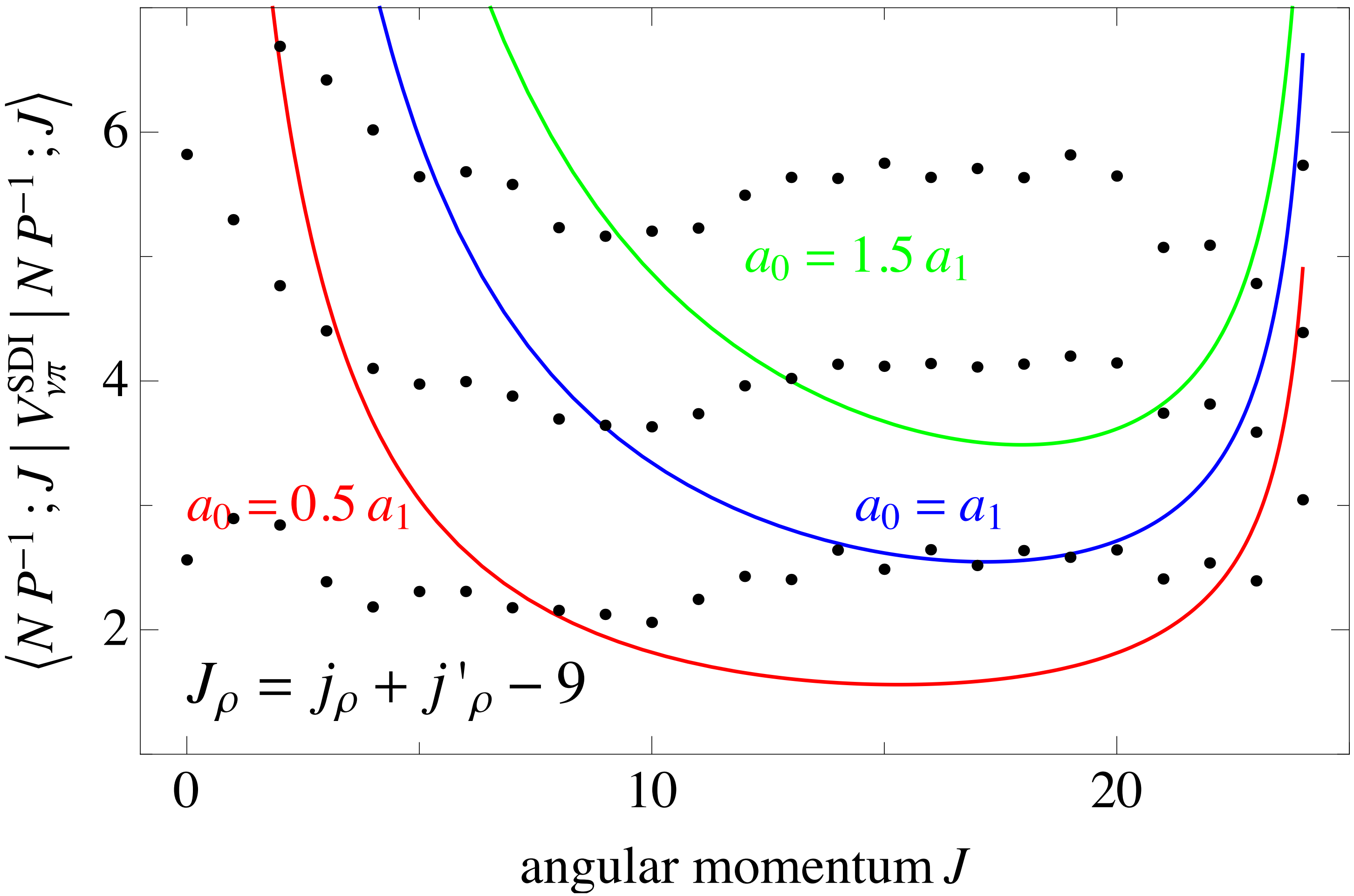
j_ϱ=21/2 & j'_ϱ=21/2 & J_ϱ=20



j_ϱ=11/2 & j'_ϱ=13/2 & J_ϱ=12



j_ϱ=21/2 & j'_ϱ=21/2 & J_ϱ=12



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1p-1h matrix element

Recall the well-known classical interpretation of a short-range nuclear matrix element.

For MSDI:

$$\left\langle j_\nu j_\pi^{-1};J\left|\hat{V}_{\nu\pi}^{\mathrm{MSDI}}\right|j_\nu j_\pi^{-1};J\right\rangle \approx (b-c)+\frac{\alpha_{\mathrm{s}}}{2\pi\sin\theta_{\nu\pi}}+\frac{\alpha_{\mathrm{t}}}{2\pi\tan\theta_{\nu\pi}}$$

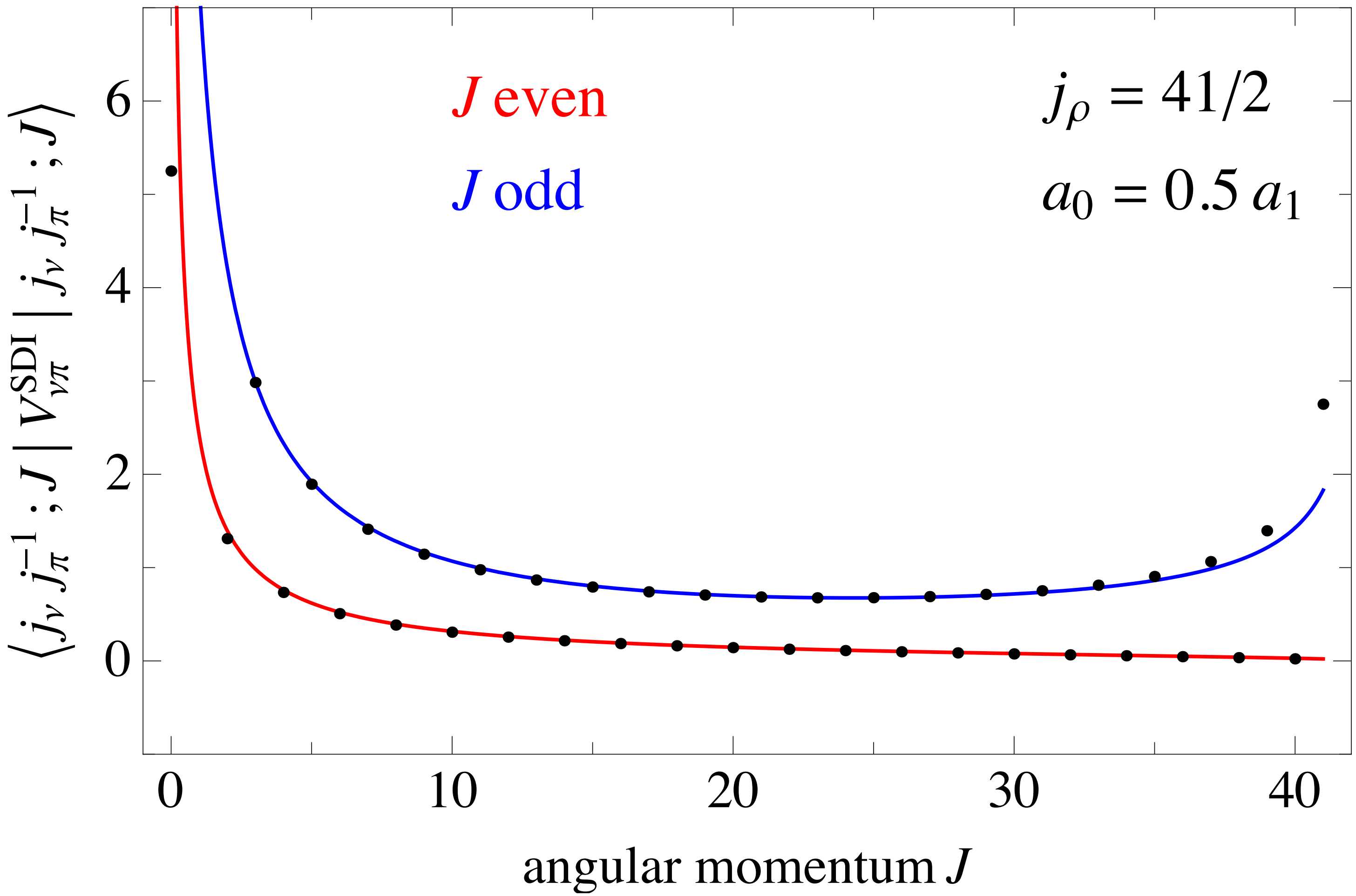
with

$$\alpha_{\mathrm{s}}=(a_0+a_1)\left[1+(-)^{j_\nu+j_\pi+J}\right]+2a_0+(-)^{\ell_\nu+\ell_\pi+J}(a_0-a_1)$$

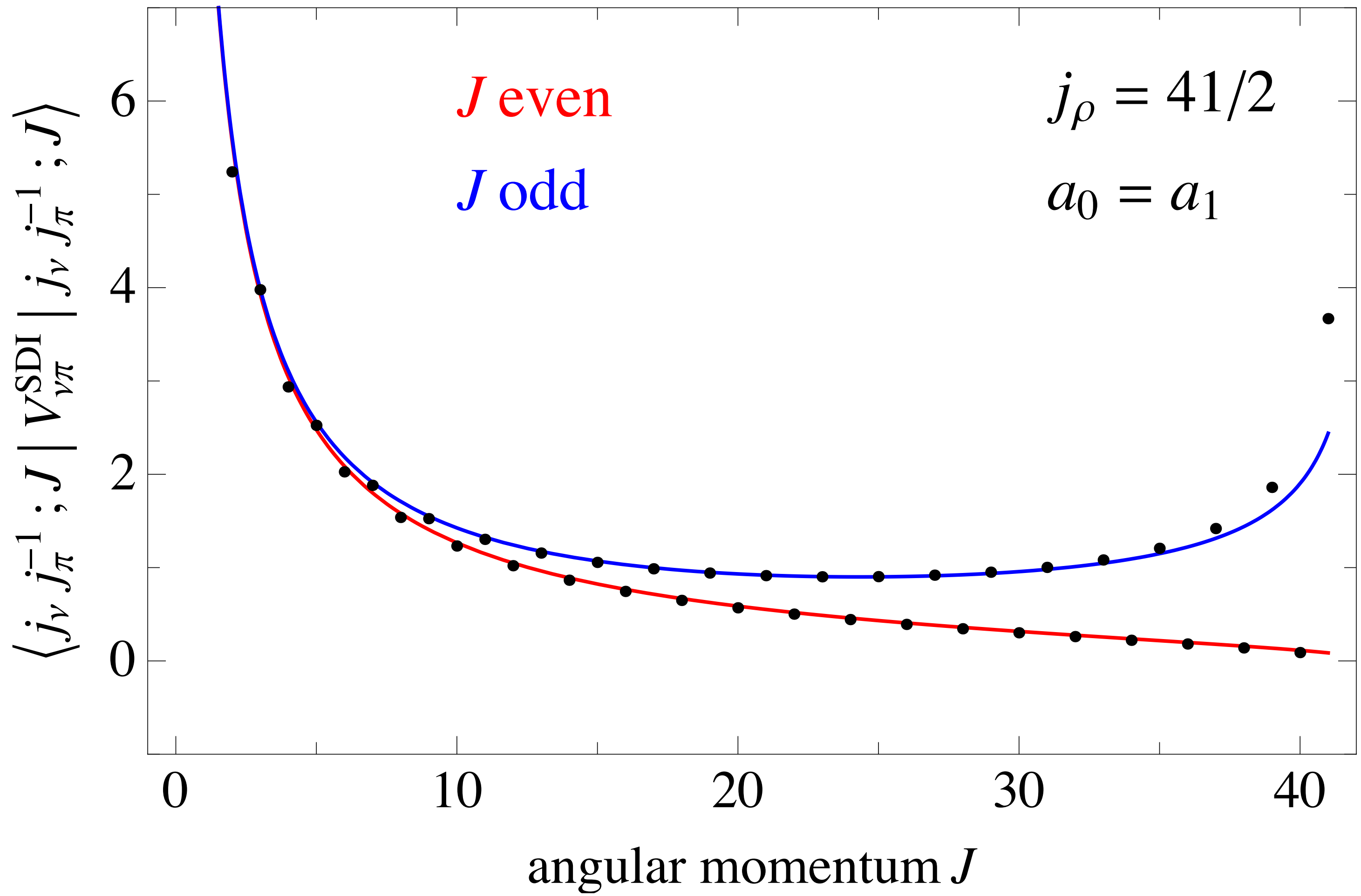
$$\alpha_{\mathrm{t}}=2(-)^{j_\nu+j_\pi+J}a_0+(-)^{\ell_\nu+\ell_\pi+j_\nu+j_\pi}(a_0-a_1)$$

J.P. Schiffer & W.W. True, Rev. Mod. Phys. **48** (1976) 191

1p-1h matrix element



1p-1h matrix element



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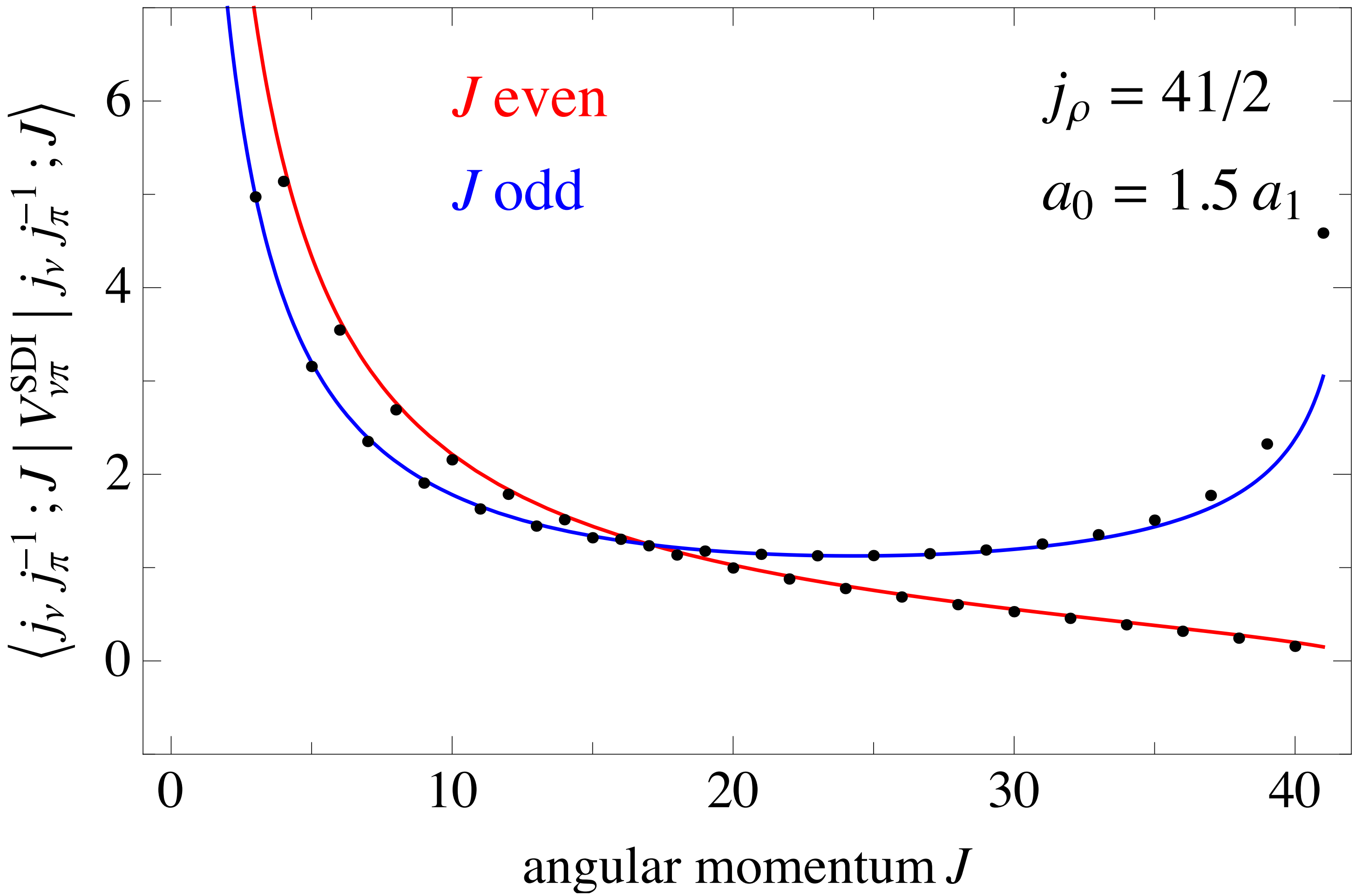
1p-1h matrix element



Conjecture

Assume *near-aligned* neutron particles and *near-aligned* proton holes:

$$\left|N\right\rangle \equiv \left|j_\nu j'_\nu j''_\nu \ldots ; J_\nu\right\rangle \quad \& \quad \left|P^{-1}\right\rangle \equiv \left|j_\pi^{-1} j'^{-1}_\pi j''^{-1}_\pi \ldots ; J_\pi\right\rangle$$

A neutron-proton short-range force has an interaction energy in the coupled state which can be approximated as

$$E(J) \equiv \left\langle NP^{-1};J\middle|\hat{V}_{\nu\pi}\middle|NP^{-1};J\right\rangle \approx \frac{\alpha_\mathrm{s}}{2\pi\sin\theta_{\nu\pi}} + \frac{\alpha_\mathrm{t}}{2\pi\tan\theta_{\nu\pi}}$$

The coefficients α_s and α_t depend on the isoscalar and isovector interaction strengths.

A simple application

Let's accept the expression for the shears energy

$$E(J) = \frac{\alpha_s}{2\pi \sin\theta_{\nu\pi}} + \frac{\alpha_t}{2\pi \tan\theta_{\nu\pi}}$$

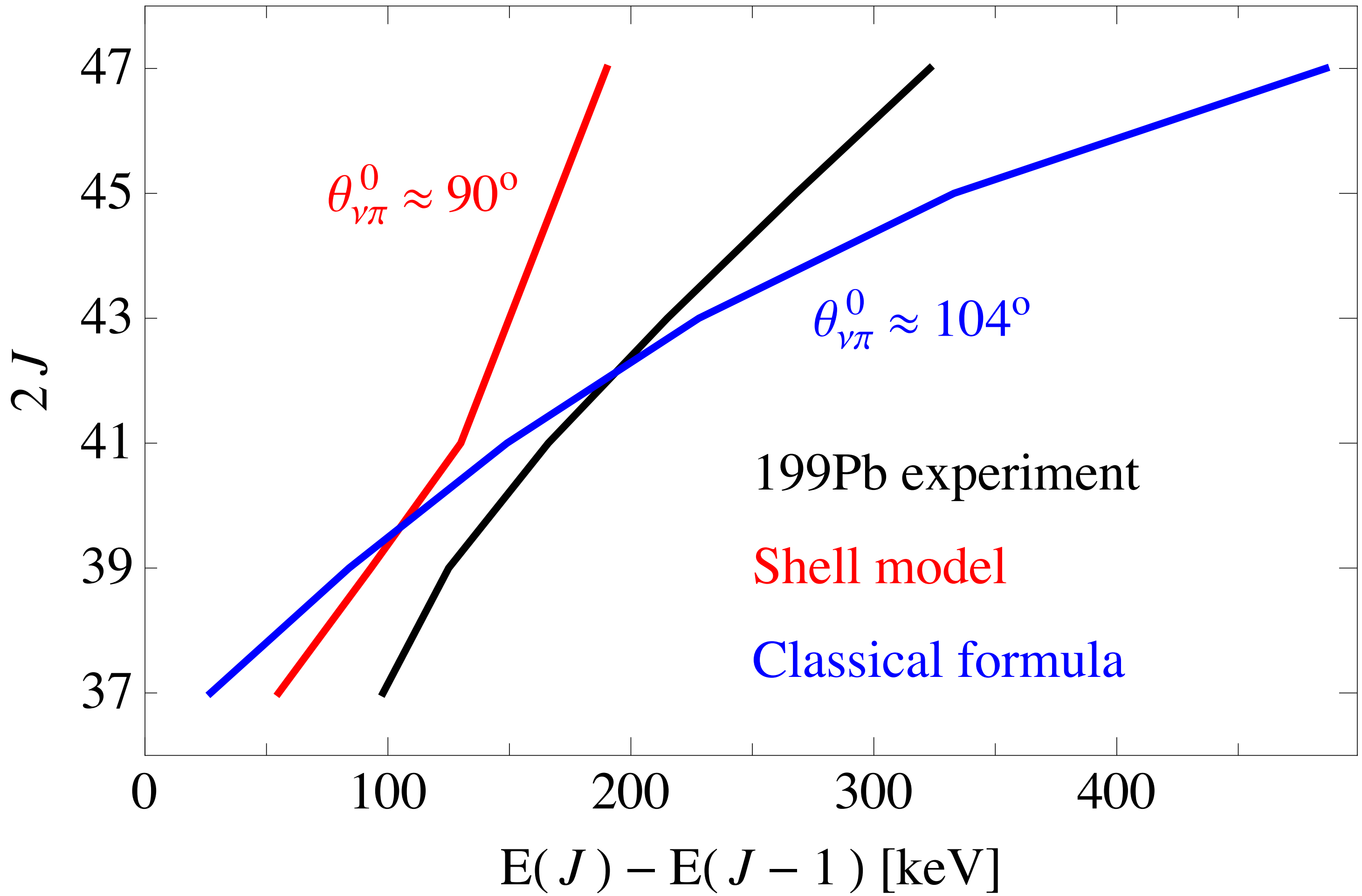
The head of the shears band follows from

$$\left.\frac{\partial E}{\partial \theta_{\nu\pi}}\right|_{\theta_{\nu\pi}=\theta^0_{\nu\pi}} = 0 \Rightarrow \cos\theta^0_{\nu\pi} = -\frac{\alpha_t}{\alpha_s} \quad \left(= \frac{a_0 - a_1}{3a_0 + a_1} \right)$$

The excitation energies of the shears-band members are given as

$$E_x(J) = \frac{\alpha_s}{2\pi \sin\theta_{\nu\pi}} \left(1 - \cos\theta^0_{\nu\pi} \cos\theta_{\nu\pi}\right) - \frac{\alpha_s \sin\theta^0_{\nu\pi}}{2\pi}$$

A simple application: ^{199}Pb



S. Frauendorf *et al.*, Nucl. Phys. A **601** (1996) 41

M1 transitions

Exact result for np-nh configurations:

$$B(\mathrm{M1}; J \to J-1)$$

$$= \frac{3}{4\pi}\left(g_{J_\nu} - g_{J_\pi}\right)^2 \frac{(C'+1)(C'-2J_\nu)(C'-2J_\pi)(C'-2J+1)}{4J(2J+1)}$$

with $C' = J_\nu + J_\pi + J$.

Classical approximation:

$$B(\mathrm{M1}; J \to J-1) \approx \frac{3}{4\pi}\left(g_{J_\nu} - g_{J_\pi}\right)^2 \frac{(2J_\nu+1)^2(2J_\pi+1)^2}{16J(2J+1)} \sin^2\theta_{\nu\pi}$$

M1 transitions in ^{199}Pb

Proposed configuration of states in band 1:

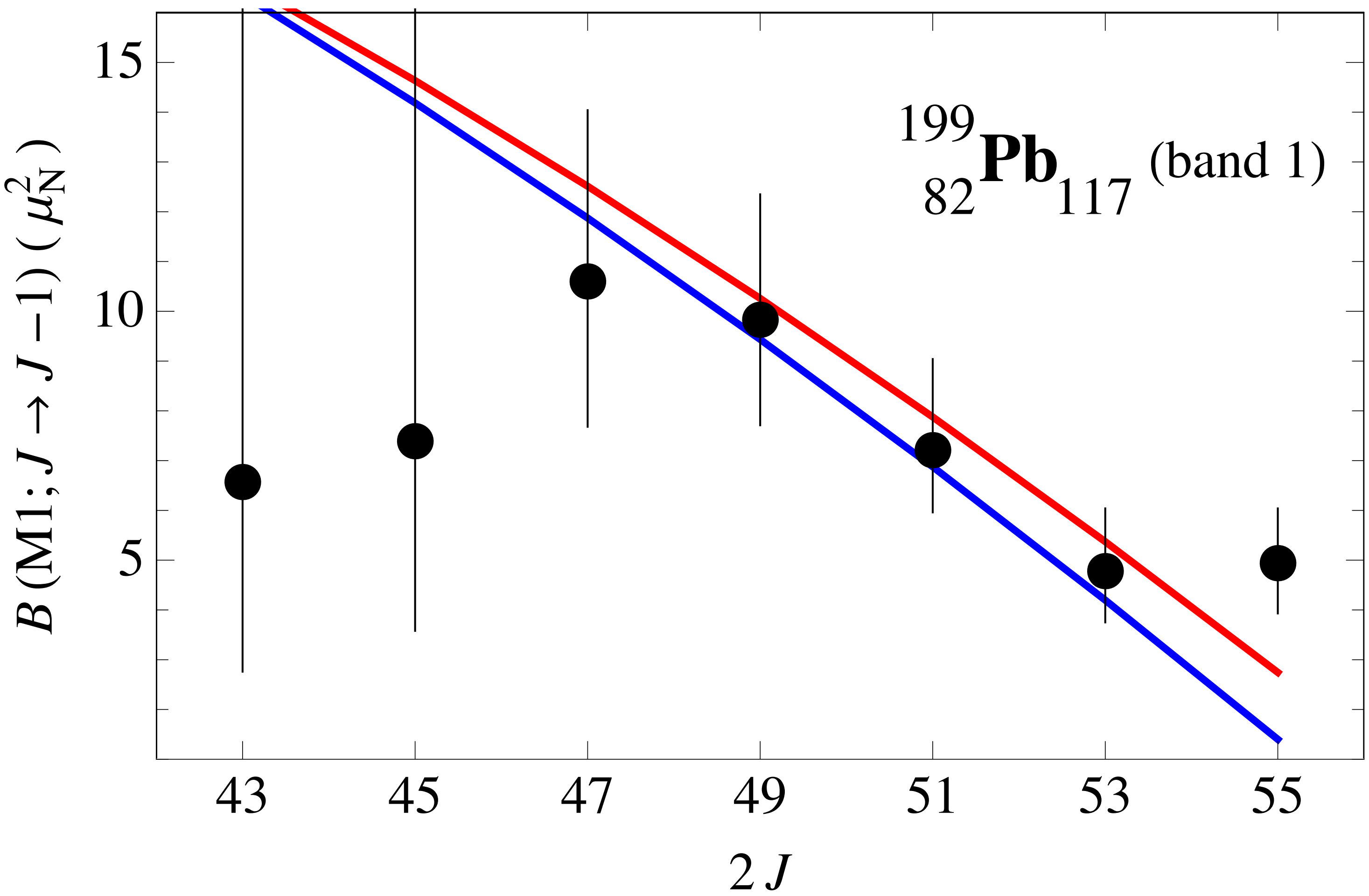
$$\left[\nu\left(1i_{13/2}^{-3}\right)^{33/2}\times\pi\left(1h_{9/2}1i_{13/2}\right)^{11}\right]^{(J)}$$

Calculation of *g* factors:

$$\nu\left(1i_{13/2}^{-3}\right)^{33/2}:g_{J_\nu}=g^{\nu}_{1i_{13/2}}=-0.29$$

$$\pi\left(1h_{9/2}1i_{13/2}\right)^{(11)}:g_{J_\pi}=\frac{9}{22}g^{\pi}_{1h_{9/2}}+\frac{13}{22}g^{\pi}_{1i_{13/2}}=1.03$$

M1 transitions in ^{199}Pb



Conclusions and outlook

The geometry of the shears mechanism can be derived from the spherical shell model.

It establishes a connection with microscopic properties of the neutron-proton interaction and indicates the limits of application.

Outlook:

Proof of the np-nh conjecture.

Analysis of other interactions (tensor…).

Treatment of mixed configurations.

P. Van Isacker & A.O. Macchiavelli, Phys. Rev. C **87** (2013) 061301(R)