

# Two-nucleon knockout; methodology and structure sensitivity

"Probing Two-Nucleon Correlations using Reactions"  
FUSTIPEN Topical Meeting  
GANIL, 18<sup>th</sup> March 2011

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Supported by the Science and Technology  
Facilities Council (STFC) Grant: ST/F012012.



# Most recent publications that relate:

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PHYSICAL REVIEW C **82**, 044616 (2010)

## **Correlations probed in direct two-nucleon removal reactions**

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(Received 25 July 2010; published 29 October 2010)

PHYSICAL REVIEW C **83**, 014605 (2011)

## **Two-nucleon correlation effects in knockout reactions from $^{12}\text{C}$**

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(Received 7 December 2010; published 26 January 2011)

# Two-nucleon removal (knockout) reaction sensitivity

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Probe of (spatial) two-nucleon correlations (g.s.)

angular  $\longleftrightarrow$  orbital angular momentum

Bottom line:

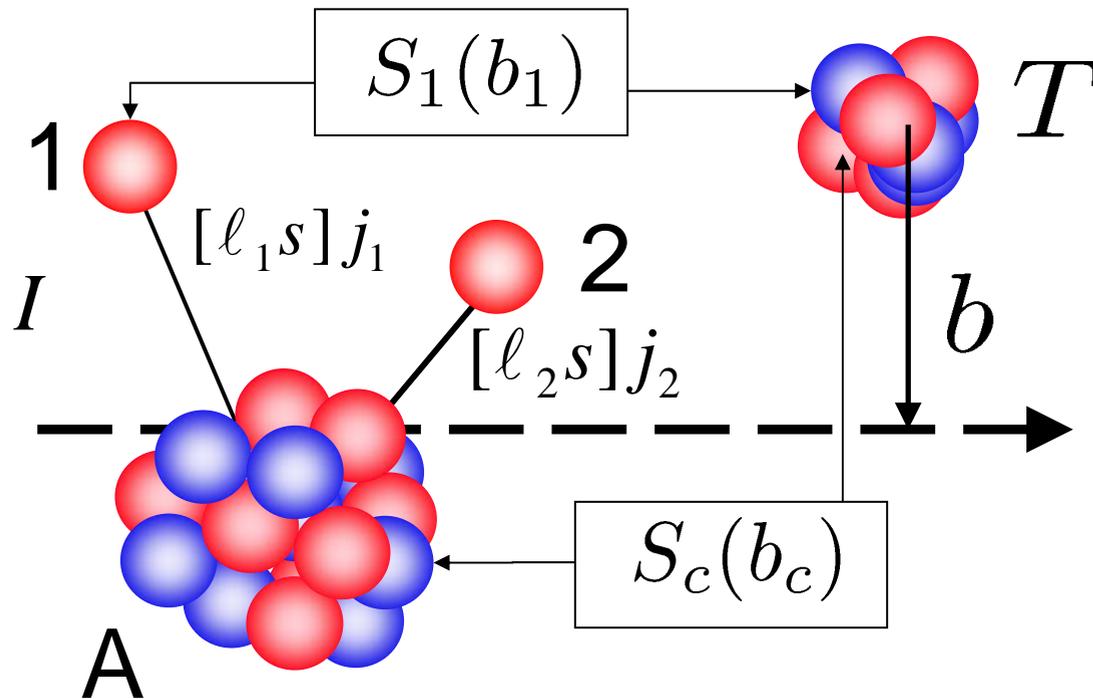
There is a need for data to benchmark and validate predictions for more exclusive final-state observables

# Outline of this contribution

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1. Removal (knockout) reactions – essentials:  
Spatial selectivity - near surface dominance  
Thresholds: direct vs indirect (two-step) pathways
2. 2N overlaps, two-particle density – angular correlations – value of the LS representation
3. Limited data sets so far – status/raincheck
4. The case of  $^{12}\text{C}(-2\text{N})$  – asks several questions
5. Summary comments (Ed Simpson will expand on other interesting aspects, good test cases and the latest ideas/results.)

# Sudden removal – eikonal reaction dynamics



Inclusive wrt target, 1 and 2 final states. Cross sections [and the  $S(b)$ ] account for 2N removal by both elastic and stripping (absorptive) events. Must add these.

State of residue using gamma-ray spectroscopy

$$\sigma_f = \frac{1}{2I+1} \sum_M \int d\vec{b} \langle F_{IM} | \hat{O}(c, 1, 2) | F_{IM} \rangle$$

# Structure interface – via the two-nucleon overlaps

$$\begin{aligned}\Psi_{J_i M_i}^{(f)}(1,2) &\equiv \langle \Phi_{J_f M_f}(A) | \Psi_{J_i M_i}(A, 1, 2) \rangle \\ &= \sum_{I \mu \alpha} C_{\alpha}^{J_i J_f I}(I \mu J_f M_f | J_i M_i) [\overline{\phi_{j_1}(1) \otimes \phi_{j_2}(2)}]_{I \mu}\end{aligned}$$

$$[\overline{\phi_{j_1}(1) \otimes \phi_{j_2}(2)}]_{I \mu} = -N_{12} \langle 1, 2 | [a_{j_1}^{\dagger} \otimes a_{j_2}^{\dagger}]_{I \mu} | 0 \rangle$$

$$D_{\alpha} = N_{12} / \sqrt{2} = 1 / \sqrt{2(1 + \delta_{12})}$$

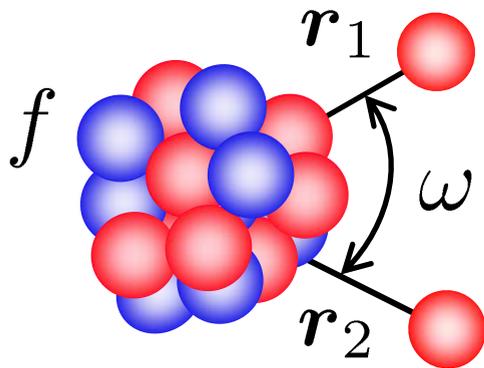
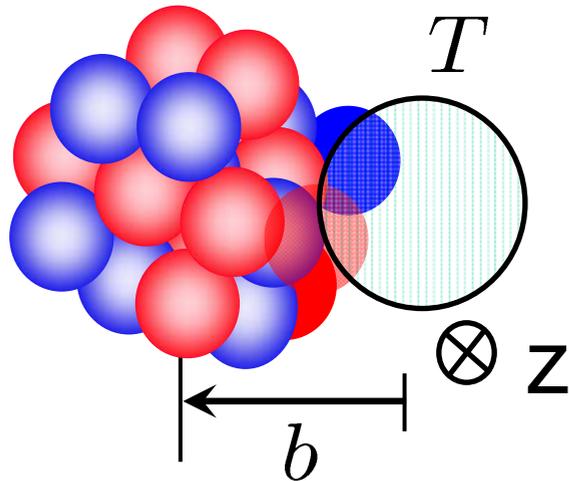
We use this AS IS – no Moshinsky, NN relative s-states projection ... no light-ion vertex restrictions

with  $J_i = 0^+$   $jj$ -two-nucleon amplitudes – TNA

$$F_{IM}(1, 2) = \sum_{j_1 j_2} (-)^{I+M} \boxed{C(j_1 j_2 I)} / \hat{I} [\overline{\phi_{j_1 m_1} \otimes \phi_{j_2 m_2}}]_{I-M}$$

# Sample a cylindrical volume at projectile surface

$$2N \text{ stripping} : \hat{O}(c, 1, 2) = |S_c|^2 (1 - |S_1|^2) (1 - |S_2|^2)$$



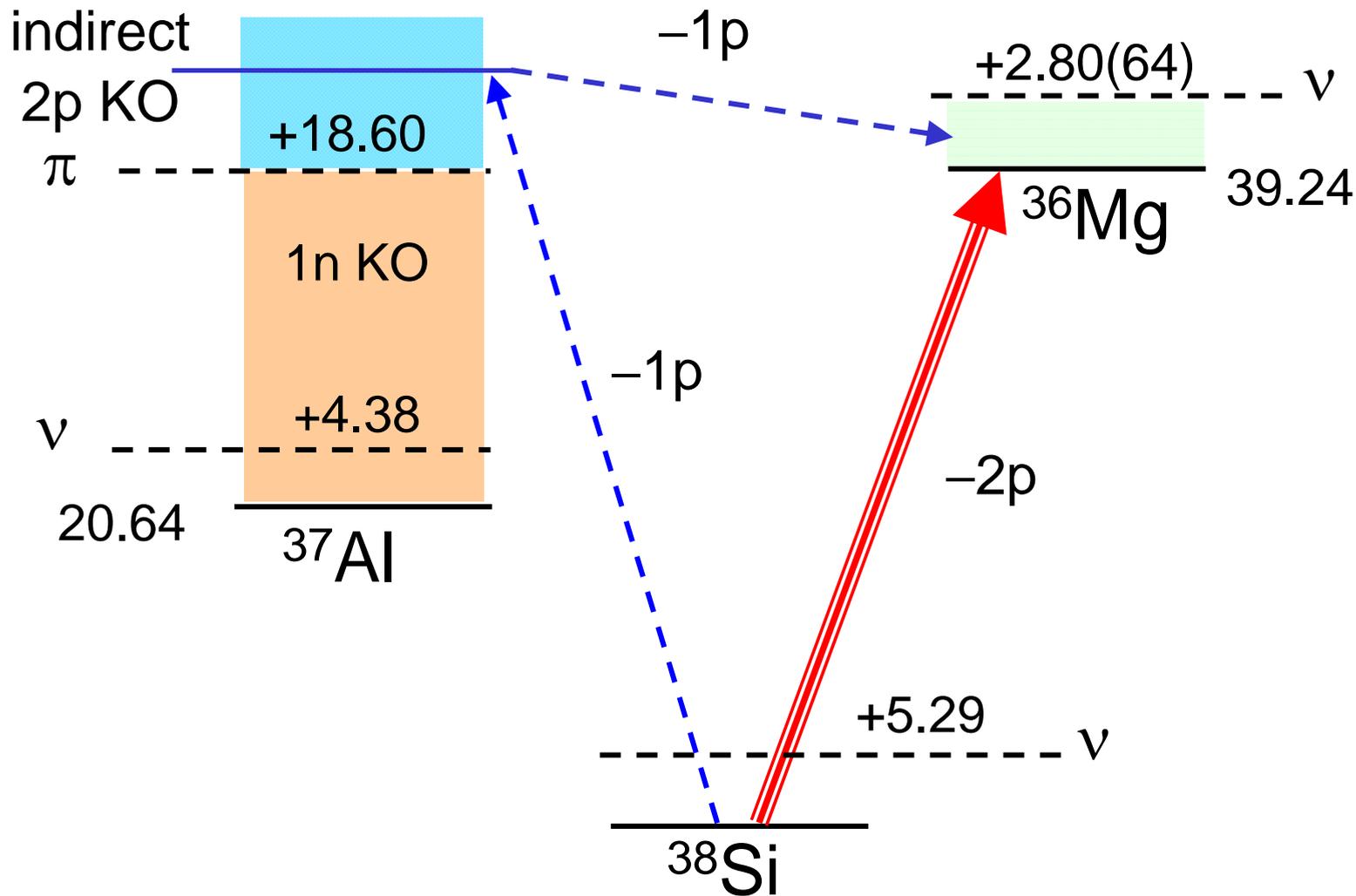
(i) 2N removal cross sections will be sensitive to the spatial correlations of pairs of nucleons near the surface

(ii) No (iso)spin bias (of  $(T)S=0$  versus  $(T)S=1$  pairs) in this 2N removal reaction mechanism

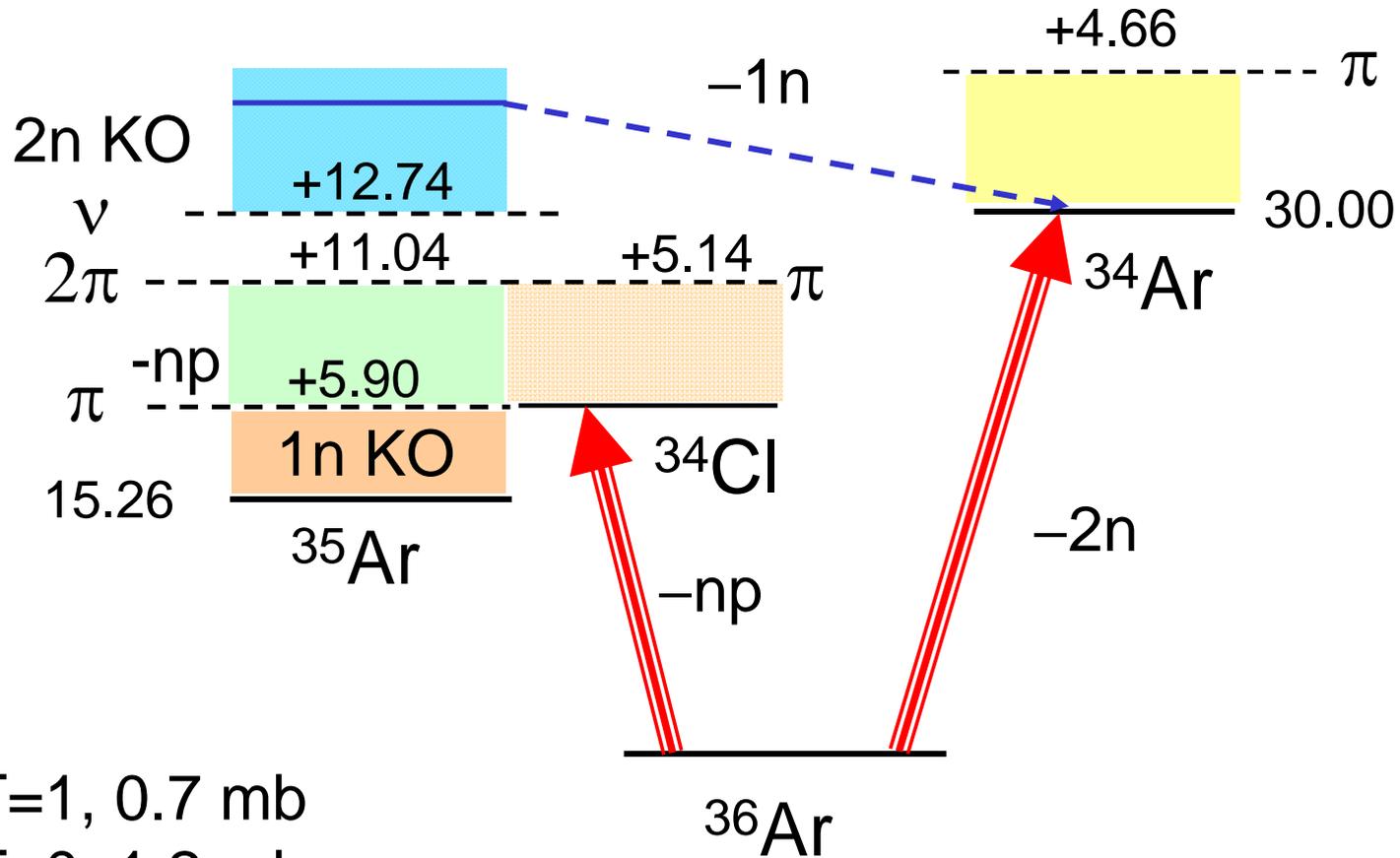
(iii) Expectation of the sensitivity to correlations can be predicted from 2N overlaps in the sampled volume

(iv) No linear or angular momentum mismatch – mechanism ‘sees’ ALL hole-like-state configurations

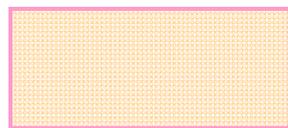
# Direct two-proton removal reaction mechanism



# -np exploratory: messy direct + indirect contributions



$0^+, T=1, 0.7 \text{ mb}$   
 $1^+, T=0, 1.2 \text{ mb}$



14 mb



19 mb

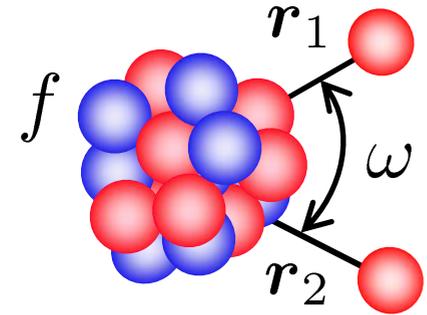


1.8 mb

# Two-nucleon position correlations

Summing over spins (to which we are insensitive) the two nucleon joint-position probability is:

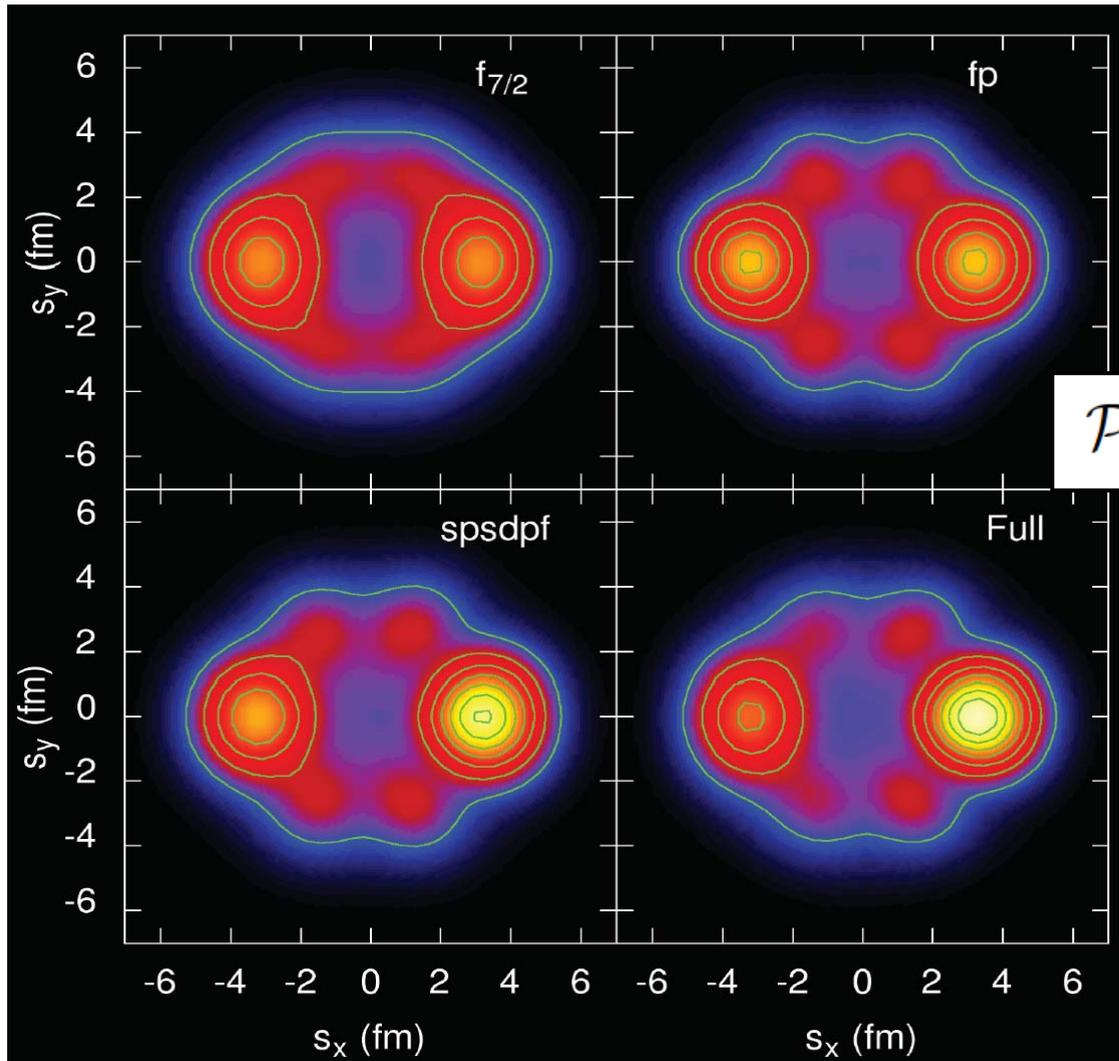
$$\begin{aligned} \rho_f(\mathbf{r}_1, \mathbf{r}_2) &= \frac{1}{\hat{J}_i^2} \sum_{M_i M_f} \langle \Psi_i^{(F)} | \Psi_i^{(F)} \rangle_{sp} \\ &= \sum_{LST} \sum_{I\alpha\alpha'} \frac{\mathfrak{C}_{\alpha LS}^{IT} \mathfrak{C}_{\alpha' LS}^{IT} D_\alpha D_{\alpha'}}{\hat{L}^2} (T\tau T_f \tau_f | T_i \tau_i)^2 \\ &\quad \times [U_{\alpha\alpha'}^D(\mathbf{r}_1, \mathbf{r}_2) \Gamma^{L,D}(\omega) \\ &\quad - (-)^{S+T} U_{\alpha\alpha'}^E(\mathbf{r}_1, \mathbf{r}_2) \Gamma^{L,E}(\omega)] , \end{aligned}$$



$$\begin{aligned} \Gamma_{\ell_1 \ell_2 \ell'_1 \ell'_2}^L(\omega) &= (-1)^L \frac{\hat{\ell}_1 \hat{\ell}'_1 \hat{\ell}_2 \hat{\ell}'_2 \hat{L}^2}{(4\pi)^2} \sum_k W(\ell_1 \ell_2 \ell'_1 \ell'_2; Lk) \\ &\quad \times (-1)^k (\ell_1 0 \ell'_1 0 | k 0) (\ell_2 0 \ell'_2 0 | k 0) P_k(\cos \omega) \end{aligned}$$

# Perturbative extended basis: 48Ca(-2n, gs)

$$\Psi = \psi((nlj)^2) + \sum_{n_1 n_2 l' j'} \psi(n_1 l' j', n_2 l' j') \frac{\langle n_1 l' j', n_2 l' j' | V | (nlj)^2 \rangle}{2\epsilon(nlj) - \epsilon(n_1 l' j') - \epsilon(n_2 l' j')}$$



$$\mathcal{P}_f(\mathbf{s}_1, \mathbf{s}_2)$$

$$= \int dz_1 \int dz_2 \rho_f(\mathbf{r}_1, \mathbf{r}_2)$$

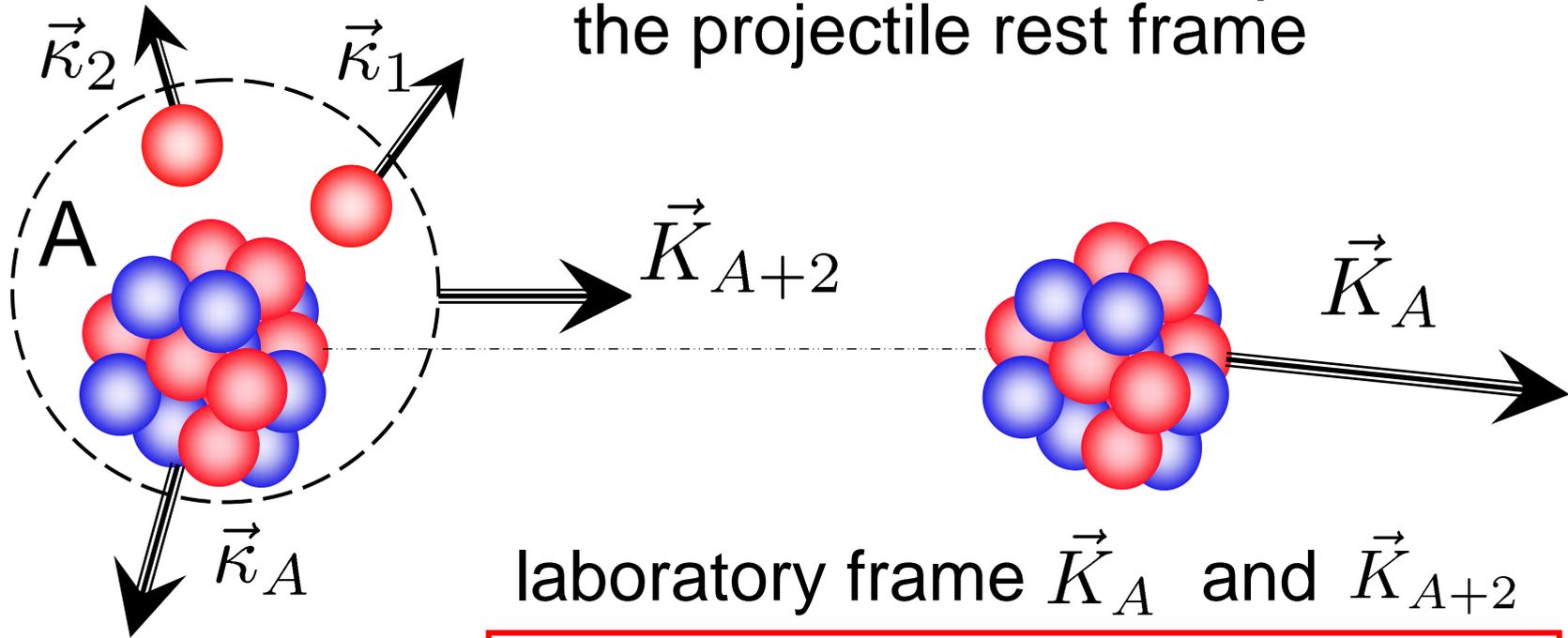
$$\vec{s}_1 = (3.8, 0.0)$$

$$\sigma \rightarrow \approx 2\sigma$$

$$[f_{7/2}]^2$$

# Sudden 2N removal from the mass A residue

Sudden removal: residue momenta probe the summed momenta of pair in the projectile rest frame

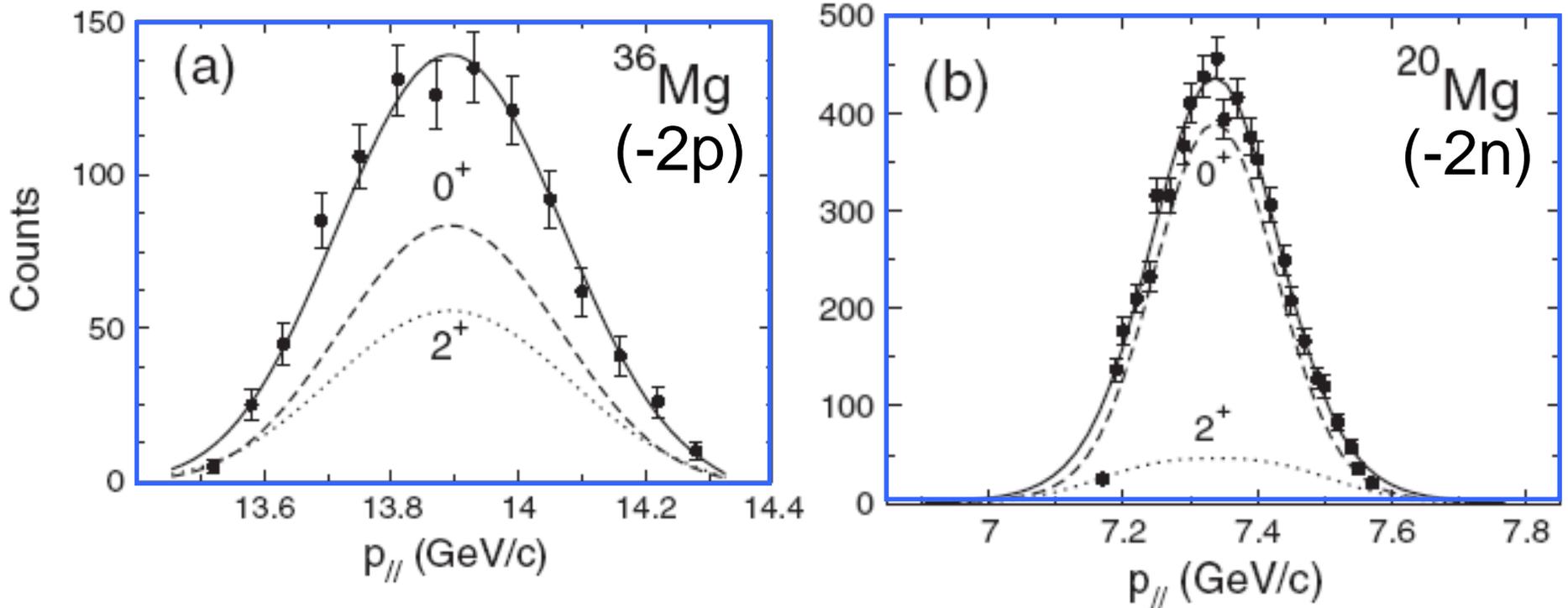


Projectile rest frame

$$\vec{K}_A = \frac{A}{A+2} \vec{K}_{A+2} - [\vec{k}_1 + \vec{k}_2]$$

and component equations

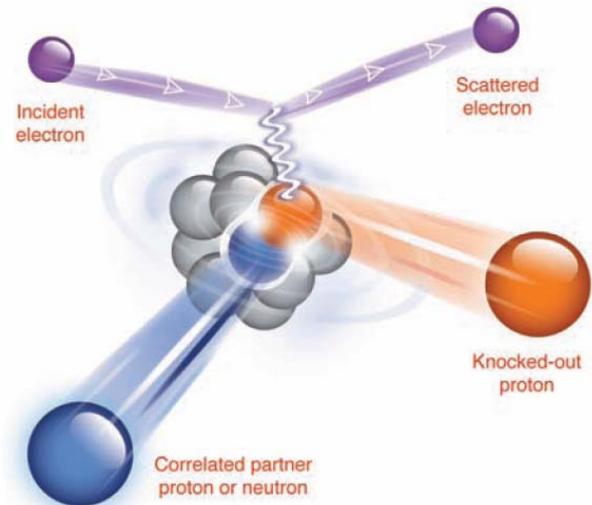
# “Inclusive” two-nucleon removal $p_{//}$ distributions



$$\bar{\mathcal{P}}_f(\vec{s}_1, \vec{s}_2, \kappa_c) = \frac{1}{\hat{J}_i^2} \sum_{M_i M_f} \int d\kappa_1 \int d\kappa_2 \frac{\delta(\kappa_c + \kappa_1 + \kappa_2)}{(2\pi)^2} \times \left\langle \left| \int dz_1 \int dz_2 e^{i\kappa_1 z_1} e^{i\kappa_2 z_2} \Psi_{J_i M_i}^{(F)} \right|^2 \right\rangle_{\text{sp}},$$

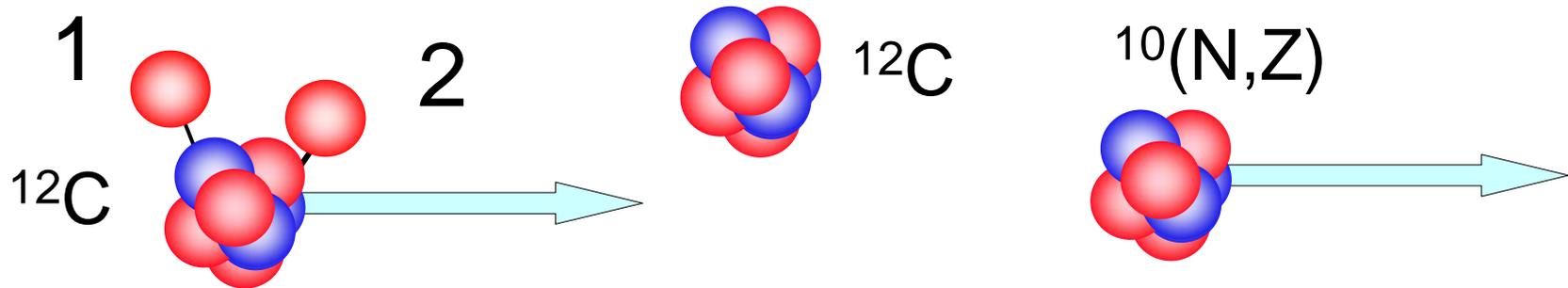
# Probing Cold Dense Nuclear Matter

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The protons and neutrons in a nucleus can form strongly correlated nucleon pairs. Scattering experiments, in which a proton is knocked out of the nucleus with high-momentum transfer and high missing momentum, show that in carbon-12 the neutron-proton pairs are nearly 20 times as prevalent as proton-proton pairs and, by inference, neutron-neutron pairs. This difference between the types of pairs is due to the nature of the strong force and has implications for understanding cold dense nuclear systems such as neutron stars.

# Two nucleon removal data – LBL measurements



Energy/nucleon	250 MeV	1.05 GeV	2.1 GeV
10Be Experimental	5.88	5.30 (30)	5.81 (29)
10C Experimental	5.33 (81)	4.44 (24)	4.11 (22)
.....	.....	.....	.....
10B Experimental	47.5 (24)	27.9 (22)	35.1 (34)

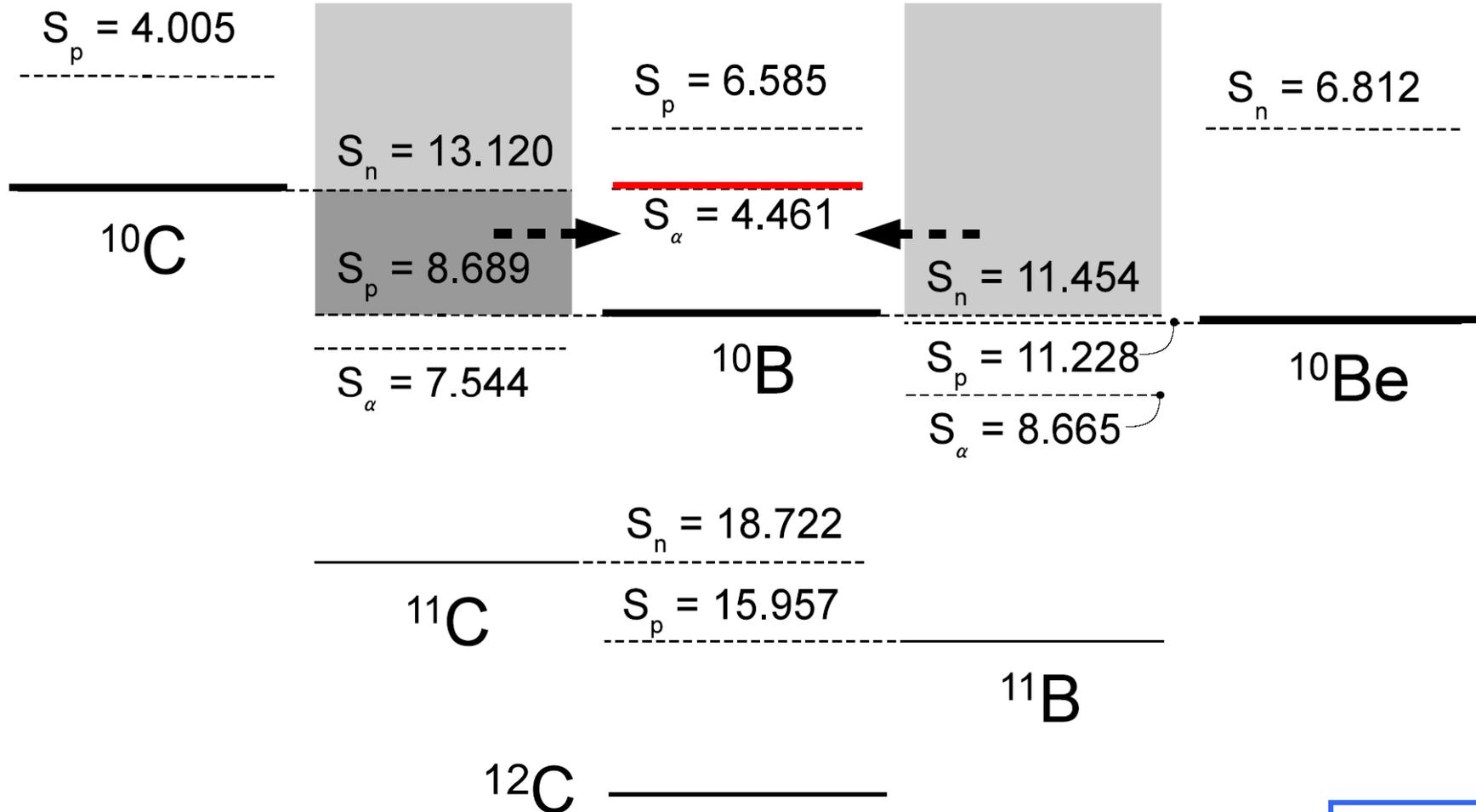
Based simply on combinatorics - and removal of the (4) p<sub>3/2</sub> nucleons we might expect scalings of:

$$\begin{aligned} \sigma \text{ (pp or nn)} &= [4 \times 3] / 2 = 6 \\ \sigma \text{ (np)} &= 4 \times 4 = 16 \end{aligned}$$

Cross sections: J.M. Kidd et al. PRC **37**, 2613 (1988)

Momentum distributions: D.E. Greiner et al., PRL **35**, 152 (1975)

# The $^{12}\text{C}$ case – direct 2n, 2p and np removal?

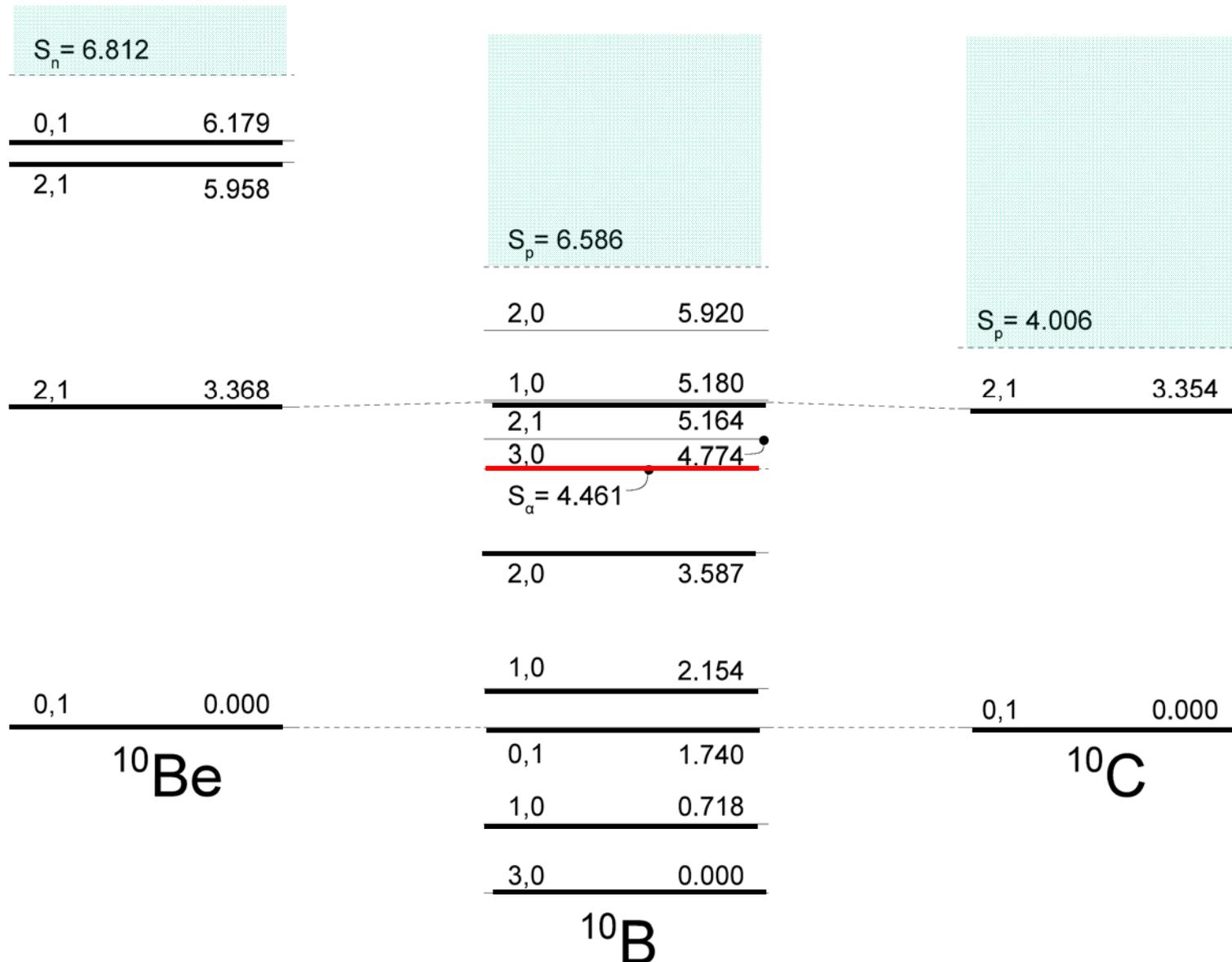


Indirect paths?

3.93

$C^2S \langle 11|12 \rangle : (3/2^-, \text{g.s.}) 3.16 (1/2^-, 2.0) 0.58 (3/2^-, 4.8) 0.19$

# Final states – rather few in all A=10 systems



# Cross sections at 2.1 GeV/u – exclusive breakdown

Residue	$J_f^\pi$	$T$	$\sigma_{str}$	$\sigma_{ds}$	$\sigma_{dif}$	$\sigma_{-2n}$
$^{10}\text{C}$	$0^+$	1	1.59	0.64	0.06	2.30
	$2^+$	1	1.96	0.71	0.06	2.74
					sum	5.04
					exp.	$4.11 \pm 0.22$
$^{10}\text{Be}$	$0^+$	1	1.65	0.68	0.07	2.40
	$2^+$	1	2.02	0.74	0.07	2.83
	$2^+$	1	0.88	0.32	0.03	1.23
	$0^+$	1	0.04	0.01	0.00	0.06
					sum	6.52
				exp.	$5.81 \pm 0.29$	
$^{10}\text{B}$	$3^+$	0	5.11	2.00	0.20	7.30
	$1^+$	0	2.47	1.01	0.10	3.58
	$0^+$	1	1.62	0.66	0.07	2.35
	$1^+$	0	1.81	0.69	0.07	2.57
	$2^+$	0	0.63	0.24	0.02	0.89
	$2^{+\dagger}$	1	1.99	0.72	0.07	2.33
					sum	19.02
				exp.	$35.10 \pm 3.40$	

$\longleftrightarrow B_\alpha = 16\%$

# Comparison to (inclusive) cross section data

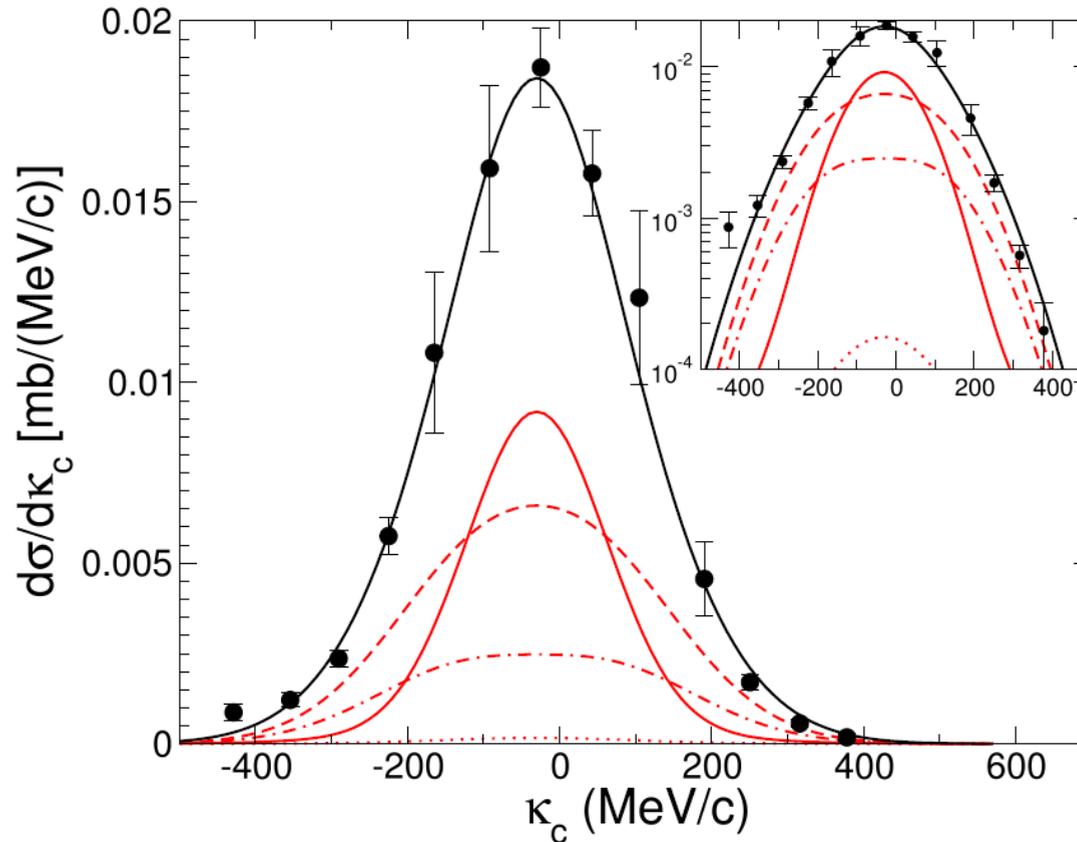
Energy MeV/u	$^{10}\text{Be}$			$^{10}\text{C}$		
	$\sigma_{th}$	$\sigma_{exp}$	$\sigma_{exp}/\sigma_{th}$	$\sigma_{th}$	$\sigma_{exp}$	$\sigma_{exp}/\sigma_{th}$
250 [5]	7.25	$5.88 \pm 9.70$	$0.81 \pm 1.34$	5.80	$5.33 \pm 0.81$	$0.92 \pm 0.14$
1050 [13]	6.62	$5.30 \pm 0.30$	$0.80 \pm 0.05$	5.13	$4.44 \pm 0.24$	$0.87 \pm 0.05$
2100 [13]	6.52	$5.81 \pm 0.29$	$0.89 \pm 0.04$	5.04	$4.11 \pm 0.22$	$0.82 \pm 0.04$

$^{10}\text{B}$		
$\sigma_{th}$	$\sigma_{exp}$	$\sigma_{exp}/\sigma_{th}$
21.57	$47.50 \pm 2.42$	$2.20 \pm 0.11$
19.27	$27.90 \pm 2.20$	$1.45 \pm 0.11$
19.03	$35.10 \pm 3.40$	$1.84 \pm 0.18$

Cross sections: J.M. Kidd et al. PRC **37**, 2613 (1988)

Momentum distributions: D.E. Greiner et al., PRL **35**, 152 (1975)

# Inclusive 2p removal momentum distribution



E.C. Simpson, JAT,  
PRC **83**, 014605  
(2011)

FIG. 4: Comparison of  $^{10}\text{Be}$  residue momentum distributions. Note the data is for a  $^9\text{Be}$  target whereas the calculations use a  $^{12}\text{C}$  target. The calculations have been offset by  $-30$  MeV/c and they have been scaled to match the experimental two-proton removal cross section ( $^9\text{Be}$  target,  $5.97$  mb).

Momentum distributions: D.E. Greiner et al., PRL **35**, 152 (1975)

# Existing (inclusive and averaged) p// distributions

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TABLE V: Gaussian fits to experimental and theoretical momentum distributions. These results are shown for comparison only - the experimental results are averaged over a range of targets, whereas the theoretical results are for the carbon target only. This considered, there is good agreement between the measurements and calculations, both in terms of the relative widths of different distributions and the absolute widths of each distribution.

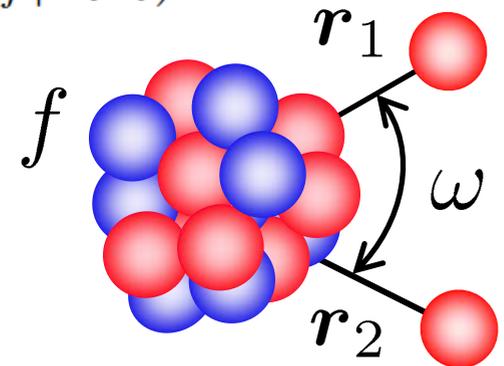
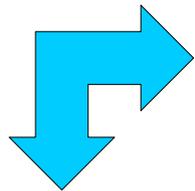
Residue	$\sigma_{exp}^{p//}$	$\sigma_{th}^{\kappa}$
$^{11}\text{B}$	$106 \pm 4$	99
$^{11}\text{C}$	$103 \pm 4$	100
$^{10}\text{Be}$	$129 \pm 4$	127
$^{10}\text{B}$	$134 \pm 3$	132
$^{10}\text{C}$	$121 \pm 6$	120

# Angular correlations – and L-transfer sensitivity

After summing over the nucleon spins (to which we are insensitive) the two nucleon joint-position probability is:

$$\rho_f(\mathbf{r}_1, \mathbf{r}_2) = \sum_{LST} \sum_{I\alpha\alpha'} \frac{\mathfrak{e}_{\alpha LS}^{IT} \mathfrak{e}_{\alpha' LS}^{IT} D_\alpha D_{\alpha'}}{\hat{L}^2} (T\tau T_f \tau_f | T_i \tau_i)^2$$

$$\times \left[ U_{\alpha\alpha'}^D(r_1, r_2) \Gamma^{L,D}(\omega) - (-)^{S+T} U_{\alpha\alpha'}^E(r_1, r_2) \Gamma^{L,E}(\omega) \right]$$



depends only on  $L (=l_1+l_2)$  of the two nucleons.

Structure calculation tells us strength of the L-content of the 2N overlap via the LS coupled two-nucleon amplitudes:

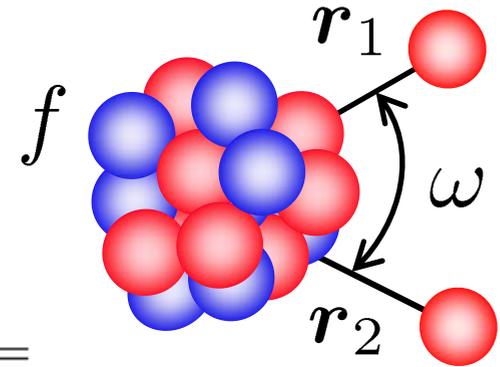
$$\mathfrak{e}_{\alpha LS}^{IT} = \hat{j}_1 \hat{j}_2 \hat{L} \hat{S} \left\{ \begin{array}{ccc} \ell_1 & s & j_1 \\ \ell_2 & s & j_2 \\ L & S & I \end{array} \right\} C_\alpha^{IT} \longrightarrow \text{predict p// distribution}$$

# Two-nucleon position correlations

The two nucleon joint-position probability is:

$$\rho_f(\mathbf{r}_1, \mathbf{r}_2) = \frac{1}{\hat{J}_i^2} \sum_{M_i M_f} \langle \Psi_i^{(F)} | \Psi_i^{(F)} \rangle_{sp}$$

$$\mathcal{P}_f(\mathbf{s}_1, \mathbf{s}_2) = \int dz_1 \int dz_2 \rho_f(\mathbf{r}_1, \mathbf{r}_2)$$



$J_f^\pi$	$[1p_{3/2}]^2$	$[1p_{1/2}, 1p_{3/2}]$	$[1p_{1/2}]^2$
$1_1^+$	0.69899	0.97868	-0.01067
$1_2^+$	-1.13385	0.22886	0.36314

$J_f^\pi$	$\sigma_{01}$	$\sigma_{10}$	$\sigma_{11}$	$\sigma_{21}$	$\sigma_{str}$
$1_1^+$	2.41	0.00	0.00	0.06	2.47
$1_2^+$	0.60	0.59	0.00	0.63	1.81

$^{12}\text{C}(-np) \rightarrow$   
 $^{10}\text{B}(1^+, T=0)$

$\sigma_{LS}$  (mb)

# Two-nucleon (spatial) correlations

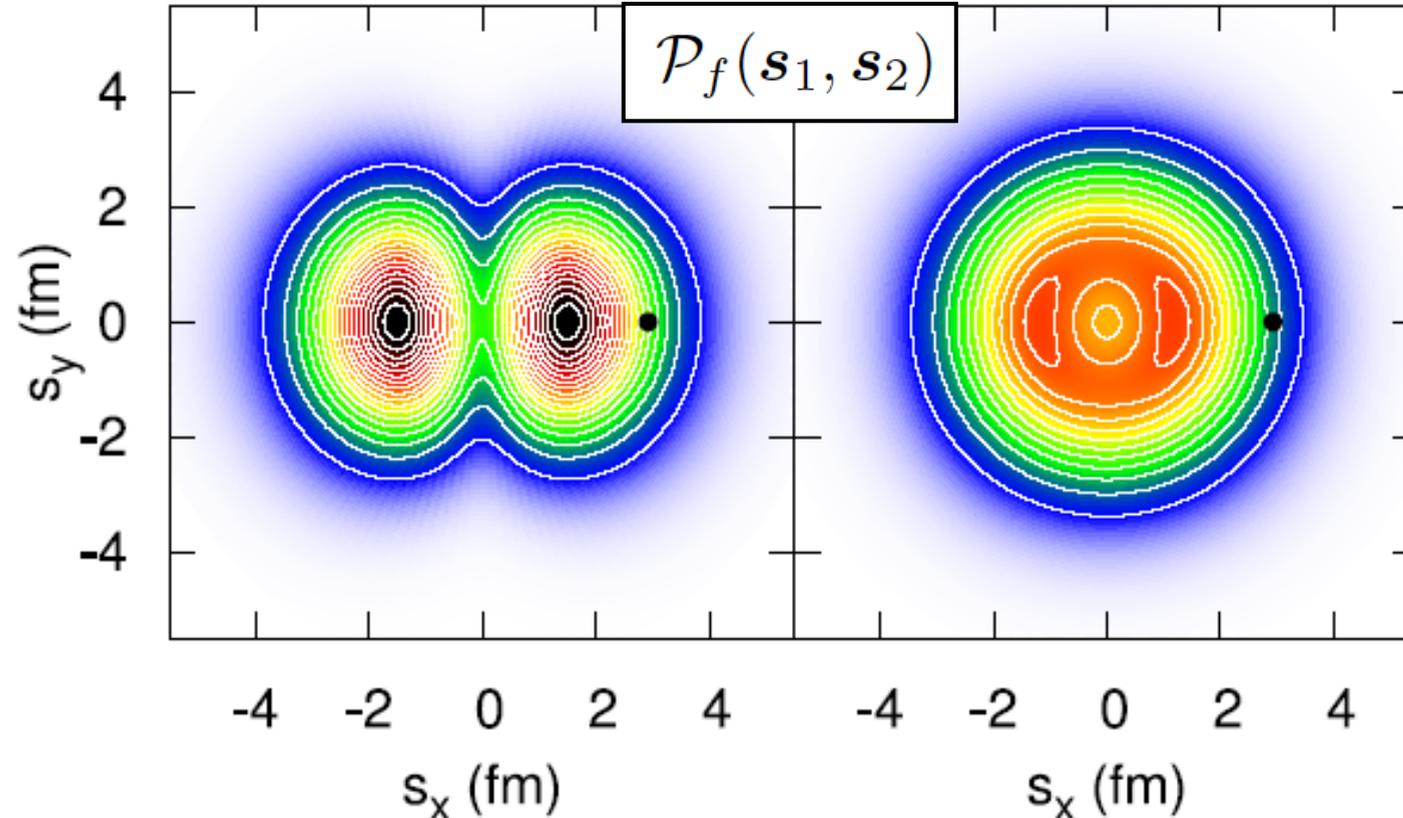


FIG. 4: Impact parameter plane-projected joint position probabilities for the first (left) and second (right)  $T = 0$   $^{10}\text{B}(1^+)$  states populated via  $np$  knockout from  $^{12}\text{C}$ .

# np-removal – specific predictions

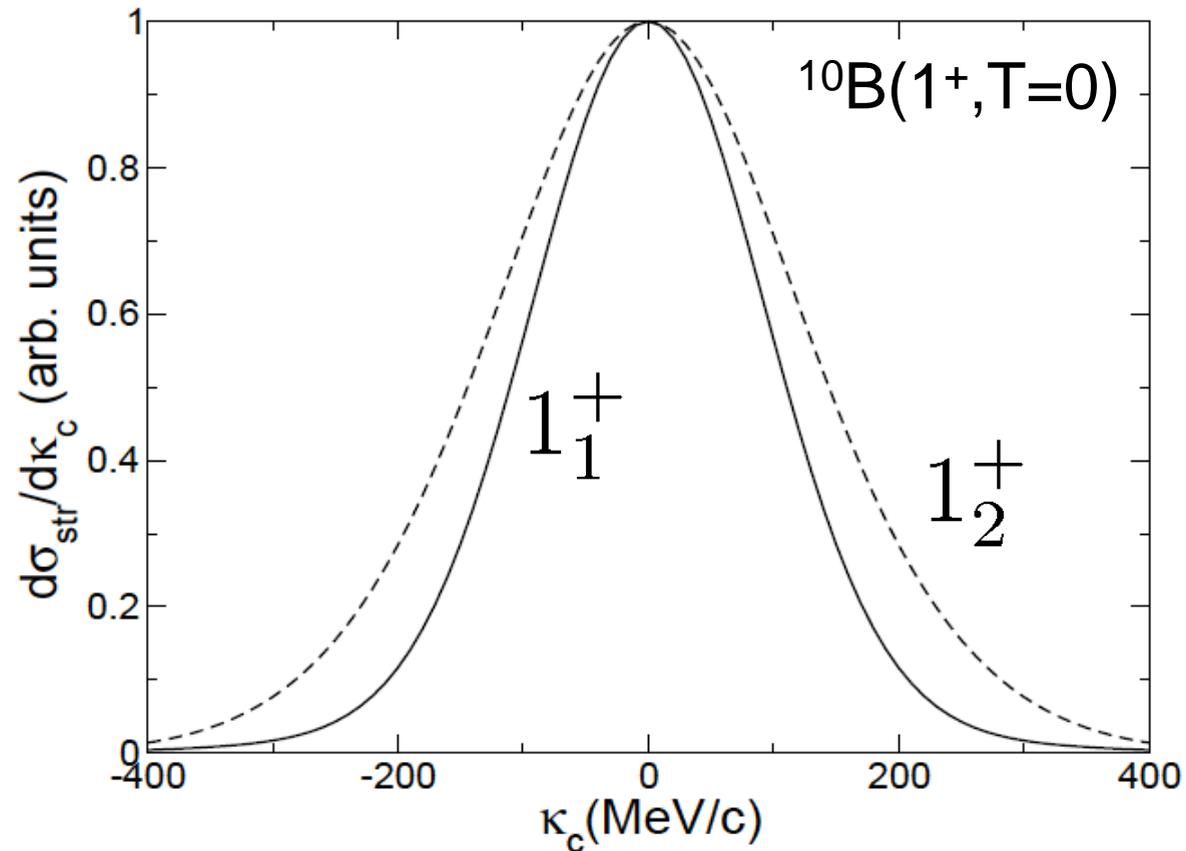
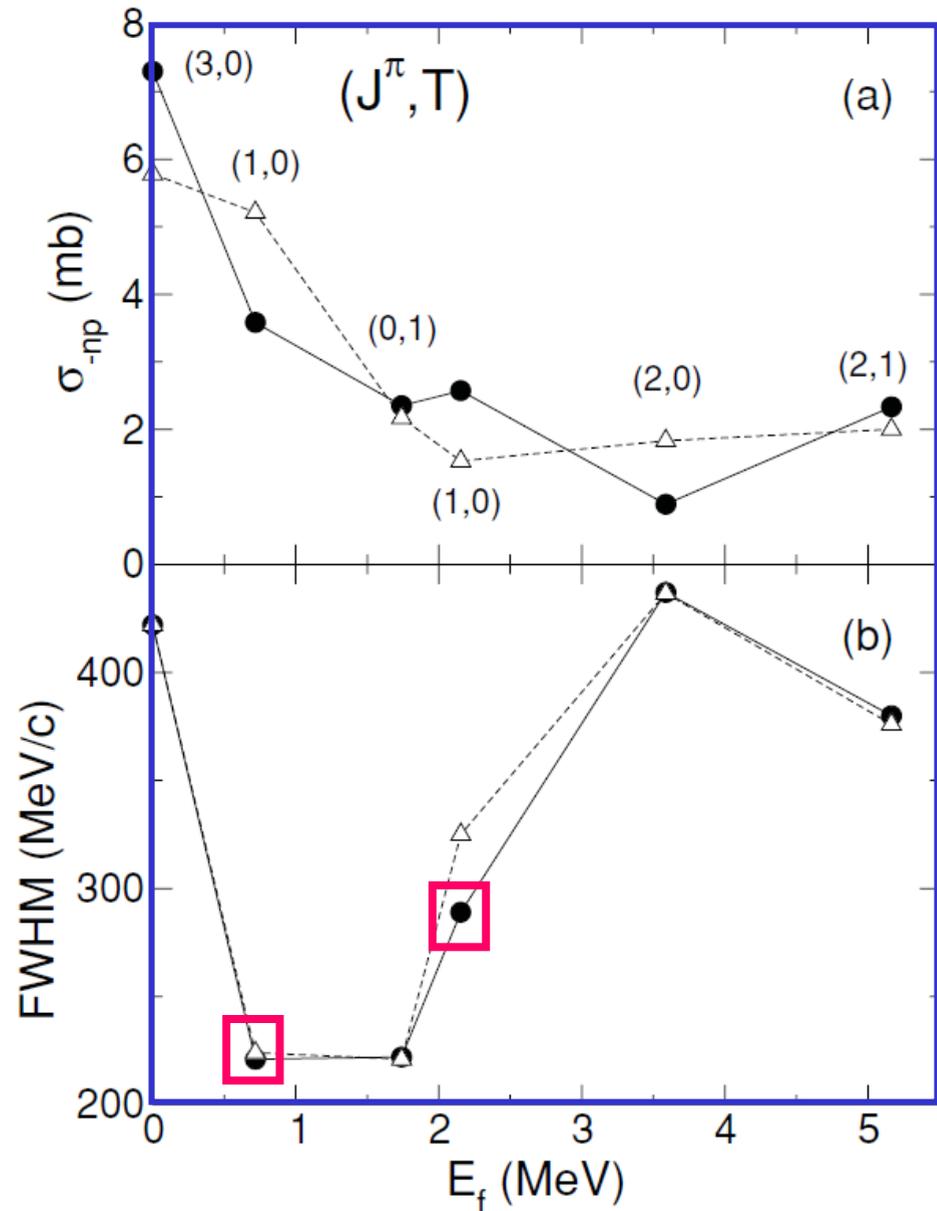


FIG. 3: Normalized residue momentum distributions for the first (solid) and second (dashed)  $^{10}\text{B}(J_f=1^+)$  states populated in  $np$  knockout from  $^{12}\text{C}$  at 2100 MeV per nucleon.

# Exclusive observables: $^{12}\text{C}(-np)$ case at 2.1 GeV/u

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(2011)

———— WBP  
----- PJT



## Summary comments – and a wish list

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1. At energies of fragmentation beams ( $\sim 100$  MeV per nucleon and greater) 2N removal calculations appear to be robust and can return quantitative information (certainly on relative strengths)
2. Exclusive final-state  $\sigma$  and  $p_{//}$  distributions after 2N removal can test the 2N correlations predicted by theoretical models in the two-particle overlaps
3. **There is still very little data** (np, but also nn and pp) to really validate the methodology - which can now make detailed, and exclusive predictions
4. **We need overlaps from non-shell model sources!**
5. **Test cases as well as the more exotic are needed, (e.g. in light systems and using stable beams?)**