Self-consistent Gorkov-Green's function calculations from Chiral-EFT interactions

V. Somà[1], T. Duguet[1,2], C. Barbieri[3]

[1]CEA/IRFU/SPhN, France
[2]NSCL and Michigan State University, USA
[3]University of Surrey, UK

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Outline

- Introduction and goals
- Elements of Green’s function methods
- Preliminary results
- Outlook

Introduction and goals

Goals

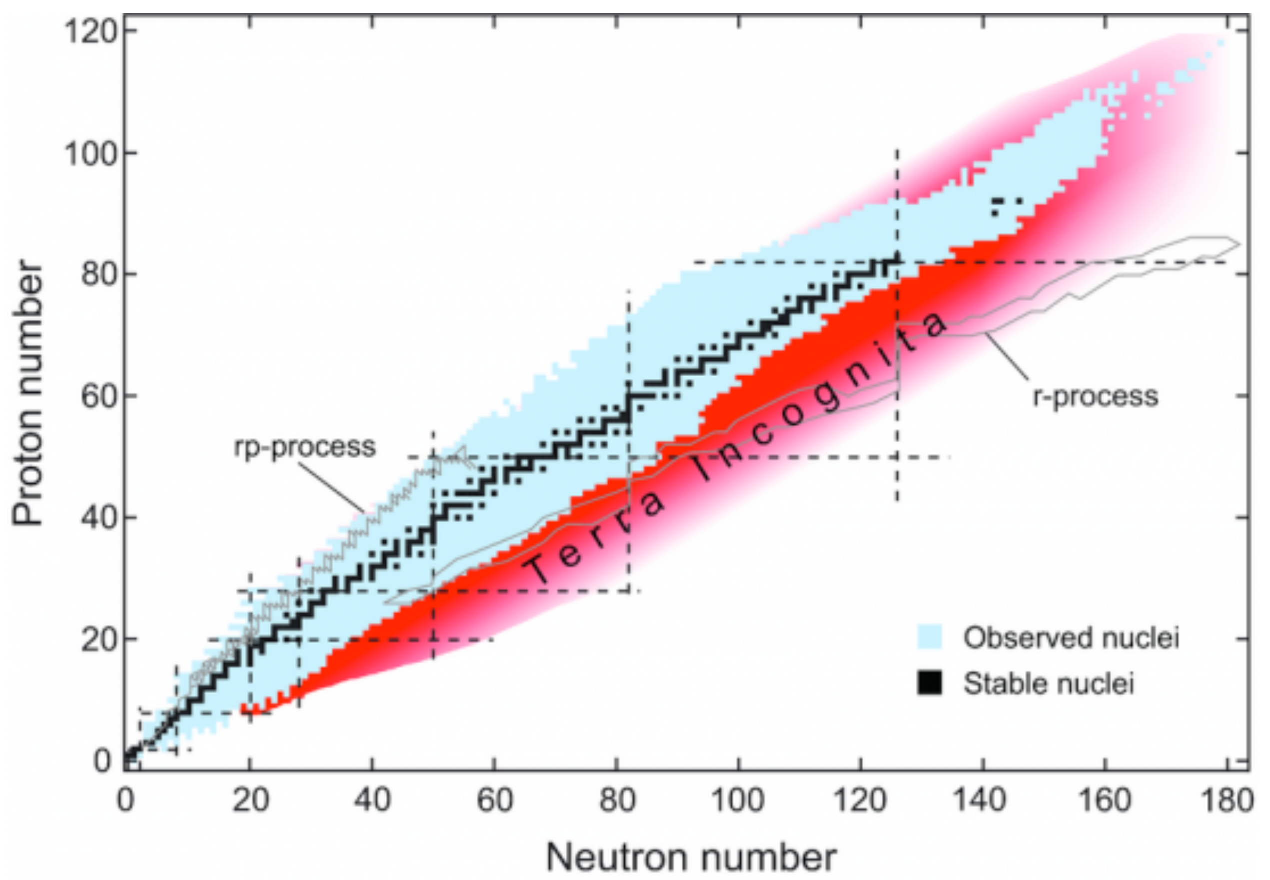
1) Tackle finite-nuclei superfluidity in an ab-initio fashion

2) Revisit many-body techniques using low-momentum interactions

3) Study the effect of NNN forces

4) Connect with (non-empirical) energy density functionals

Towards a unified description of nuclei



“Exact” methods (GFMC, NCSM, ...)

Ab-initio approaches (CC, SCGF, IM-SRG)

Shell model

SR and MR energy density functionals

State-of-the-art ab-initio nuclear structure theory

- Methods for an ab-initio description of medium-mass nuclei as of 2010

  (1) Coupled-cluster [Dean, Papenbrock, Hagen, ...]

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➟ ^{3}He, ^{4}He, ^{16}O, ^{40}Ca [Hagen *et al.* 2007]

➟ ^{16}O, ^{22}O, ^{24}O, ^{28}O [Hagen *et al.* 2009]

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➠ ^{16}O, ^{40}Ca [Barbieri, Dickhoff 2004]

➠ ^{56}Ni [Barbieri, Hjorth-Jensen 2009]

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→ Explicit treatment of pairing correlations is missing

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→ Explicit treatment of pairing correlations is missing

→ Extensions required to tackle open-shell nuclei

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⇩

Dyson-Green's functions ⟶ Gorkov-Green's functions

Low-momentum nuclear interactions

➟ Traditional “hard core” potentials

(1) Unconstrained

- short-range
- low-*k* - high-*k* coupling

(2) Low-*k* ⟷ high-*k* couplings make N-body calculations unbearable

(3) Details of high-*k* physics irrelevant to low-energy nuclear structure

Low-momentum nuclear interactions

RG

➟ Traditional "hard core" potentials ⇒ ➟ "Soft" NN and NNN interactions

(1) Unconstrained { short-range low-*k* - high-*k* coupling	(1) Universal low-k physics unchanged
(2) Low-*k* ⟷ high-*k* couplings make N-body calculations unbearable	(2) Low-*k* ⟷ high-*k* decoupled
(3) Details of high-*k* physics irrelevant to low-energy nuclear structure	(3) High-*k* physics screened out

Low-momentum nuclear interactions

RG

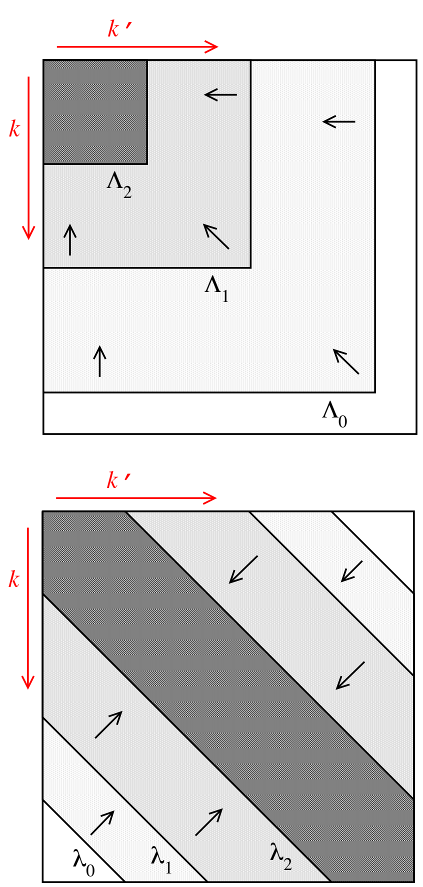
➟ Traditional “hard core” potentials ➡ ➟ “Soft” NN and NNN interactions

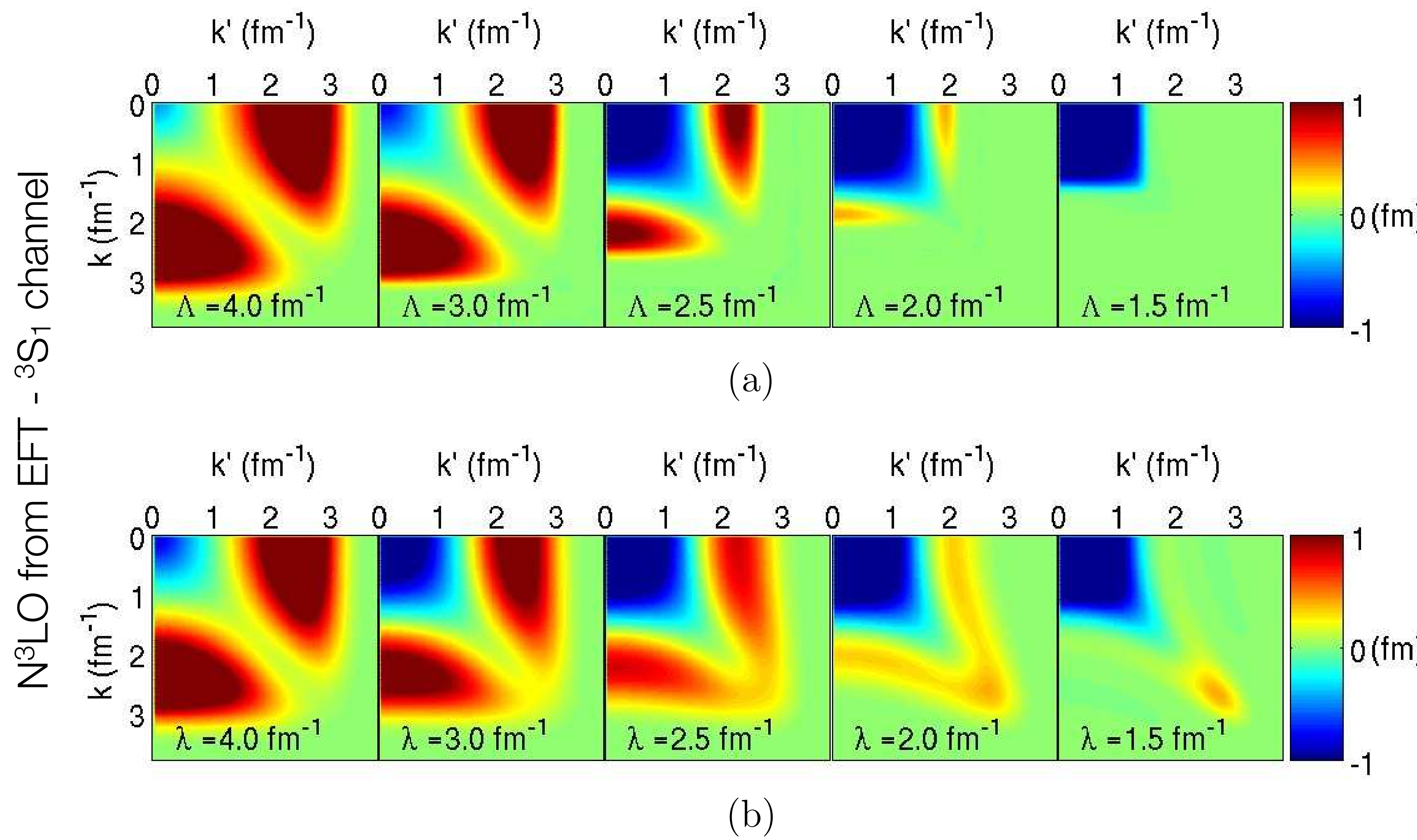
✺ Renormalization group transformations to decouple low and high momenta

(a) $V_{low\text{-}k}$

[Bogner, Furnstahl, Schwenk 2009]

(b) V_{SRG}

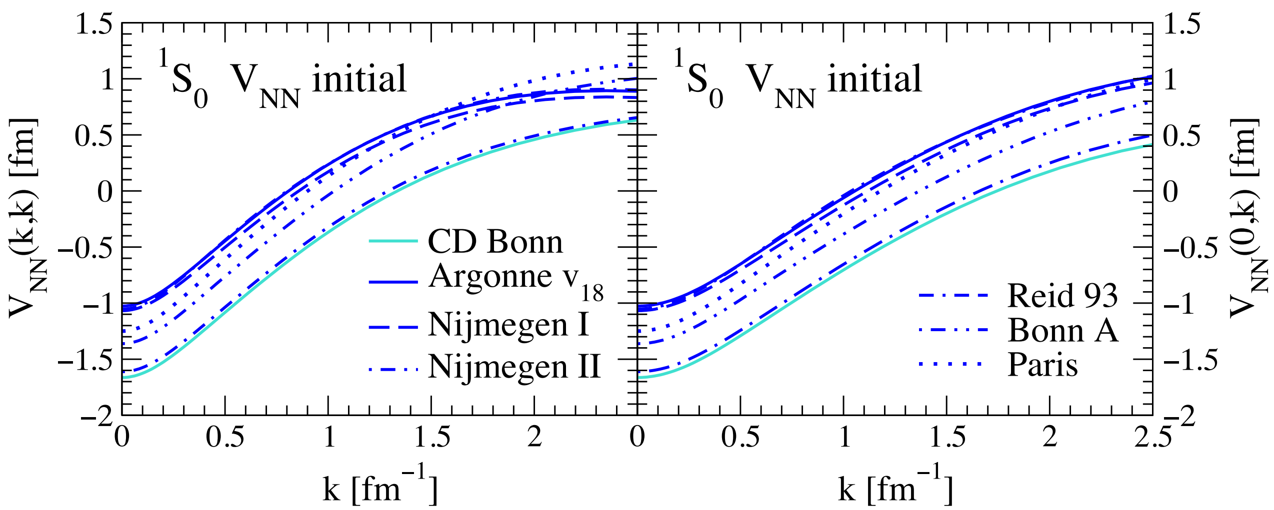


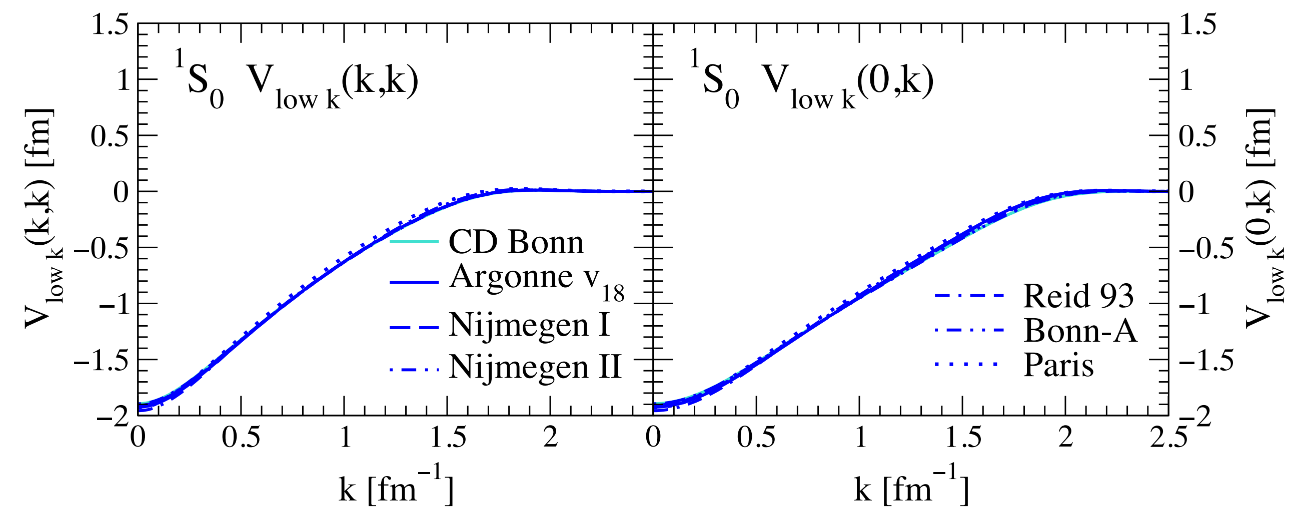


➟ NN scattering phase-shifts and deuteron binding energy conserved

Low-momentum nuclear interactions

✷ Universality





✷ RG transformations induce many-body forces

[Bogner, Furnstahl, Schwenk 2009]

✷ N-body observables are RG invariant if and only if

1) Induced N-body (e.g. 3-body) forces are kept

2) N-body calculation is exact

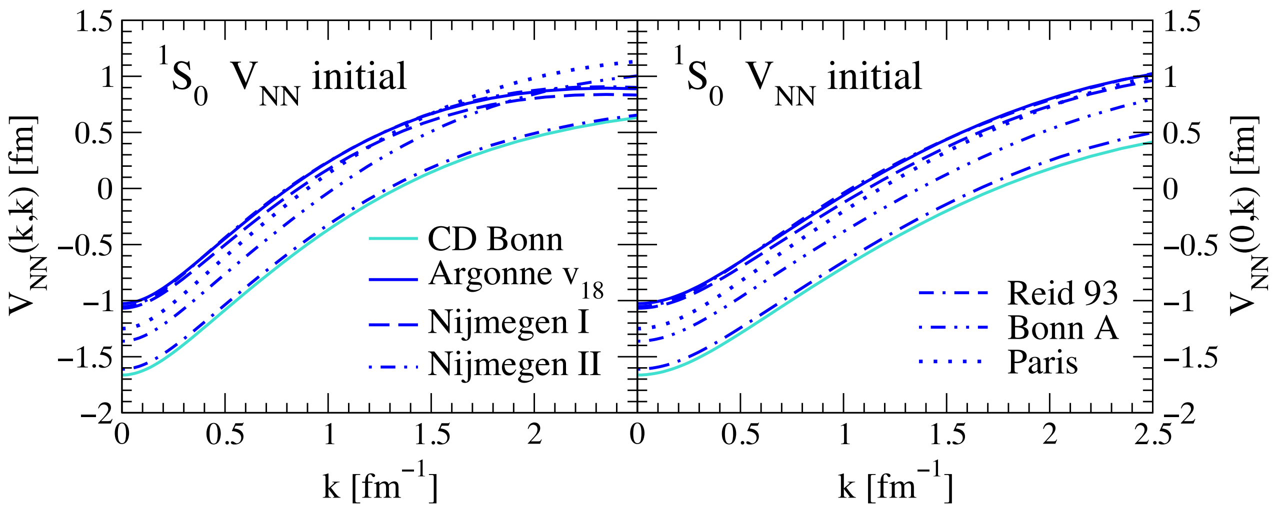
➡ RG cutoff dependence helps tracking

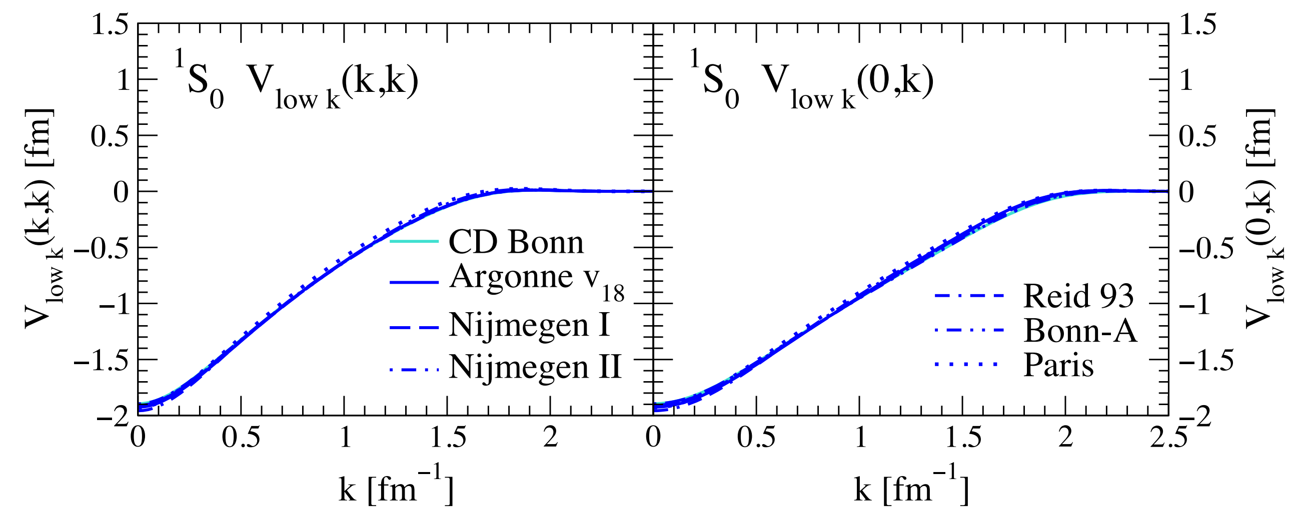
1) Importance of omitted N-body forces

2) Incomplete N-body calculation

Low-momentum nuclear interactions

- Universality

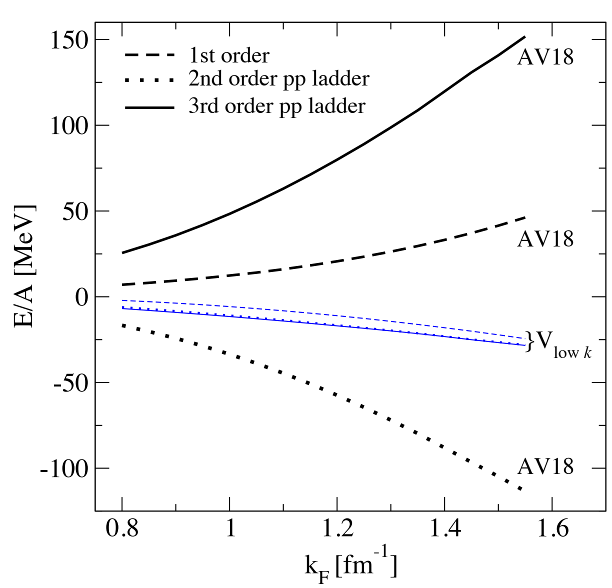


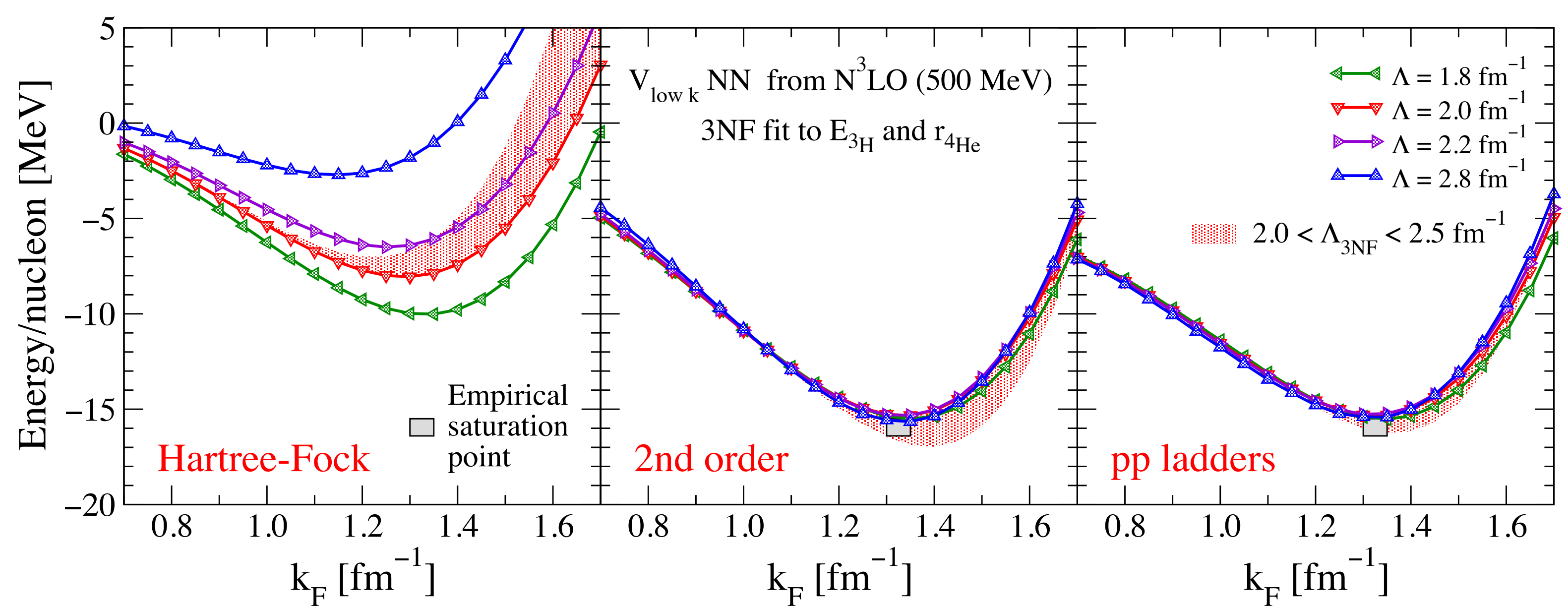


[Bogner, Furnstahl, Schwenk 2009]

- Perturbativeness?

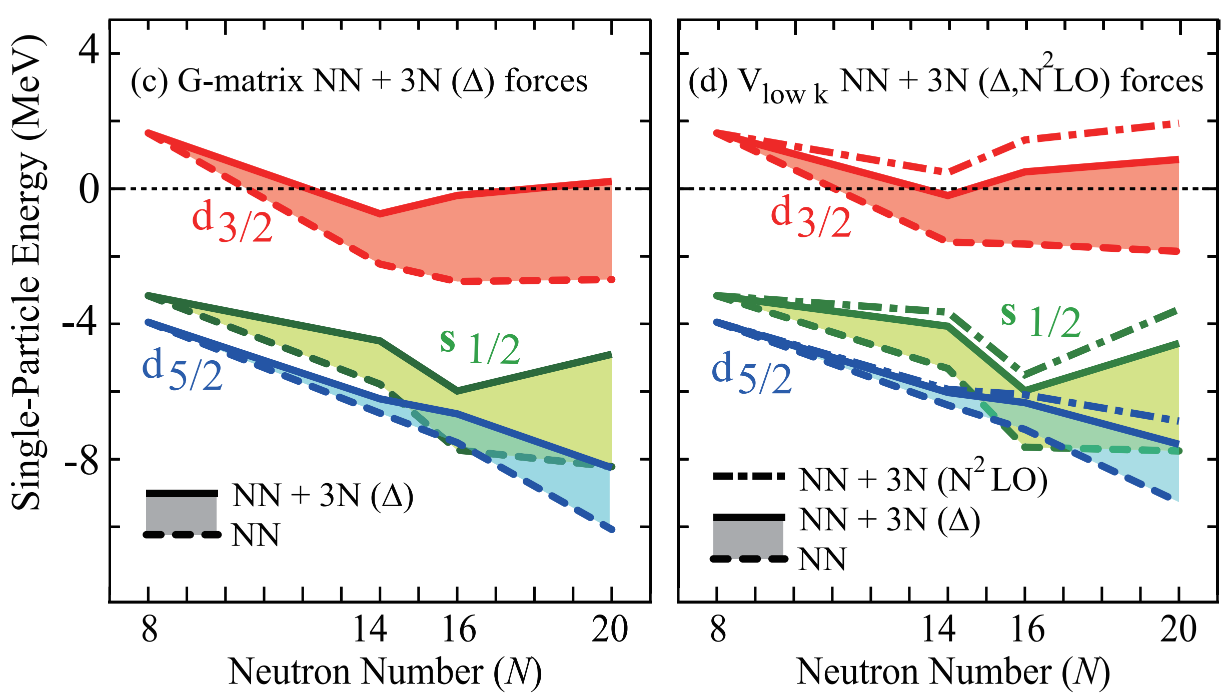
[Bogner, Furnstahl, Nogga, Schwenk 2009]





Three-body forces

- Realistic microscopic calculations cannot avoid the use of NNN forces
  - Binding energies, saturation properties and radii
  - Shell evolution
  - Spin-orbit splitting
  - Three-nucleon scattering



[Otsuka *et al.* 2010]

➟ Dripline location in O isotopes (^{24}O) possibly due to NNN physics

Three-body forces

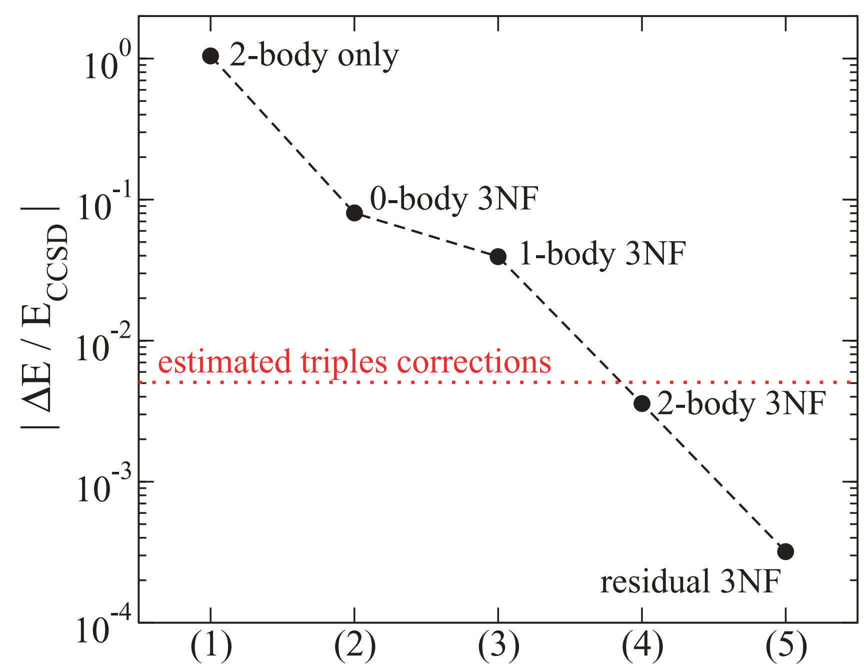
- Realistic microscopic calculations cannot avoid the use of NNN forces
  - Binding energies, saturation properties and radii
  - Shell evolution
  - Spin-orbit splitting
  - Three-nucleon scattering

- Currently: microscopic NNN interactions only in light systems and INM
  - Normal-ordered (average) part of NNN possibly sufficient

➠ Coupled-cluster in ^{4}He [Hagen *et al.* 2007]

➠ SCGF in INM [Somà, Bożek 2008]

➠ Perturbation theory in INM [Hebeler, Schwenk 2009]

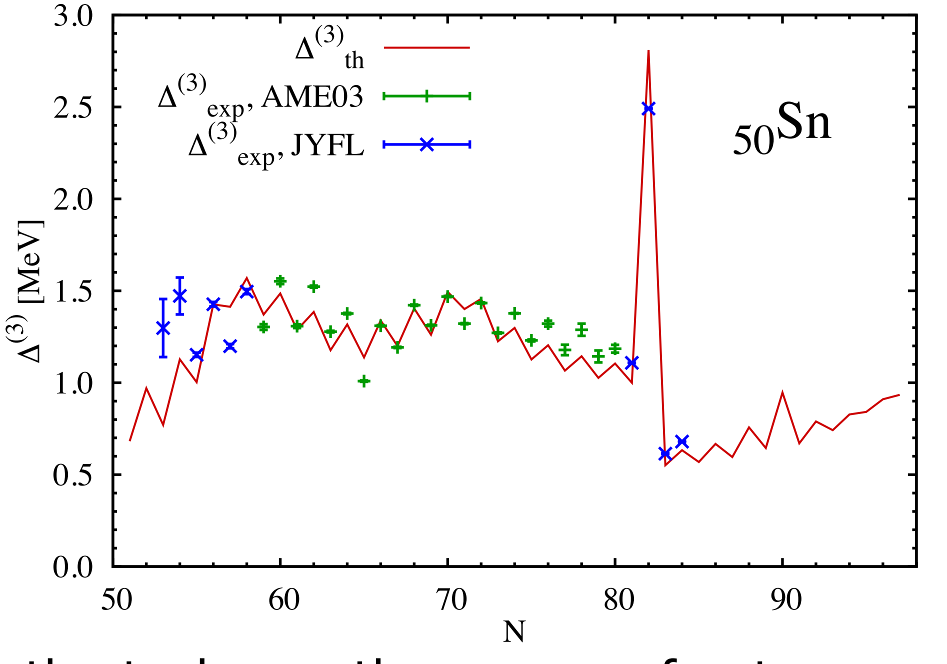


Connection to *non-empirical* EDF

- Standard EDF parameterizations (e.g. Skyrme, Gogny, relativistic)
  - Successful in major shell where adjusted
  - Lack predictive power in new regions of interest

Efforts to extend and connect with more fundamental approaches

- Non-empirical EDF from low-momentum interactions
  - Pairing channel [Duguet *et al.*]
  - Particle-hole channel [Gebremariam *et al.*]



Benchmarks needed from many-body methods that share the same features

Elements of Green's function methods

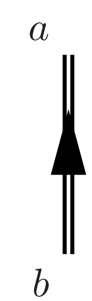
Dyson Green's functions

✷ Many-body Hamiltonian

$$H \equiv T + V^{NN} + V^{NNN} \equiv \sum_{ab} t_{ab}\, a_a^\dagger a_b + \frac{1}{(2!)^2} \sum_{abcd} \bar{V}^{NN}_{abcd}\, a_a^\dagger a_b^\dagger a_d a_c + \frac{1}{(3!)^2} \sum_{abcdef} \bar{V}^{NNN}_{abcdef}\, a_a^\dagger a_b^\dagger a_c^\dagger a_f a_e a_d$$

✷ One-body propagator or Green's function

$$i\, G_{ab}(t,t') \equiv \langle \Psi_0^N | T \left\{ a_a(t) a_b^\dagger(t') \right\} | \Psi_0^N \rangle \quad \equiv$$



✷ Heisenberg representation for creation/annihilation operators

$$\begin{aligned} a_b(t) &= \quad a_b^{(H)}(t) \quad \equiv \exp[iHt]\, a_b \exp[-iHt] \\ a_b^\dagger(t) &= \left[a_b^{(H)}(t) \right]^\dagger \equiv \exp[iHt]\, a_b^\dagger \exp[-iHt] \end{aligned}$$

✷ Fourier transform to energy domain

$$G_{ab}(\omega) = \int d\,(t-t')\, e^{i\,\omega(t-t')}\, G_{ab}(t,t')$$

✷ Hierarchy of coupled equations between 1-body, 2-body, ... N-body propagators

Observables (1)

1) Separation energy spectrum

$$G_{ab}(\omega) = \sum_k \frac{\mathcal{X}_a^{k\,(N)\,*}\,\mathcal{X}_b^{k\,(N)}}{\omega - E_k^{+\,(N)} + i\eta} + \sum_k \frac{\mathcal{Y}_a^{k\,(N)}\,\mathcal{Y}_b^{k\,(N)\,*}}{\omega - E_k^{-\,(N)} - i\eta}$$

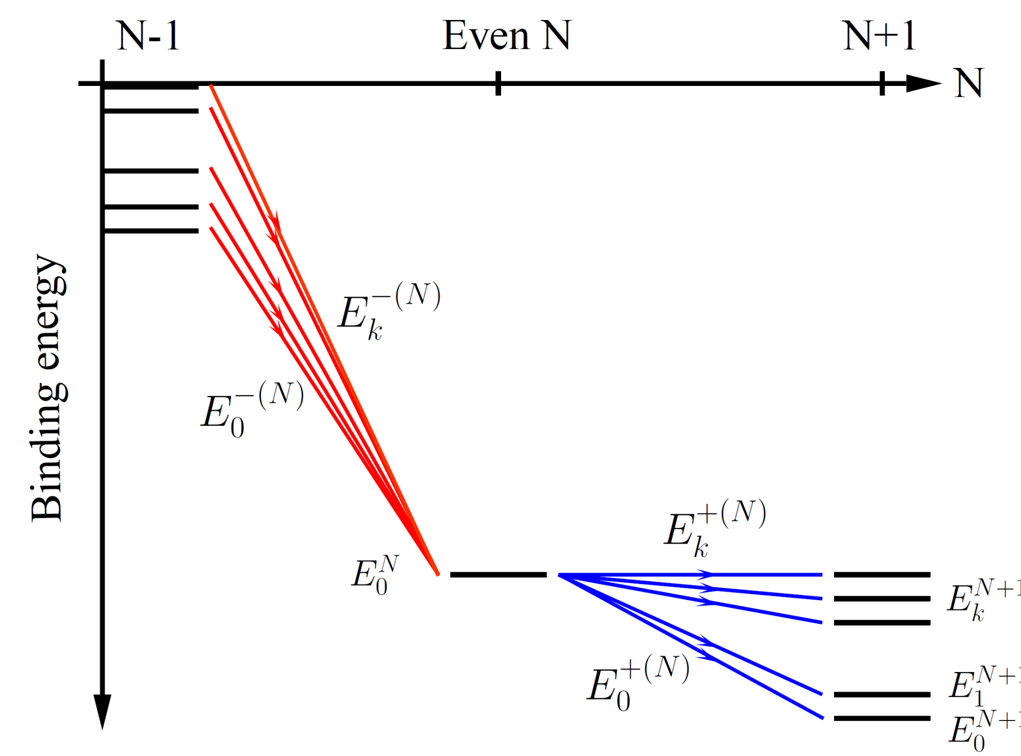
Lehmann representation

where

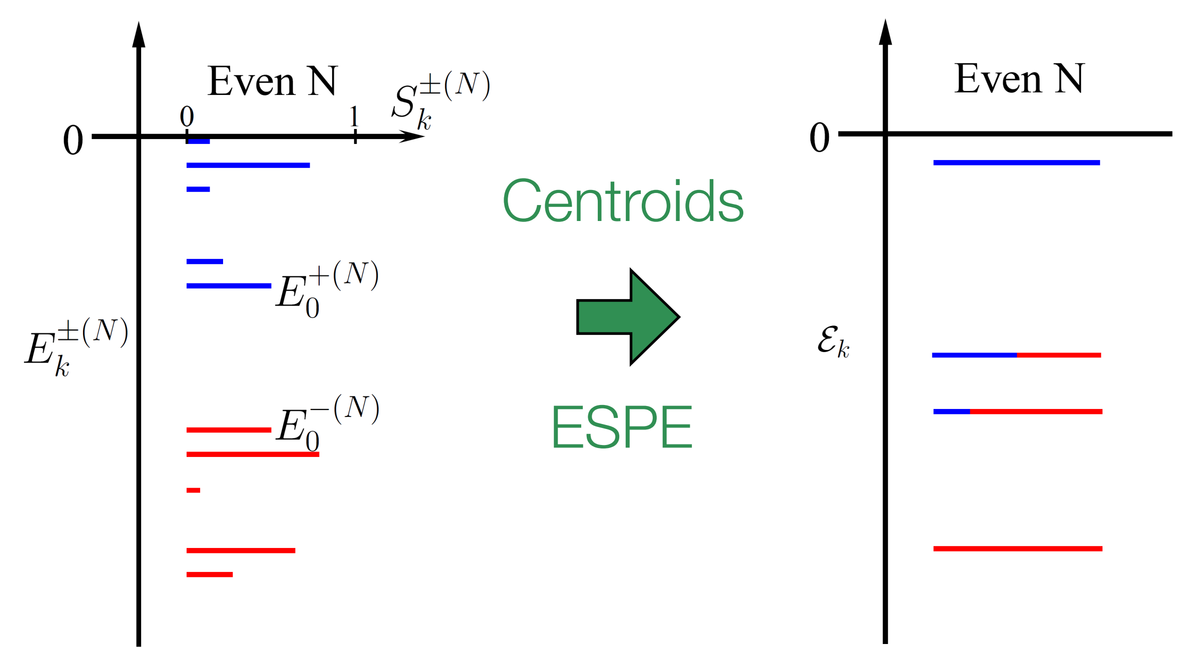
$$\begin{cases} \mathcal{X}_a^{k\,(N)} \equiv \langle \psi_k^{N+1} | a_a^\dagger | \psi_0^N \rangle \\ \mathcal{Y}_a^{k\,(N)} \equiv \langle \psi_k^{N-1} | a_a | \psi_0^N \rangle \end{cases}$$

and

$$\begin{cases} E_k^{+\,(N)} \equiv E_k^{N+1} - E_0^N \\ E_k^{-\,(N)} \equiv E_0^N - E_k^{N-1} \end{cases}$$



Separation energies



2) Spectroscopic factors

$$S_k^{N+1} \equiv \sum_a \left| \langle \psi_k^{N+1} | a_a^\dagger | \psi_0^N \rangle \right|^2 = \sum_a \left| \mathcal{X}_a^{k\,(N)} \right|^2$$

$$S_k^{N-1} \equiv \sum_a \left| \langle \psi_k^{N-1} | a_a | \psi_0^N \rangle \right|^2 = \sum_a \left| \mathcal{Y}_a^{k\,(N)} \right|^2$$

Observables (2)

3) One-body observables with $\hat{O} = \sum_{ab} O_{ab}\, a_a^\dagger\, a_b$

$$\langle \hat{O} \rangle = \sum_{ab} \int \frac{d\omega}{2\pi}\, O_{ab}\, G_{ab}(\omega)$$

➠ e.g. kinetic energy $\langle \hat{T} \rangle = \sum_{ab} \int \frac{d\omega}{2\pi}\, t_{ab}\, G_{ab}(\omega)$

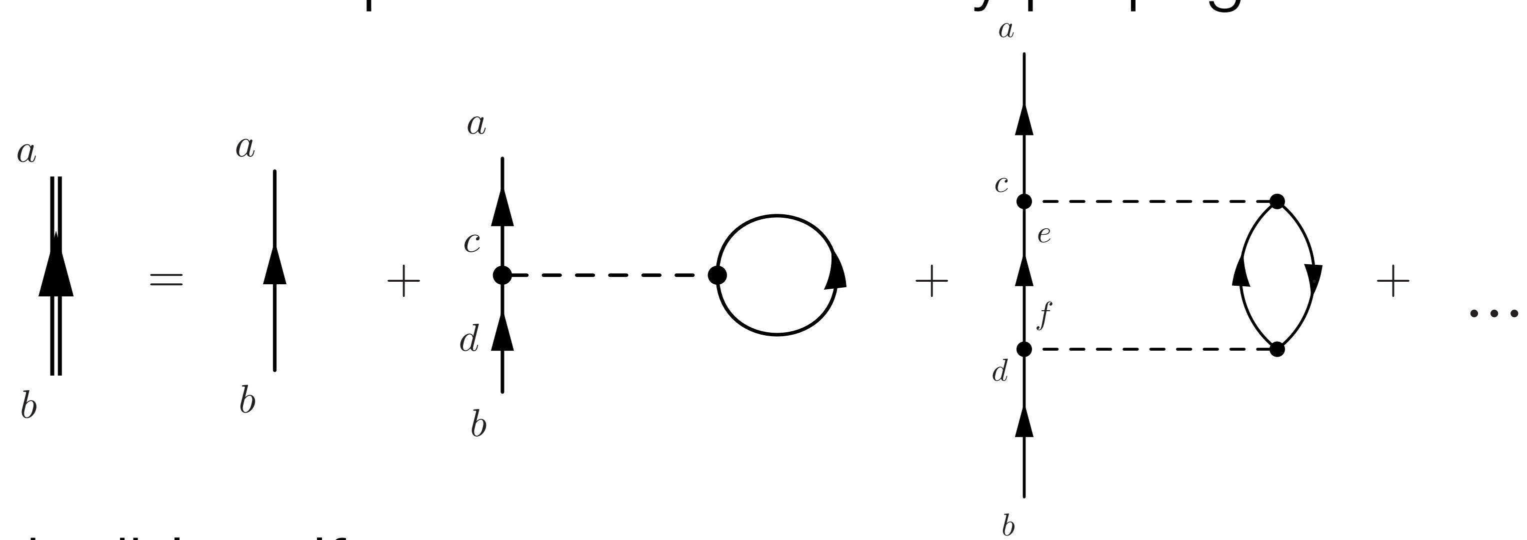
4) Koltun sum rule

$$\langle \hat{H} \rangle = E_0 = \sum_{ab} \int \frac{d\omega}{2\pi}\, [t_{ab} + \omega\, \delta_{ab}]\; G_{ab}(\omega)$$

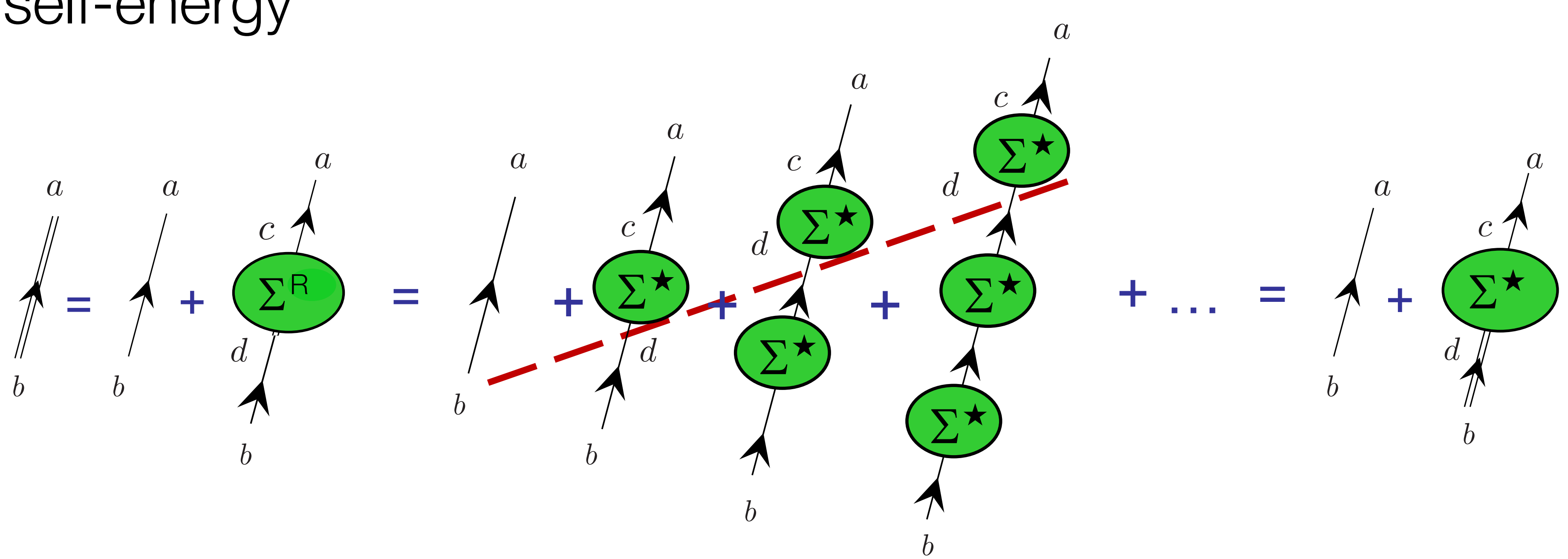
➠ two-body observable computed from the one-body propagator

Dyson equation & self-energy

✵ Perturbative expansion of one-body propagator



✵ Irreducible self-energy



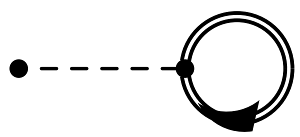
✵ Dyson equation

$$G_{ab}(\omega) = G^{(0)}_{ab}(\omega) + \sum_{cd} G^{(0)}_{ac}(\omega)\, \Sigma^{\star}_{cd}(\omega)\, G_{db}(\omega)$$

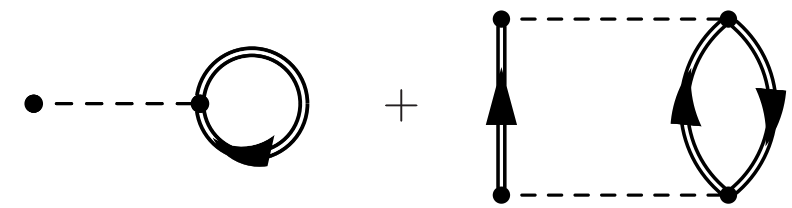
Solving Dyson equation

✳ Different approximations to the self-energy (**self-consistent** approaches)

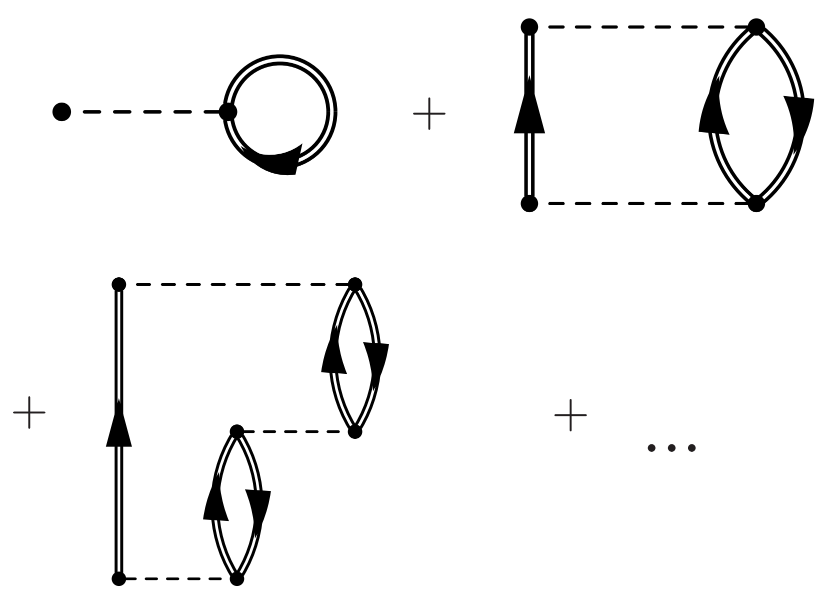
➡ Hartree-Fock



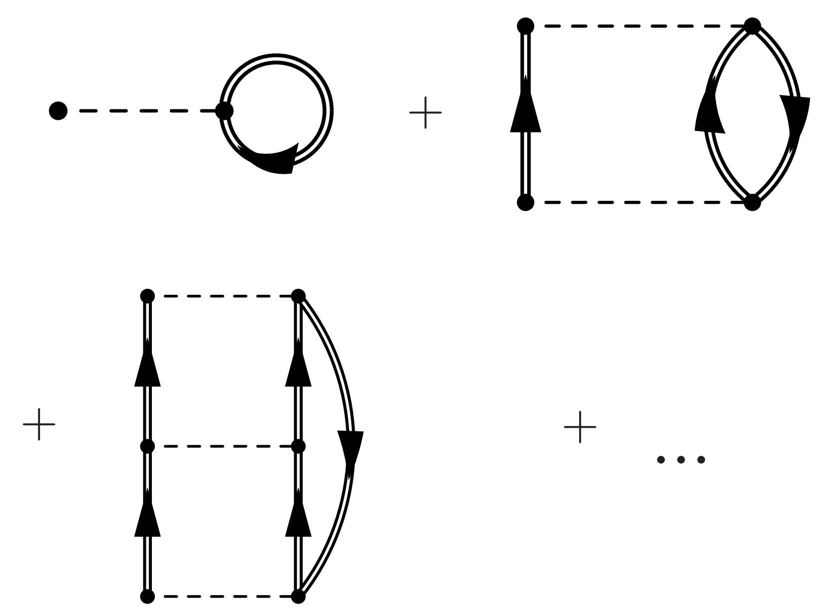
➡ Second order



➡ RPA



➡ Ladder (or T-matrix)



➡ (Nearly) degenerate systems: breakdown of truncated expansions

⤷ e.g. *pairing correlations:* approximation schemes face Cooper instability

➡ Non-perturbative treatment of such an instability needed

Gorkov ansatz

- Ansatz $$... \approx E_0^{N+2} - E_0^N \approx E_0^N - E_0^{N-2} \approx ... \approx 2\mu$$

- Auxiliary many-body state $|\Psi_0\rangle \equiv \sum_N^{\text{even}} c_N \, |\psi_0^N\rangle$

  → Mixes various particle numbers

  → Introduce a "grand-canonical" potential $\Omega = H - \mu N$

➡ $|\Psi_0\rangle$ minimizes $\Omega_0 = \langle\Psi_0|\Omega|\Psi_0\rangle$

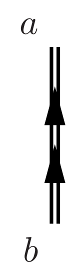
under the constraint $N = \langle\Psi_0|N|\Psi_0\rangle$

➡ $$\Omega_0 = \sum_{N'} |c_{N'}|^2 \Omega_0^{N'} \approx E_0^N - \mu N$$

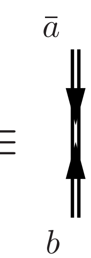
Gorkov Green's functions and equations

✺ Set of 4 Green's functions

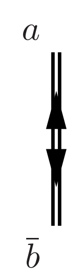
$$i\, G^{11}_{ab}(t,t') \equiv \langle \Psi_0 | T \left\{ a_a(t) a^{\dagger}_b(t') \right\} | \Psi_0 \rangle \quad \equiv$$



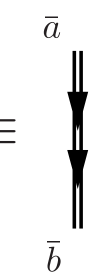
$$i\, G^{21}_{ab}(t,t') \equiv \langle \Psi_0 | T \left\{ \bar{a}^{\dagger}_a(t) a^{\dagger}_b(t') \right\} | \Psi_0 \rangle \quad \equiv$$



$$i\, G^{12}_{ab}(t,t') \equiv \langle \Psi_0 | T \left\{ a_a(t) \bar{a}_b(t') \right\} | \Psi_0 \rangle \quad \equiv$$



$$i\, G^{22}_{ab}(t,t') \equiv \langle \Psi_0 | T \left\{ \bar{a}^{\dagger}_a(t) \bar{a}_b(t') \right\} | \Psi_0 \rangle \quad \equiv$$



[Gorkov 1958]

$$\mathbf{G}_{ab}(\omega) = \mathbf{G}^{(0)}_{ab}(\omega) + \sum_{cd} \mathbf{G}^{(0)}_{ac}(\omega)\, \mathbf{\Sigma}^{\star}_{cd}(\omega)\, \mathbf{G}_{db}(\omega)$$

Gorkov equations

$$\mathbf{\Sigma}^{\star}_{ab}(\omega) \equiv \begin{pmatrix} \Sigma^{\star\,11}_{ab}(\omega) & \Sigma^{\star\,12}_{ab}(\omega) \\ \Sigma^{\star\,21}_{ab}(\omega) & \Sigma^{\star\,22}_{ab}(\omega) \end{pmatrix}$$

$$\mathbf{\Sigma}^{\star}_{ab}(\omega) \equiv \mathbf{\Sigma}_{ab}(\omega) - \mathbf{U}_{ab}$$

Lehmann representation

✳ Set eigenstates of Ω $\qquad \Omega|\Psi_k\rangle = \Omega_k|\Psi_k\rangle$

→ define

$$\mathcal{U}_a^{k*} \equiv \langle\Psi_k|\bar{a}_a^\dagger|\Psi_0\rangle \qquad \bar{\mathcal{U}}_a^{k*} \equiv \langle\Psi_k|a_a^\dagger|\Psi_0\rangle$$
$$\mathcal{V}_a^{k*} \equiv \langle\Psi_k|a_a|\Psi_0\rangle \qquad \bar{\mathcal{V}}_a^{k*} \equiv \langle\Psi_k|\bar{a}_a|\Psi_0\rangle$$

✳ Lehmann representation

$$G_{ab}^{11}(\omega) = \sum_k \left\{ \frac{\bar{\mathcal{U}}_a^k \bar{\mathcal{U}}_b^{k*}}{\omega - \omega_k + i\eta} + \frac{\mathcal{V}_a^{k*} \mathcal{V}_b^k}{\omega + \omega_k - i\eta} \right\} \qquad G_{ab}^{21}(\omega) = \sum_k \left\{ \frac{\bar{\mathcal{V}}_a^k \bar{\mathcal{U}}_b^{k*}}{\omega - \omega_k + i\eta} + \frac{\mathcal{U}_a^{k*} \mathcal{V}_b^k}{\omega + \omega_k - i\eta} \right\}$$

$$G_{ab}^{12}(\omega) = \sum_k \left\{ \frac{\bar{\mathcal{U}}_a^k \bar{\mathcal{V}}_b^{k*}}{\omega - \omega_k + i\eta} + \frac{\mathcal{V}_a^{k*} \mathcal{U}_b^k}{\omega + \omega_k - i\eta} \right\} \qquad G_{ab}^{22}(\omega) = \sum_k \left\{ \frac{\bar{\mathcal{V}}_a^k \bar{\mathcal{V}}_b^{k*}}{\omega - \omega_k + i\eta} + \frac{\mathcal{U}_a^{k*} \mathcal{U}_b^k}{\omega + \omega_k - i\eta} \right\}$$

where $\omega_k \equiv \Omega_k - \Omega_0$ and

$$E_k^+ \equiv +\omega_k + \mu$$
$$E_k^- \equiv -\omega_k + \mu$$

✳ Generalized spectroscopic factors

$$\mathcal{S}_k^+ \equiv \sum_a \left|\langle\psi_k|a_a^\dagger|\psi_0\rangle\right|^2 = \sum_a \left|\mathcal{U}_a^k\right|^2$$

$$\mathcal{S}_k^- \equiv \sum_a \left|\langle\psi_k|a_a|\psi_0\rangle\right|^2 = \sum_a \left|\mathcal{V}_a^k\right|^2$$

Gorkov equations (2)

✺ Gorkov equations $\xrightarrow{\text{Lehmann}}$ energy-dependent eigenvalue problem

$$\sum_b \begin{pmatrix} t_{ab} - \mu_{ab} + \Sigma^{11}_{ab}(\omega) & \Sigma^{12}_{ab}(\omega) \\ \Sigma^{21}_{ab}(\omega) & -t_{ab} + \mu_{ab} + \Sigma^{22}_{ab}(\omega) \end{pmatrix}\Bigg|_{\omega_k} \begin{pmatrix} \mathcal{U}^k_b \\ \mathcal{V}^k_b \end{pmatrix} = \omega_k \begin{pmatrix} \mathcal{U}^k_a \\ \mathcal{V}^k_a \end{pmatrix}$$

- $\Sigma^{g_1 g_2}(\omega)$ play the role of energy-dependent potentials
- Iterative problem: the number of poles ω_k grows with iterations
- Constraint: correct number of particles in average $N = \sum_{a,k} \left|\mathcal{V}^k_a\right|^2$
- Normalization condition $\sum_a \left(\mathcal{V}^k_a \; \mathcal{U}^k_a \right) \begin{pmatrix} \mathcal{V}^{k*}_a \\ \mathcal{U}^{k*}_a \end{pmatrix} = 1 + \sum_{ab} \left(\mathcal{V}^k_a \; \mathcal{U}^k_a \right) \frac{\partial \mathbf{\Sigma}_{ab}(\omega)}{\partial \omega}\Big|_{-\omega_k} \begin{pmatrix} \mathcal{V}^{k*}_a \\ \mathcal{U}^{k*}_a \end{pmatrix}$

Gorkov equations (2)

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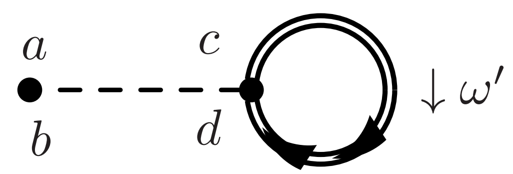
- $\Sigma^{g_1 g_2}(\omega)$ play the role of energy-dependent potentials
- Iterative problem: the number of poles ω_k grows with iterations
- Constraint: correct number of particles in average $N = \sum_{a,k} \left|\mathcal{V}^k_a\right|^2$
- Normalization condition $\sum_a \left(\mathcal{V}^k_a \; \mathcal{U}^k_a \right) \begin{pmatrix} \mathcal{V}^{k*}_a \\ \mathcal{U}^{k*}_a \end{pmatrix} = 1 + \sum_{ab} \left(\mathcal{V}^k_a \; \mathcal{U}^k_a \right) \frac{\partial \Sigma_{ab}(\omega)}{\partial \omega}\Big|_{-\omega_k} \begin{pmatrix} \mathcal{V}^{k*}_a \\ \mathcal{U}^{k*}_a \end{pmatrix}$

Objective
- Short term ➟ **Self-consistent** second order
- Longer term ➟ **Self-consistent** Faddeev-QRPA

1st order diagrams and HFB limit

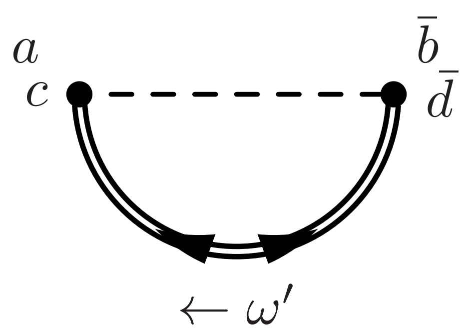
- Energy-independent self-energy

$\Sigma^{11\,(1)}_{ab} =$



$$\Sigma^{11\,(1)}_{ab} = \sum_{cd,k} \bar{V}_{acbd}\, \mathcal{V}^{k*}_{d}\, \mathcal{V}^{k}_{c} \equiv \Lambda_{ab} = -\Sigma^{22\,(1)}_{ab}$$

$\Sigma^{12\,(1)}_{ab} =$



$$\Sigma^{12\,(1)}_{ab} = \frac{1}{2} \sum_{cd,k} \bar{V}_{a\bar{b}c\bar{d}}\, \mathcal{V}^{k*}_{c}\, \mathcal{U}^{k}_{d} \equiv \tilde{h}_{ab} = \left[\Sigma^{21\,(1)}_{ba}\right]^*$$

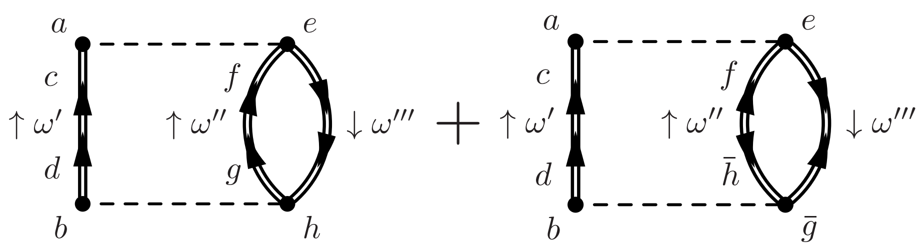
- HFB problem is recovered ⟶ energy-independent eigenvalue problem

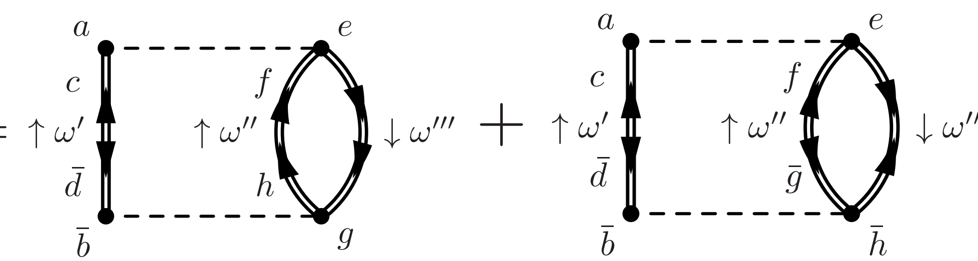
$$\sum_b \begin{pmatrix} t_{ab} + \Lambda_{ab} - \mu\,\delta_{ab} & \tilde{h}_{ab} \\ \tilde{h}^{\dagger}_{ab} & -t_{ab} - \Lambda_{ab} + \mu\,\delta_{ab} \end{pmatrix} \begin{pmatrix} U^k_b \\ V^k_b \end{pmatrix} = \omega_k \begin{pmatrix} U^k_a \\ V^k_a \end{pmatrix}$$

with the normalization condition $\sum_a \left|U^k_a\right|^2 + \sum_a \left|V^k_a\right|^2 = 1$

2nd order diagrams

- Energy-dependent self-energy

$\Sigma^{11\,(2)}_{ab}(\omega) =$ 

$\Sigma^{12\,(2)}_{ab}(\omega) =$ 

$$\Sigma^{11\,(2)}_{ab}(\omega) = \sum_{k_1 k_2 k_3} \left\{ \frac{\mathcal{C}_a^{k_1 k_2 k_3}\, \mathcal{C}_b^{k_1 k_2 k_3\,\dagger}}{\omega - E_{k_1 k_2 k_3} + i\eta} + \frac{\mathcal{D}_a^{k_1 k_2 k_3\,\dagger}\, \mathcal{D}_b^{k_1 k_2 k_3}}{\omega + E_{k_1 k_2 k_3} + i\eta} \right\}$$

$$\Sigma^{12\,(2)}_{ab}(\omega) = -\sum_{k_1 k_2 k_3} \left\{ \frac{\mathcal{C}_a^{k_1 k_2 k_3}\, \mathcal{D}_b^{k_1 k_2 k_3}}{\omega - E_{k_1 k_2 k_3} + i\eta} + \frac{\mathcal{D}_a^{k_1 k_2 k_3\,\dagger}\, \mathcal{C}_b^{k_1 k_2 k_3\,\dagger}}{\omega + E_{k_1 k_2 k_3} + i\eta} \right\}$$

$$\mathcal{C}_a^{k_1 k_2 k_3} \equiv \frac{1}{\sqrt{6}} \sum_{\{1,2,3\}} \sum_{ijk} \bar{V}_{akij}\, \bar{\mathcal{U}}_i^{k_1} \bar{\mathcal{U}}_j^{k_2} \mathcal{V}_k^{k_3}$$

$$\mathcal{D}_a^{k_1 k_2 k_3} \equiv \frac{1}{\sqrt{6}} \sum_{\{1,2,3\}} \sum_{ijk} \bar{V}_{akij}\, \mathcal{V}_i^{k_1} \mathcal{V}_j^{k_2} \bar{\mathcal{U}}_k^{k_3}$$

- Recast known energy dependence into new quantities

$$(\omega_k - E_{k_1 k_2 k_3})\, \mathcal{W}_k^{k_1 k_2 k_3} \equiv \sum_a \left[\mathcal{C}_a^{k_1 k_2 k_3\,\dagger}\, \mathcal{U}_a^k - \mathcal{D}_a^{k_1 k_2 k_3}\, \mathcal{V}_a^k \right]$$

$$(\omega_k + E_{k_1 k_2 k_3})\, \mathcal{Z}_k^{k_1 k_2 k_3} \equiv \sum_a \left[-\mathcal{D}_a^{k_1 k_2 k_3}\, \mathcal{U}_a^k + \mathcal{C}_a^{k_1 k_2 k_3\,\dagger}\, \mathcal{V}_a^k \right]$$

Gorkov equations (3)

$$\sum_b \begin{pmatrix} t_{ab} - \mu_{ab} + \Sigma^{11}_{ab}(\omega) & \Sigma^{12}_{ab}(\omega) \\ \Sigma^{21}_{ab}(\omega) & -t_{ab} + \mu_{ab} + \Sigma^{22}_{ab}(\omega) \end{pmatrix} \Bigg|_{\omega_k} \begin{pmatrix} \mathcal{U}^k_b \\ \mathcal{V}^k_b \end{pmatrix} = \omega_k \begin{pmatrix} \mathcal{U}^k_a \\ \mathcal{V}^k_a \end{pmatrix}$$

Expand in terms ⬇ of *W* and *Z*

$$\begin{pmatrix} T - \mu + \Lambda & \tilde{h} & \mathcal{C} & -\mathcal{D}^\dagger \\ \tilde{h}^\dagger & -T + \mu - \Lambda & -\mathcal{D}^\dagger & \mathcal{C} \\ \mathcal{C}^\dagger & -\mathcal{D} & E & 0 \\ -\mathcal{D} & \mathcal{C}^\dagger & 0 & -E \end{pmatrix} \begin{pmatrix} \mathcal{U}^k \\ \mathcal{V}^k \\ \mathcal{W}_k \\ \mathcal{Z}_k \end{pmatrix} = \omega_k \begin{pmatrix} \mathcal{U}^k \\ \mathcal{V}^k \\ \mathcal{W}_k \\ \mathcal{Z}_k \end{pmatrix}$$

Energy *independent* eigenvalue problem

with the normalization condition $$\sum_a \left[\left|\mathcal{U}^k_a\right|^2 + \left|\mathcal{V}^k_a\right|^2 \right] + \sum_{k_1 k_2 k_3} \left[\left|\mathcal{W}^{k_1 k_2 k_3}_k\right|^2 + \left|\mathcal{Z}^{k_1 k_2 k_3}_k\right|^2 \right] = 1$$

Results

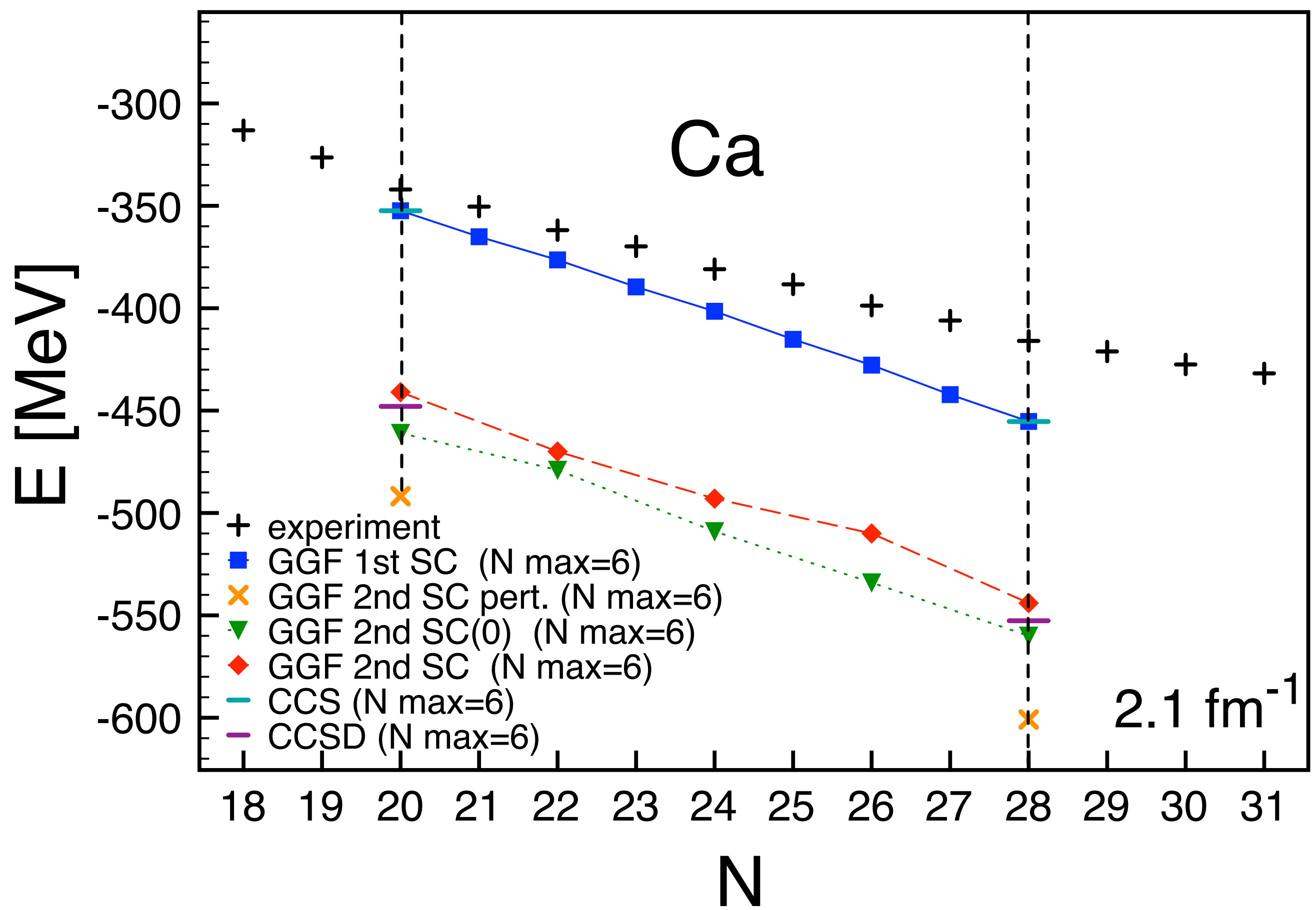
Results

- Calculations of $^{40-48}$Ca isotopes
  - ➠ Spherical harmonic oscillator basis
  - ➠ V_{low-k} from Ch-EFT potential with cutoff Λ = 2.1 & 2.5 fm^{-1}
  - ➠ NN interaction only
- CEA-CCRT massively-parallel high-performance cluster
  - ➠ ~ 58 000 cores, ~ 300 Tflops total
  - ➠ Parallelized code

→ Essential for self-consistent second-order calculations

Binding energies

✷ Systematic along isotopic/isotonic chains become available



➟ Correlation energy “consistent” with CCSD (quantitative analysis in progress)

➟ Overbinding with A: traces need for (at least) NNN forces

➟ Second-order SCGF qualitatively different from second-order MBPT

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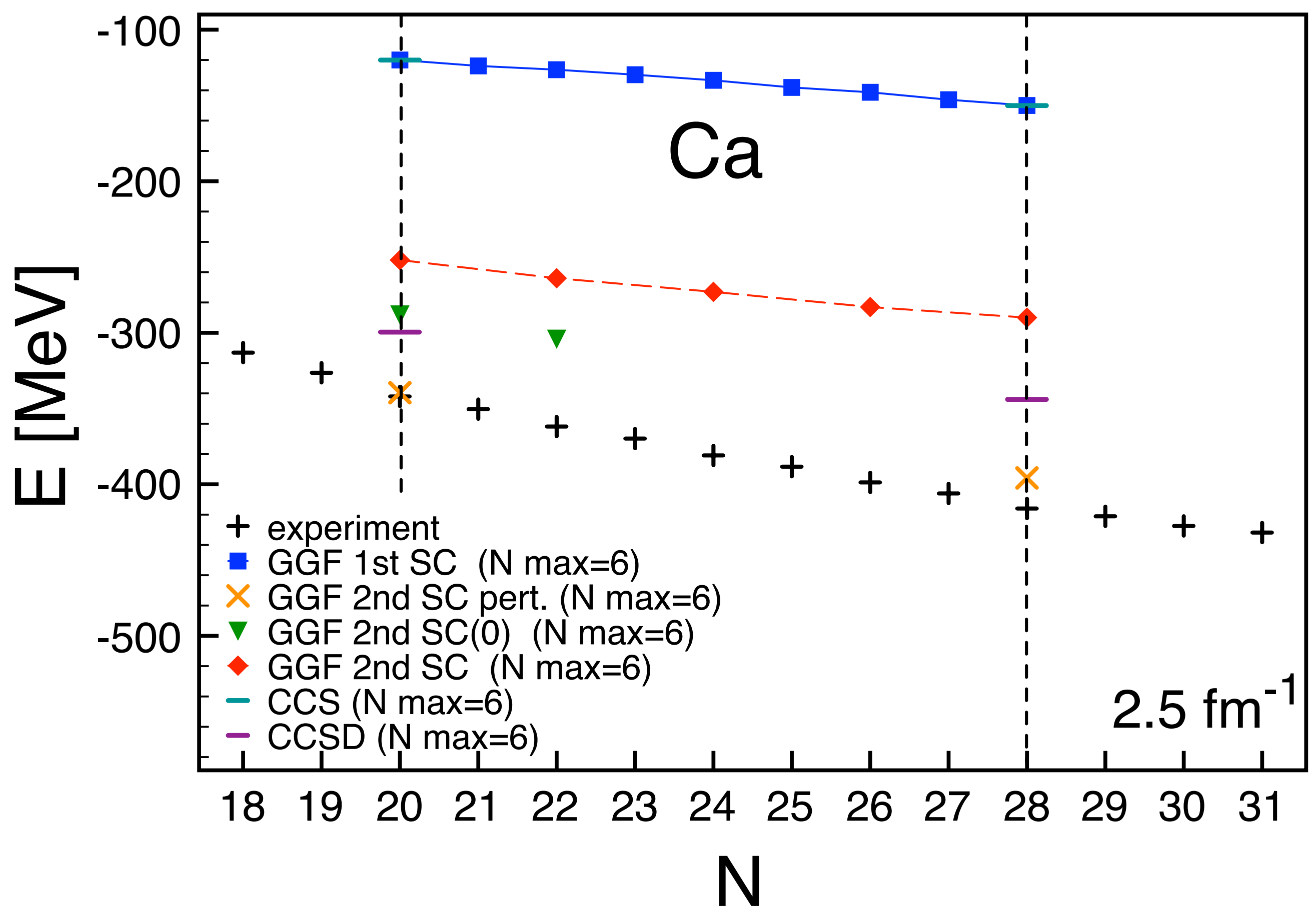

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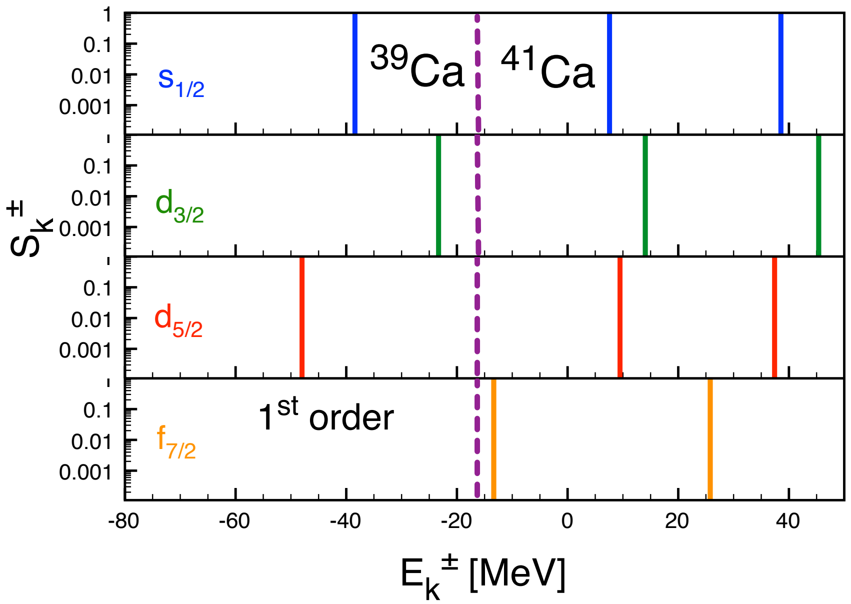
Spectral function

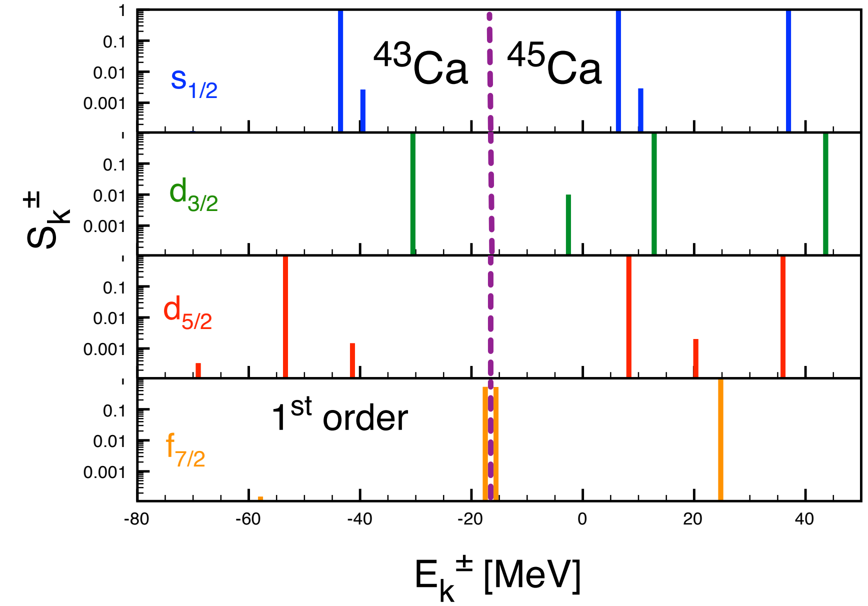
Dyson 1^{st} order (HF)



Fragmentation

Static pairing

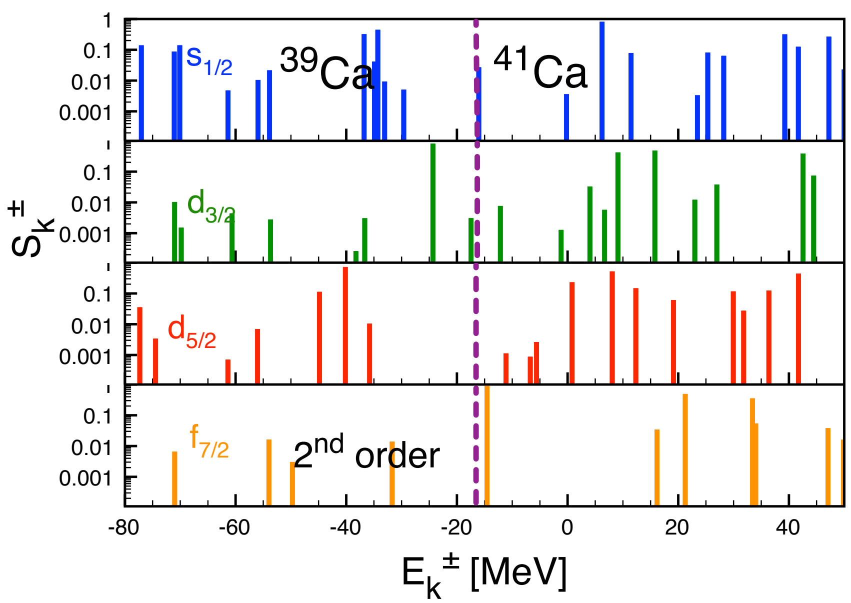
Gorkov 1^{st} order (HFB)

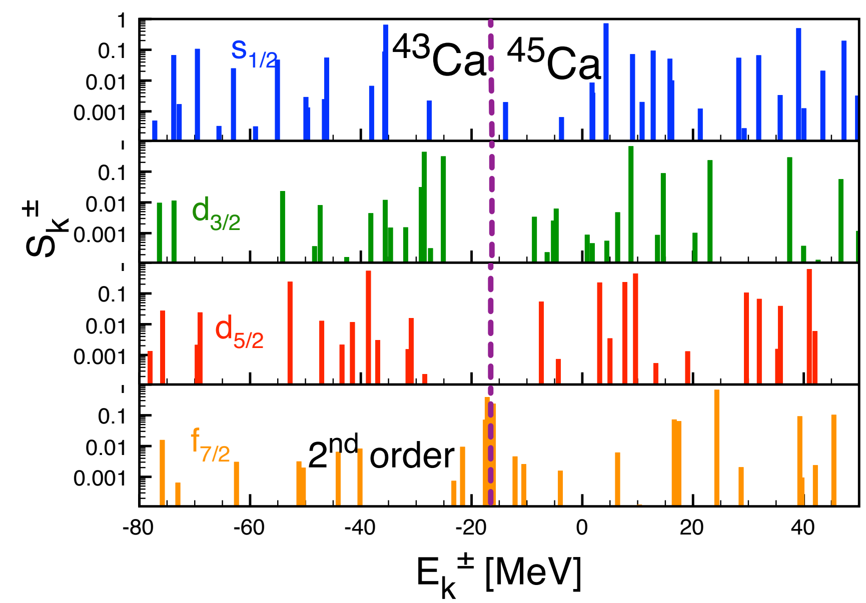


Dyson 2^{nd} order

Dynamical fluctuations

Gorkov 2^{nd} order



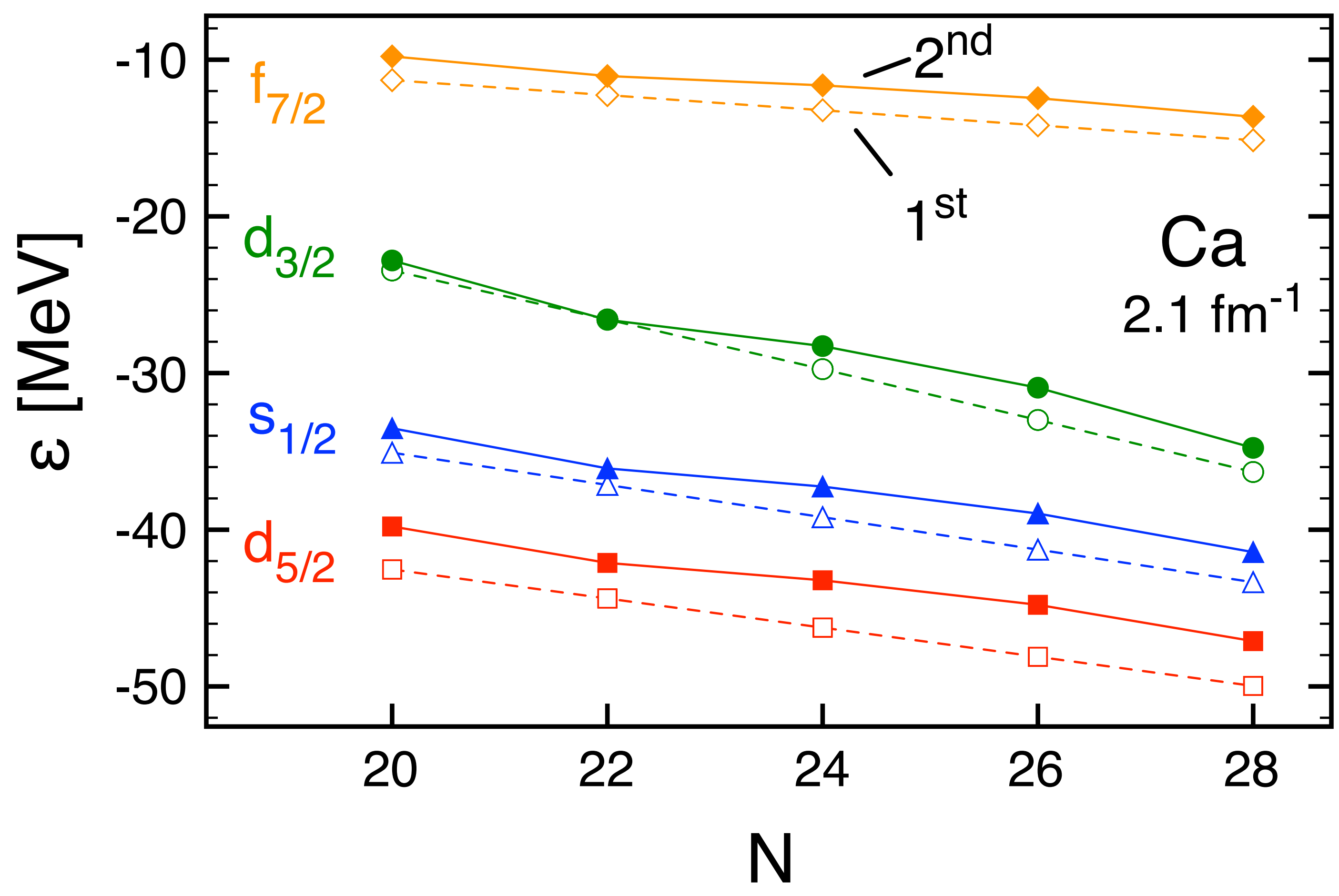


Shell structure evolution

✺ ESPE collect fragmentation of “single-particle” strengths from both N±1

$$\epsilon_a = \sum_k \mathcal{S}_k^{+a} E_k^+ + \sum_k \mathcal{S}_k^{-a} E_k^- = t_{aa} + \sum_{cd} \bar{V}_{acad}\, \rho_{dc}$$

[Baranger 1970]



➟ Correlations shift ESPE up in a non-uniform manner

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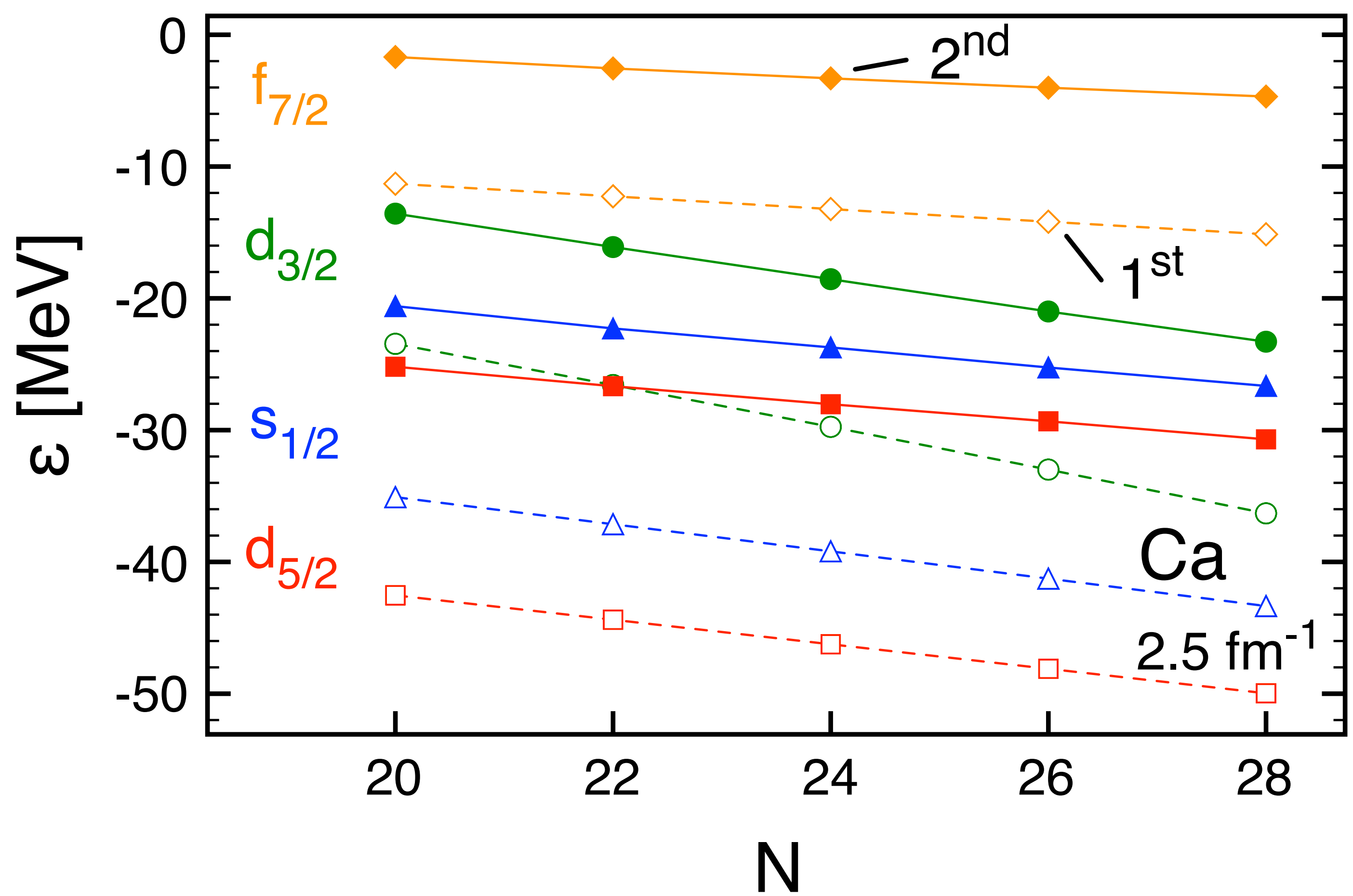

➟ Correlations shift ESPE up in a non-uniform manner

Next

- ✺ Systematic comparison with CC in doubly magic ±1 and ±2 nuclei

  [in collaboration with G. Hagen]

- ✺ Implementation of NNN forces

- ✺ Formulation of particle-number restored Gorkov theory